PennDOT Agreement E03134

EXAMPLE CONFIGURATIONS OF THE PA FLEXBEAM BRIDGE SYSTEM

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ABSTRACT

Revision January 2020: The example configuration calculations for flexure and shear strength limit states have been updated to include a concrete overlay with a unit weight of 140 lb/ft^3 and a uniform 2 in. thickness. The allowance for a 2 in. overlay thickness accommodates potential variations in the overlay thicknesss to provide cross-slope to the bridge deck, since a typical overlay is 1-1/4 in. thick. The example configurations calculations for flexure and shear have also been revised to include an additional 1 in. of deck thickness on the exterior overhang to meet the Pennsylvania Design Manual, Part 4 requirement for rebar cover on exterior overhangs. This weight is assumed to be uniformly distributed over the tributary width of the steel tee, the additional strength provided by the additional thickness is not considered. The calculations for dead load deflections have been updated to include a typical 1-1/4 in. thick uniform overlay and the future wearing surface dead load has been removed from the dead load deflection calculations.

Revision September 2019: The original calculations in the appendix and the transverse rebar spacing in Table 1 were calculated using the distribution factor for two or more loaded lanes. Subsequent research on distribution factors for the FlexBeam system demonstrated that the equation for a single loaded lane produced higher demands for all FlexBeam configurations. The appendix contains revised calculations using the PennDOT Design Manual, Part 4 (DM-4) single lane distribution factor for shear. Revised shear envelopes using the correct distribution factor were generated and the shear dowel spacing in Table 1 was adjusted to account for the increase in shear flow. In addition, an additional non-standard sample configuration is added as appendix D.

An experimental study was conducted on an economical steel/concrete composite highway bridge system for the Pennsylvania Department of Transportation. The system, referred to as the FlexBeam system, was constructed from a series of T-shaped steel sections (e.g., standard split wide-flange shapes, known as WT sections) precast into a doubly reinforced concrete deck slab section. The study presented in this report is based on a prototype design developed in Phase 1 of the research effort. It is the intention that each composite steel-T-concrete-slab module will be precast independently, delivered to the bridge site, erected on simply supported boundary conditions, and the concrete slabs of the adjacent steel-T-concrete-slab modules will be joined with a 6 in. wide high strength concrete longitudinal closure joint.

This report provides a summary of sample calculations for the FlexBeam system. Sample calculations for 30, 40, 50, 60, and 70 ft simple span configurations with steel T beams spaced at 36 and 40 in. The supporting calculations are included in the attached sheets. The main calculations were conducted using Mathcad version 15. The shear dowel sample calculations were conducted in Matlab version r2017b and Excel 2016.

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1. BACKGROUND

This report summarizes example configurations for the PA FlexBeam system. Included in this report are examples of 30, 40, 50, 60, and 70 ft simple span configurations with steel T beams spaced at 36 and 40 in. The example dimensions are included below in Table 1. The supporting calculations are included in the attached sheets. The main calculations were conducted using Mathcad version 15. The shear dowel sample calculations were conducted in Matlab version r2017b and Excel 2016.

1.1. DISCLAIMER

1.2. EXAMPLE CONFIGURATION ASSUMPTIONS

The following assumptions are used in the example configurations of the PA FlexBeam system.

- Design Specifications
  - AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  - PennDOT Design Manual, Part 4 (DM-4) June 30, 2015

- Material Properties
  - Structural steel: ASTM A709 Grade 50
  - Concrete reinforcement: ASTM A615 or A706 Grade 60, fy = 60 ksi
  - Concrete deck f'c = 4 ksi (Class AAAP Concrete)
  - Concrete barriers, f'c = 3.5 ksi (Class AA Concrete)

- Loads and Distribution
  - Normal weight concrete = 150 lb/ft^3
  - Future wearing surface = 30 lb/ft^2 (evenly distributed to all beams)
  - 45 in. tall F-shape barrier, weight = 700 lb/ft (distributed to exterior two beams)
  - Live load distribution factor assumed to be 0.3 for flexure limit states and using DM-4 equation for shear limit states

- Bridge Dimensions
  - Minimum clear widths including 24, 28, 32, 36, and 40 ft
  - Spans including 30, 40, 50, 60, 70 ft
  - Configurations are double T modules (i.e., precast deck with two steel T beams embedded) with a 6 in. UHPC closure joint.

- The beams are evaluated for the following cases:
  1. Strength I (flexure and shear design due to design truck or design tandem)
  2. Strength II (flexure of shear design due to Permit Load)
  3. Service II (stress check due to design truck or design tandem)
  4. Strength V (combined flexural demand from wind and vehicle loading)
  5. Optional L/800 Live Load Deflection Check (in accordance with AASHTO 3.6.1.3.2)

- The supporting research for the PA FlexBeam example configurations are summarized in ATLSS Report 15-01 and ATLSS Report 16-01, and ATLSS Report 18-04.

  Cercone, C., Naito, C., Sause, R., "PA Flex Beam Shear Strength Evaluation and Construction Methods," ATLSS Report No. 16-01, ATLSS Center, Lehigh University, February 2016, 84 pages.

Aghl, P. P., Naito, C., Sause, R., "PA Flex Beam Preliminary Analysis," ATLSS Report No. 15-01, ATLSS Center, Lehigh University, January 2015, 26 pages.

Naito, C., Hendricks, R., Sause, R., "Full-Scale Evaluation of the PA FlexBeam Bridge System," ATLSS Report No. 18-04, ATLSS Center, Lehigh University, December 2018.

1.3. Example Configurations

The dimensions of the PA FlexBeam example configurations provided below include the structural steel dimensions and shear dowel spacing for the simple spans and bridge widths outlined above. The sections were developed for 36 and 40 in. steel T beam spacing, however, the configurations were also checked for a smaller beam spacing. The 40 in. T beam spacing configurations are also applicable to a 38 in. beam spacing. The 36 in. T beam spacing configurations are also applicable to a 34 in. spacing with the caveat that the optional Service I deflection criteria of L/800 is exceeded by approximately 1% for a 34 in. spacing. The example configurations are summarized in Table 1. The nomenclature used for the various section dimensions are illustrated in Figure 1.

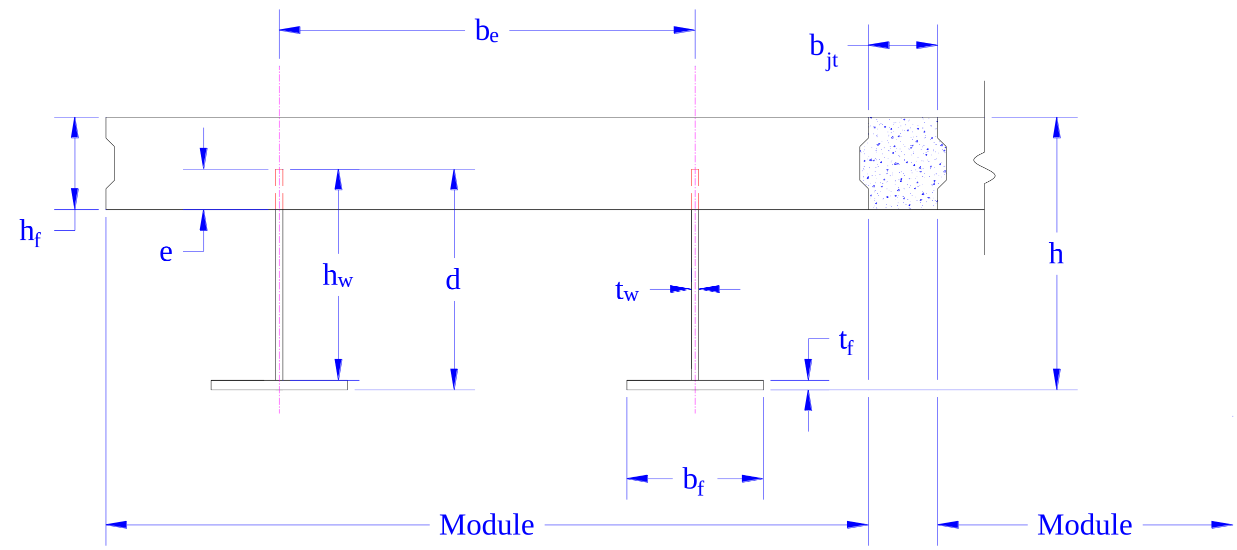


Figure 1: Module nomenclature

Table 1: PA FlexBeam Example Configurations

Span Length, L [ft]	Design Roadway [ft]	Superstructure Height, h [in.]	Number of Modules	Beam Spacing, b_e [in.]	Bottom Flange		Web		Stiffener Height [in.]	Revised Shear Dowel Spacing on Half Span from Support to Midspan		
					Thickness, t_f [in.]	Width, b_f [in.]	Depth, h_w [in.]	Thickness, t_w [in.]		Number of Spaces @ 4 in.	Number of Spaces @ 8 in.	Number of Spaces @ 12 in.
30	24	22	5	36	0.500	12.000	17.500	0.375	13.400	25	7	2
	28	22	5	40	0.500	12.000	17.500	0.375	13.400	26	8	1
	32	22	6	36	0.500	12.000	17.500	0.375	13.400	25	7	2
	36	22	6	40	0.500	12.000	17.500	0.375	13.400	26	8	1
	40	22	7	40	0.500	12.000	17.500	0.375	13.400	26	8	1
40	24	25	5	36	0.500	12.000	20.500	0.375	16.400	30	12	2
	28	25	5	40	0.750	12.000	20.250	0.375	16.150	31	10	3
	32	25	6	36	0.500	12.000	20.500	0.375	16.400	30	12	2
	36	25	6	40	0.750	12.000	20.250	0.375	16.150	31	10	3
	40	25	7	40	0.750	12.000	20.250	0.375	16.150	31	10	3
50	24	29	5	36	0.625	12.000	24.375	0.500	20.275	30	12	7
	28	29	5	40	1.000	12.000	24.000	0.500	19.900	31	13	6
	32	29	6	36	0.625	12.000	24.375	0.500	20.275	30	12	7
	36	29	6	40	1.000	12.000	24.000	0.500	19.900	31	13	6
	40	29	7	40	1.000	12.000	24.000	0.500	19.900	31	13	6
60	24	35	5	36	0.750	12.000	30.250	0.625	26.150	30	15	10
	28	35	5	40	0.875	12.000	30.125	0.625	26.025	33	18	7
	32	35	6	36	0.750	12.000	30.250	0.625	26.150	30	15	10
	36	35	6	40	0.875	12.000	30.125	0.625	26.025	33	18	7
	40	35	7	40	0.875	12.000	30.125	0.625	26.025	33	18	7
70	24	38	5	36	0.750	12.000	33.250	0.625	29.150	38	17	11
	28	38	5	40	1.125	12.000	32.875	0.625	28.775	39	15	12
	32	38	6	36	0.750	12.000	33.250	0.625	29.150	38	17	11
	36	38	6	40	1.125	12.000	32.875	0.625	28.775	39	15	12
	40	38	7	40	1.125	12.000	32.875	0.625	28.775	39	15	12

2. Appendix A – Example Configuration Calculations

The example configuration calculations are included in this section. The calculations were conducted with Mathcad version 15. Calculations include checks on Service II, Strength I, and Strength II limit states as well as the optional check on live load deflections meeting the L/800 criteria for non-pedestrian bridges.

Flexbeam Example Configuration Calculations

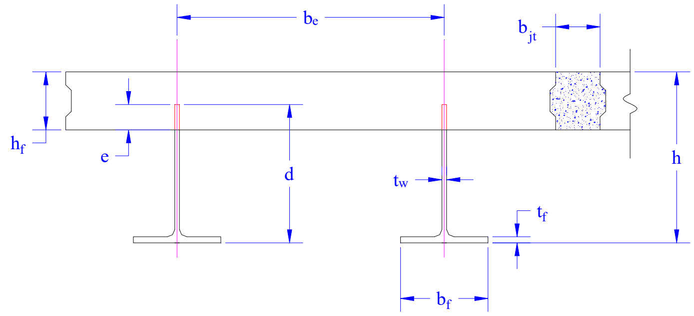
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The Fflexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 30ft$ Span length

$b_e := 34in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 22in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 215.136 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 86.04 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 334 \cdot in$ $W = 27.833\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 24.458\,ft$

$D_c := (w_c \cdot h_f \cdot W) = 2.609 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot (W - 2 \cdot b_{par}) = 0.734 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.043 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{(D_c)}{N} + D_s = 0.304 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[(2b_e - b_{jt})(h_f \cdot w_c)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 17.229 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.279 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.321 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.073 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$

distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.066 \cdot \frac{kip}{ft}$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.489 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.25 \cdot in$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.417 \cdot in$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

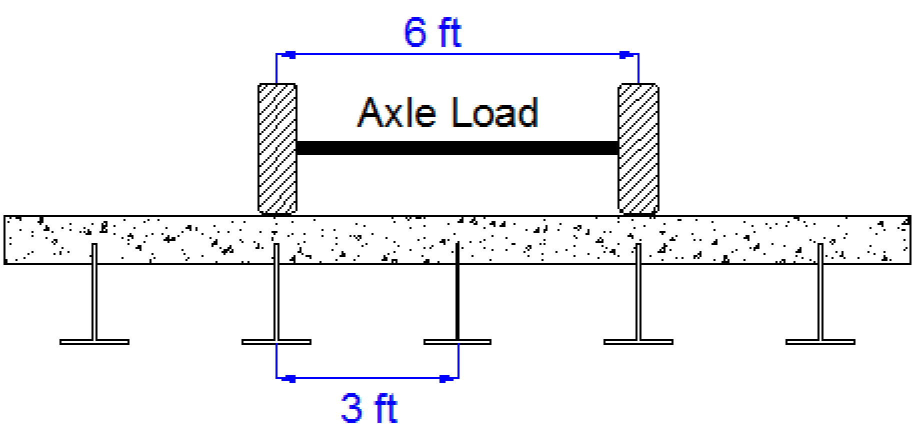
$$M_{DL} := \frac{DL \cdot L^2}{8} = 36.157 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 55.067 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 21145.34 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 14.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 2.895 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 2.895 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

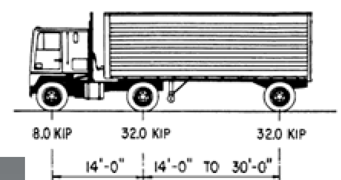
Design truck load in AASHTO (HS20 Truck) is used for this example.

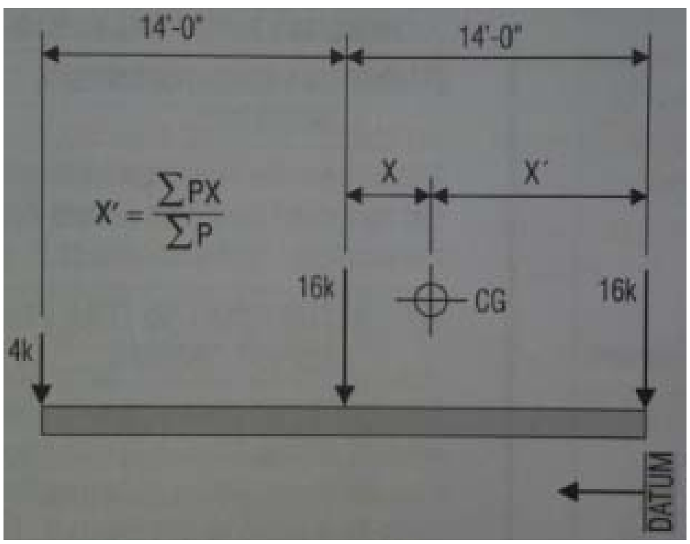
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

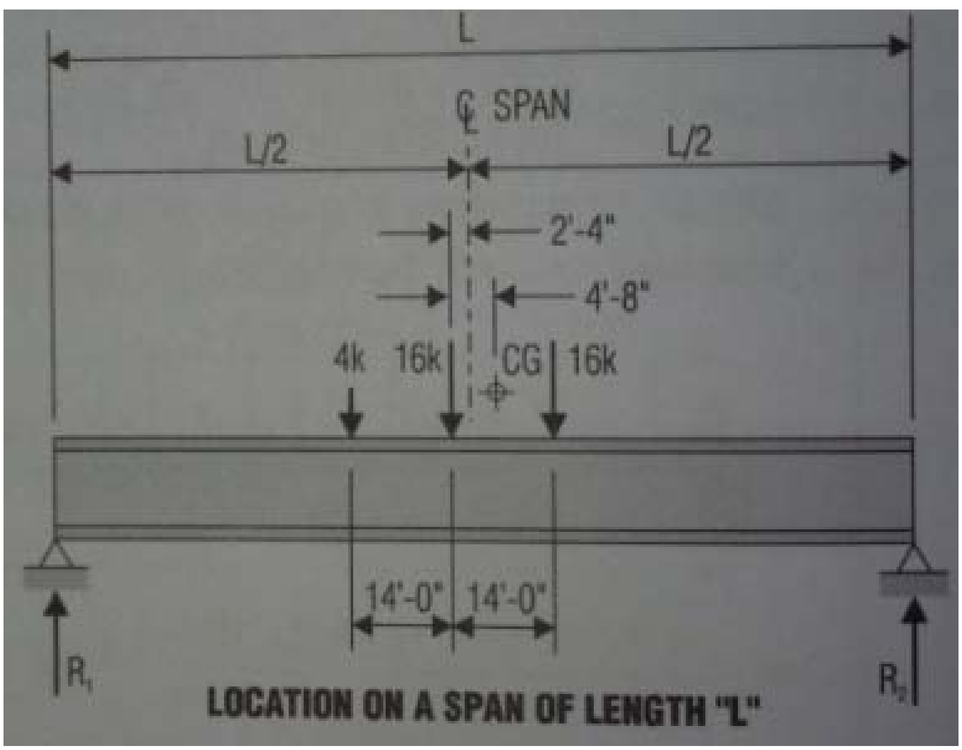
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.167\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 408.333\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 162.925\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

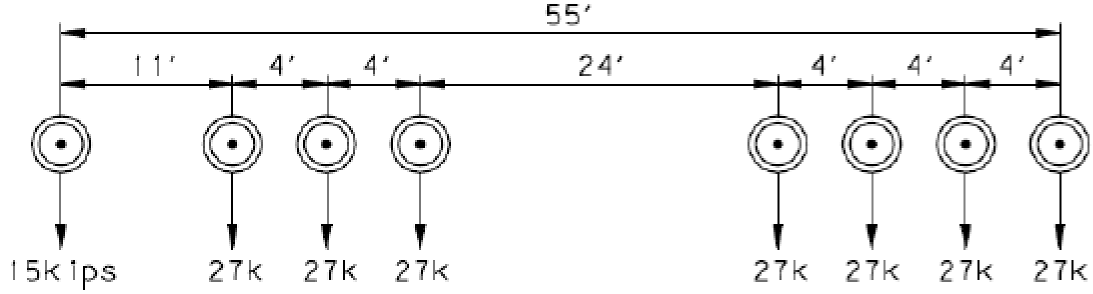
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 21.6\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



P1 P2 P3 P4 P5 P6 P7

NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 31.875 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\ 3}}{12} = 149.414 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.491 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2165 \cdot in^4$$

Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 14.491 \cdot in$$

N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 10.625 \cdot in^2$$

$$I_{tr3} := \frac{b_{tr3} \cdot h_f^{\ 3}}{12} = 49.805 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.045 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1489 \cdot in^4$$

Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 11.045 \cdot in$$

N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.082 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.231}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h - d)]] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9031.2 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8202.9 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -2.37 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.1077}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h - d)]] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 7652.03 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8764.7 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 683.6 \cdot kip \cdot ft \qquad Mn = 8202.9 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -6.552\cdot in$$

$$D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in$$

$$\frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.218\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 14.824\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.283\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 4.901\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.315\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 34.705\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.39\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.389\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.188 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.96 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.12 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.287 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.346 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 450.7 \cdot kip \cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.176 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 418 \cdot kip \cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1723.1 \cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.133 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.96 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.203 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1583.87 \cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 134.839 \cdot in^3$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 149.374 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 127.797 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}} \right) \cdot S_{ST} = 480.82 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 608.617 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 146.068 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 376.905 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 388.653 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $Mn = 683.575 \cdot kip \cdot ft$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

<u>Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)</u>

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

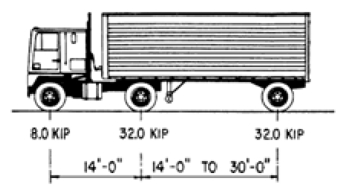
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.115 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot ft$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 0$

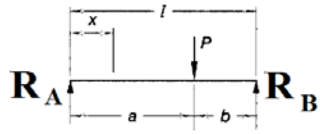
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot ft$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot ft$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.431 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

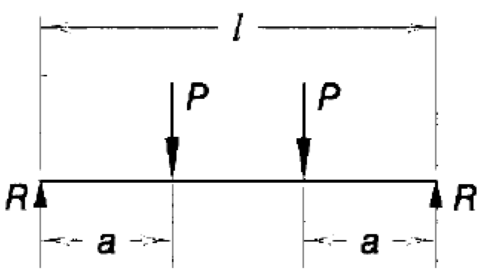
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.267 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.186 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.251 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.037 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.290 \cdot in$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.473}$ For one design lane loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 119.834 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.321 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.489 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 31.225 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 36.723 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 4.544 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 4.821 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.342 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 89.256 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 48.871 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 83.015 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 89.256 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements — $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) — $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ — Compressive capacity of bearing stiffener — $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

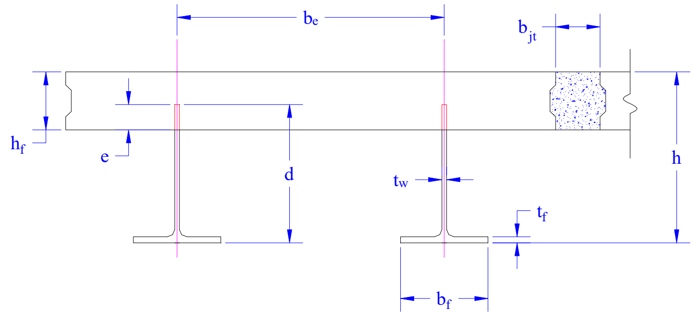
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The Fflexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 30ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 22in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 215.136kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 86.04kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$

Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$

MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\dfrac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 354 \cdot \text{in}$ $W = 29.5\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 26.125\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.766 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\dfrac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\dfrac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.784 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\dfrac{\text{lbf}}{\text{ft}^3} = 0.043 \cdot \dfrac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.319 \cdot \dfrac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\dfrac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 18.167 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\dfrac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.295 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.338 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.078 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.498 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

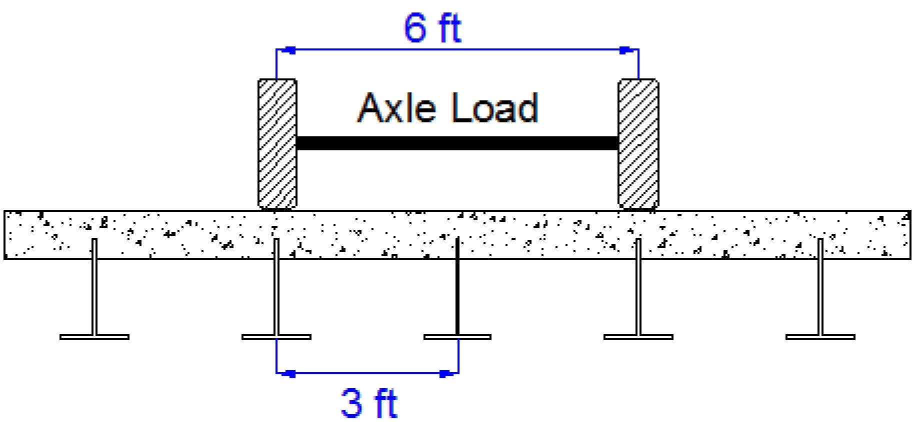
$$M_{DL} := \frac{DL \cdot L^2}{8} = 38.032 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 56.067 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^2\right) = 21145.34 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 14.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 2.895 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 2.895 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

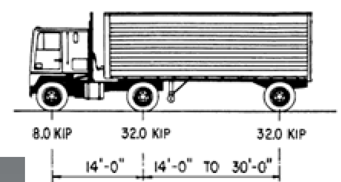
Design truck load in AASHTO (HS20 Truck) is used for this example.

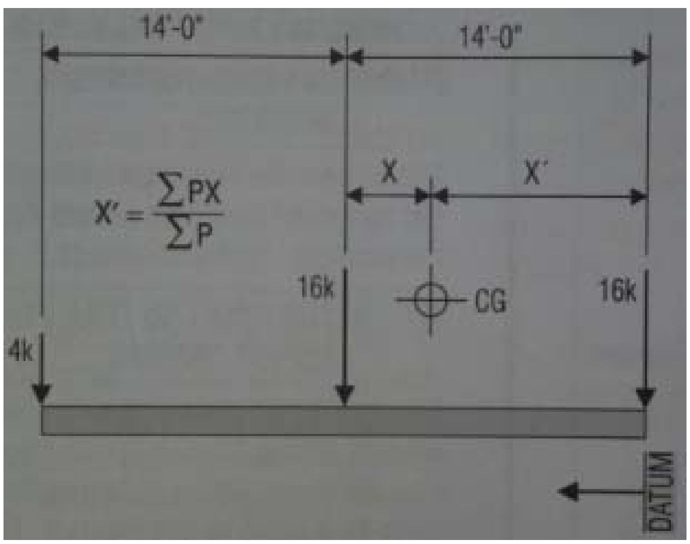
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

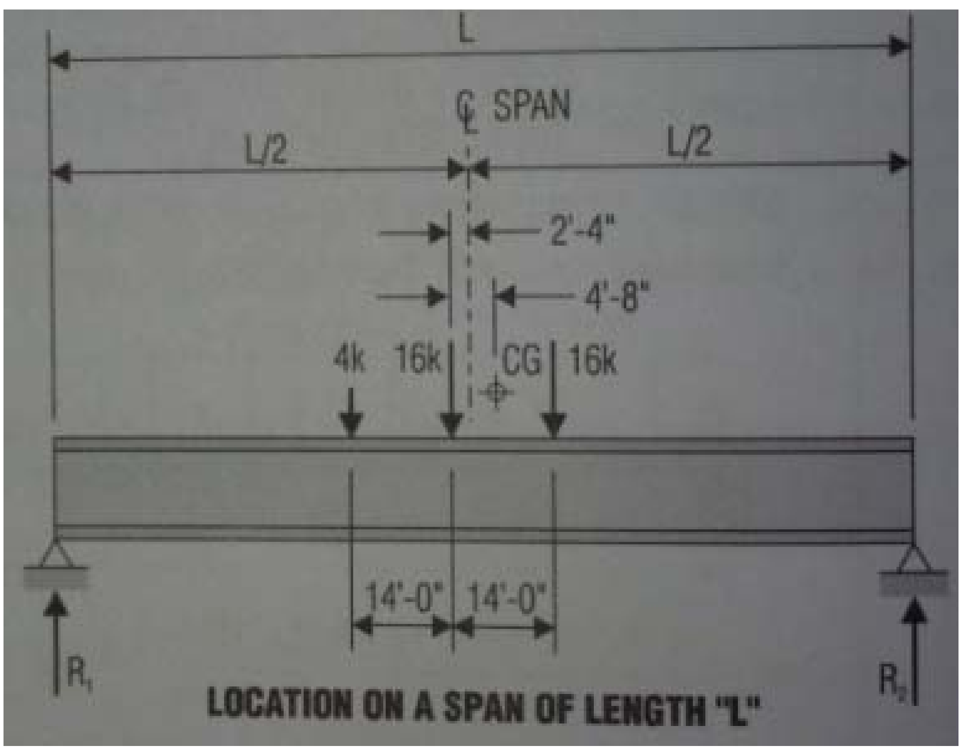
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.167\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 408.333\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 162.925\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

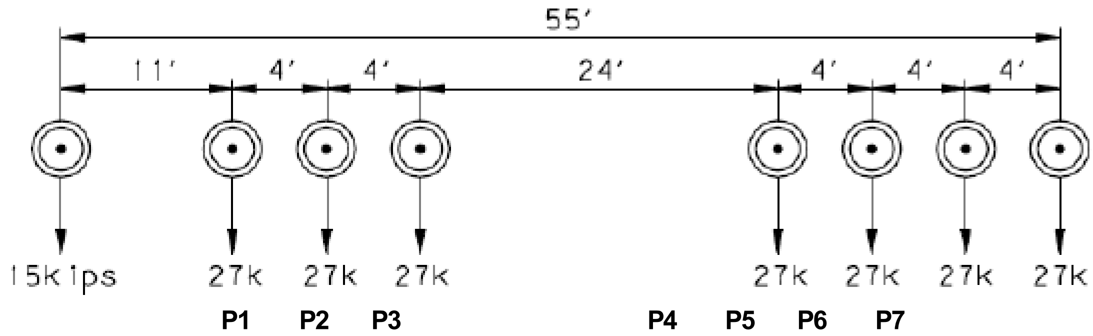
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\dfrac{F_{DLL}\cdot L^2}{8}\right) = 21.6\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot i \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 158.203 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.643 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2199 \cdot in^4$$

Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 14.643 \cdot in$$

N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$$

$$I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 52.734 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.234 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1524 \cdot in^4$$

Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 11.234 \cdot in$$

N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 4.866 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.221}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9115.29 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8342 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -3.73 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.1695}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 7305.23 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8683.6 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 695.2 \cdot kip \cdot ft \qquad Mn = 8342 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = -7.951 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 3.365 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 14.746 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 17.193 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.96 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 37.452 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 34.856 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 27.5 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.495 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.191\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.926\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 1.08\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.281\cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.282\cdot ksi$$

$$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 454.6\cdot kip\cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.118\cdot ksi$$

$$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 422\cdot kip\cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6\cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1676.33\cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.134\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.926\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.198\cdot ksi$$

$$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1536.43\cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 135.642 \cdot in^3$ $\quad$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 150.159 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 131.64 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 479.932 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 611.572 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 146.777 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 380.749 \cdot kip \cdot ft$ $\quad$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 392.554 \cdot kip \cdot ft$ $\quad$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 695.164 \cdot kip \cdot ft$ $\quad$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

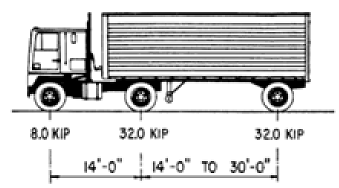
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.098 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

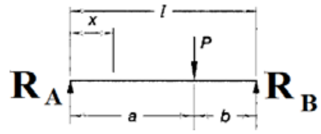
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.424 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

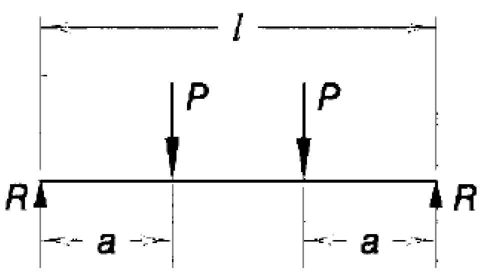
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.263 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.183 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.247 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.037 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.286 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$DF_s := 0.36 + \frac{S}{25} = 0.48$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 121.746 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.338 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.498 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left\lfloor DF_s \cdot (1 + I) \right\rfloor = 31.665 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left\lfloor DF_s \cdot (1 + I) \right\rfloor = 37.24 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 4.608 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.071 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.476 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 90.786 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 49.559 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 84.457 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 90.786 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

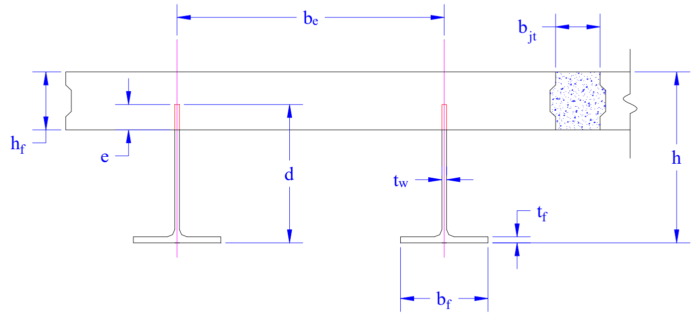
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The Fflexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $\quad$ $modules := \frac{N}{2} = 6$

$L := 30ft$ $\quad$ Span length

$b_e := 36in$ $\quad$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad$ $t_f := 0.5in$ $\quad$ $b_f := 12in$

$h := 22in$ $\quad$ Overall Height of section

$h_f := 7.5in$ $\quad$ Total thickness of deck

$e := 3.5in$ $\quad$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ $\quad$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ $\quad$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 215.136 kip \cdot ft$ $\quad$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 86.04 kip$ $\quad$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\quad \frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\quad \frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $\quad b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $\quad t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$

Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$

MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ $\quad$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 426 \cdot \text{in}$ $W = 35.5\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 32.125\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.328 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700 \frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.964 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{\text{lbf}}{\text{ft}^3} = 0.043 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.32 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 18.167 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.296 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.339 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.08 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.5 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

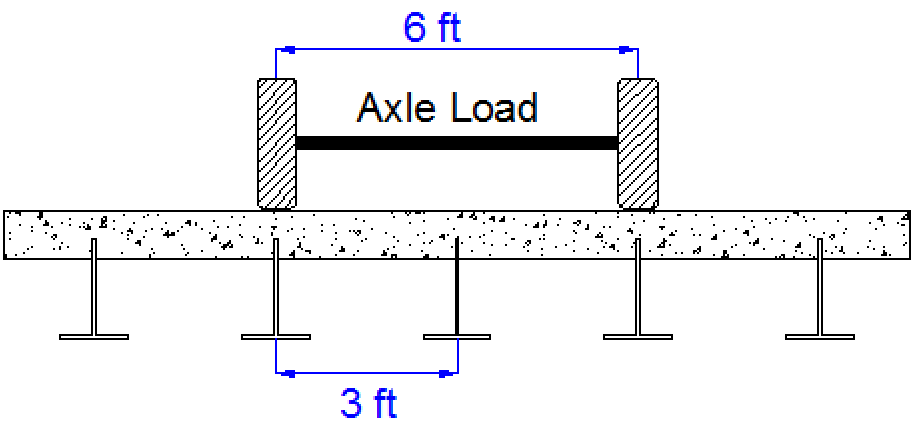
$$M_{DL} := \frac{DL \cdot L^2}{8} = 38.12 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 56.285 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot {e_g}^2\right) = 21145.34 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***<u>Per Lane</u>*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{\text{kip}}{\text{ft}^2} = 0.021 \cdot \frac{\text{kip}}{\text{ft}^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 14.5 \cdot \text{in}$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 2.895 \cdot \text{kip} \cdot \text{ft}$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot \text{in}^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 2.895 \cdot \text{ksi}$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

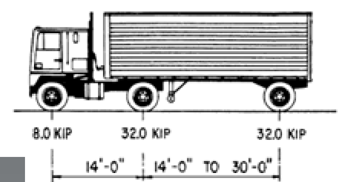
Design truck load in AASHTO (HS20 Truck) is used for this example.

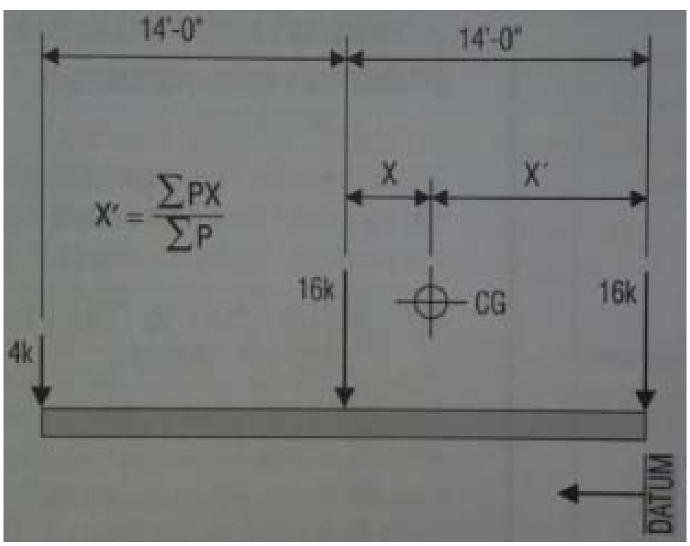
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

Find bridge reaction

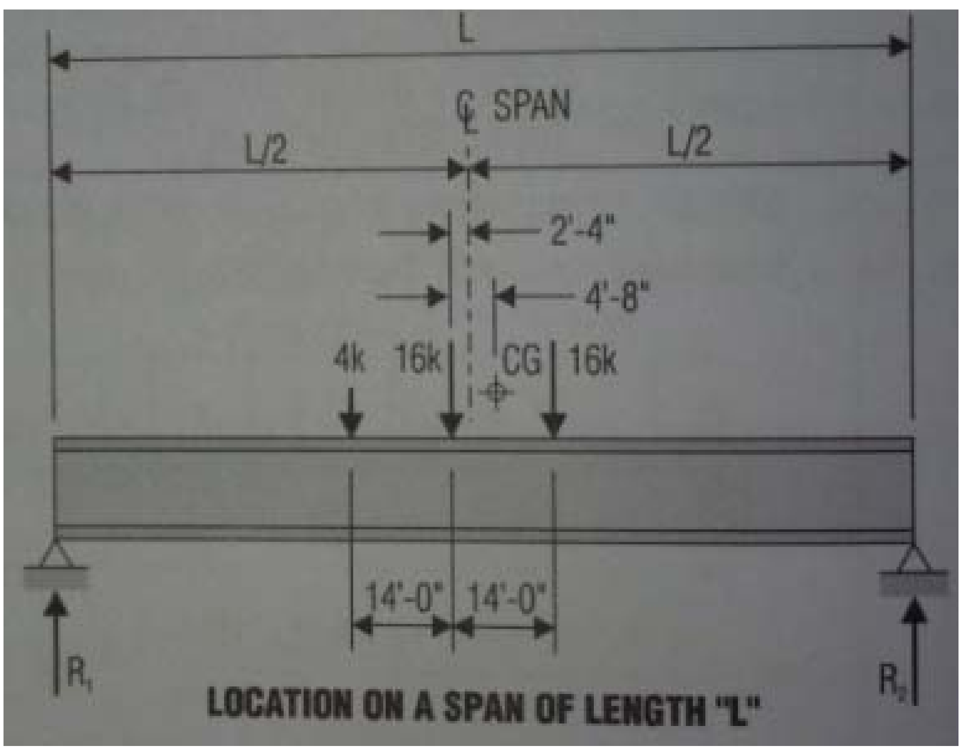
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.167\cdot\text{kip}$$

Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 408.333\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 162.925\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

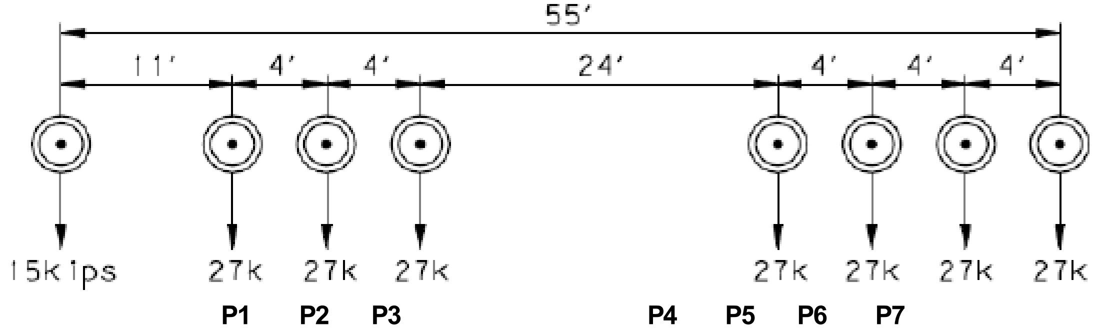
$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 21.6\cdot\text{kip}\cdot\text{ft}$$

Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot i$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 158.203 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.643 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2199 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 14.643 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 52.734 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.234 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1524 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 11.234 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f

$h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in$ $\quad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 4.866 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\quad \boxed{\frac{Dp1}{h} = 0.221}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1$ $\quad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9115.29 \cdot kip \cdot in}$ $\quad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8342 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -3.73 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\quad \boxed{\frac{Dp2}{h} = -0.1695}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0}$ $\quad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 7305.23 \cdot kip \cdot in}$ $\quad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8683.6 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 695.2 \cdot kip \cdot ft$ $\quad Mn = 8342 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = -7.951 \cdot in$ $\quad D_{cp0} := D_{cp} > 0 = 0$

$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in$ $\quad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$ $\quad$ Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 3.372 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 14.746 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 17.193 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.979 \cdot ksi$$

STRENGTH I $\quad f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 37.491 \cdot ksi$

STRENGTH II $\quad f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 34.895 \cdot ksi$

SERVICE II $\quad f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 27.52 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$f_f + \frac{f_{lSII}}{2} = 27.522 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.191 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.926 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.08 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.283 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.284 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 455 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.121 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 423 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1677.87 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.135 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.926 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.199 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1537.51 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 135.642 \cdot in^3$ $\quad$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 150.159 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 132.077 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 479.448 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 611.525 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 146.766 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 381.186 \cdot kip \cdot ft$ $\quad$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 392.99 \cdot kip \cdot ft$ $\quad$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 695.164 \cdot kip \cdot ft$ $\quad$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.167$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

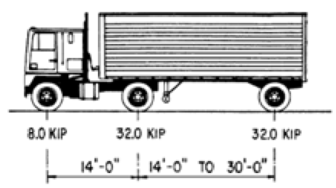
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.098 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

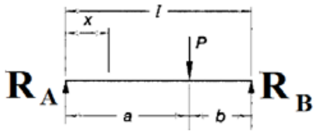
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.424 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

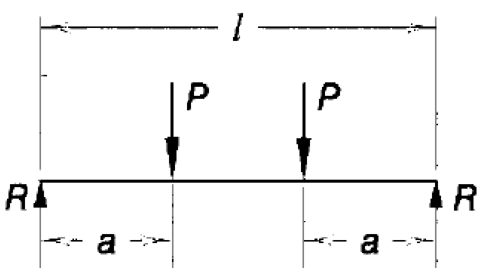
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.263 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.152 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.206 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 106.667 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.03 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.238 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.48}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 121.746 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.339 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.5 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 31.665 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.24 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 4.608 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.083 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.505 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 90.844 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 49.559 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 84.515 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 90.844 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \quad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

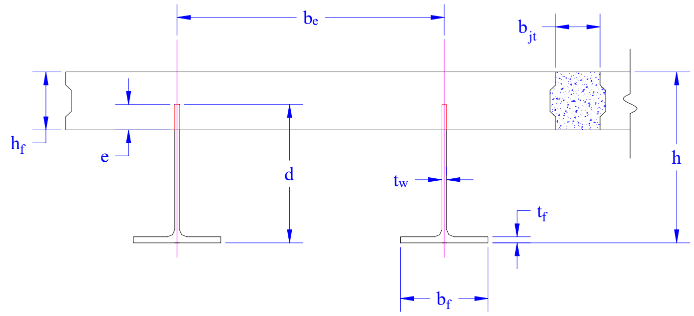
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The Fflexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 30ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 22in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 215.136 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 86.04kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\dfrac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 374 \cdot \text{in}$ $W = 31.167\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 27.792\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.922 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\dfrac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\dfrac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.834 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\dfrac{\text{lbf}}{\text{ft}^3} = 0.043 \cdot \dfrac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.335 \cdot \dfrac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\dfrac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 19.104 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\dfrac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.312 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.355 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.083 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$

distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.507 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

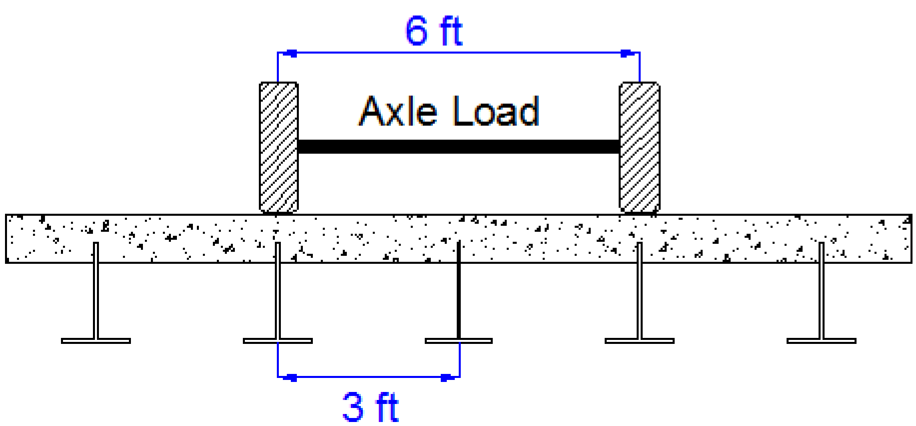
$$M_{DL} := \frac{DL \cdot L^2}{8} = 39.907 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 57.067 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 21145.34 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56\cdot 10^{-6}\cdot\left(V_w\right)^2\cdot K_z\cdot G\cdot C_D\cdot\frac{kip}{ft^2} = 0.021\cdot\frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 14.5\cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z\cdot h_{exp}\cdot L^2}{8} = 2.895\cdot kip\cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{{b_f}^3\cdot t_f}{12} = 72\cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL}\cdot\frac{b_f}{2}}{I_{yy}} = 2.895\cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

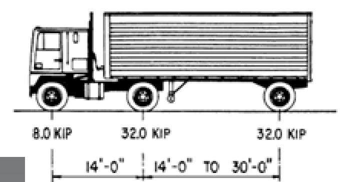
Design truck load in AASHTO (HS20 Truck) is used for this example.

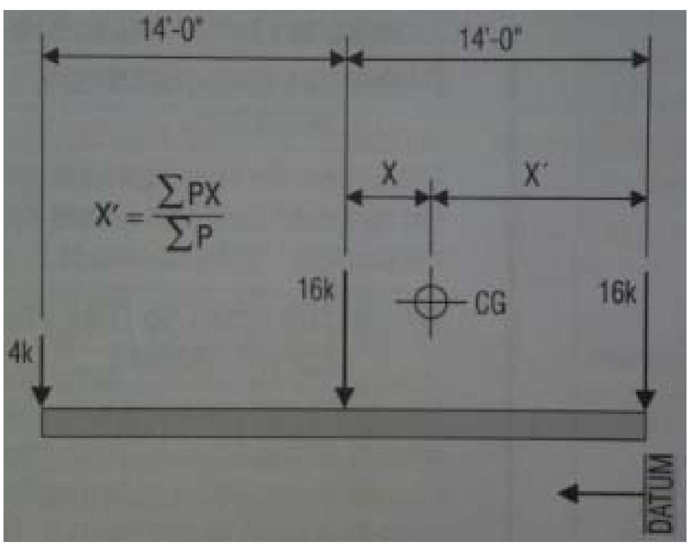
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

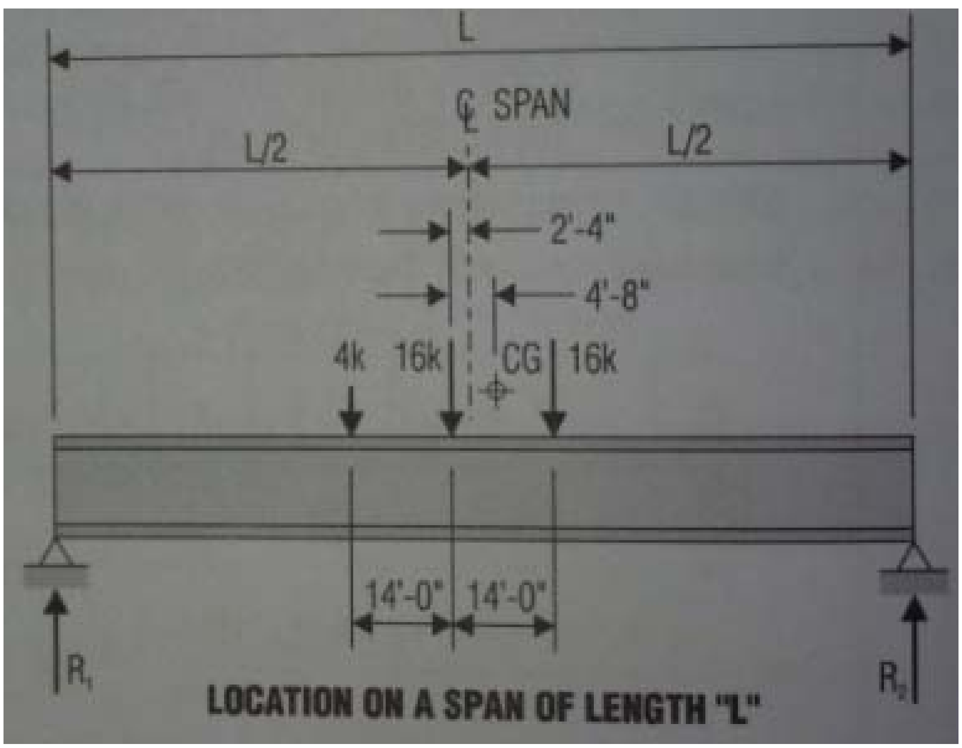
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.167\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 408.333\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 162.925\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

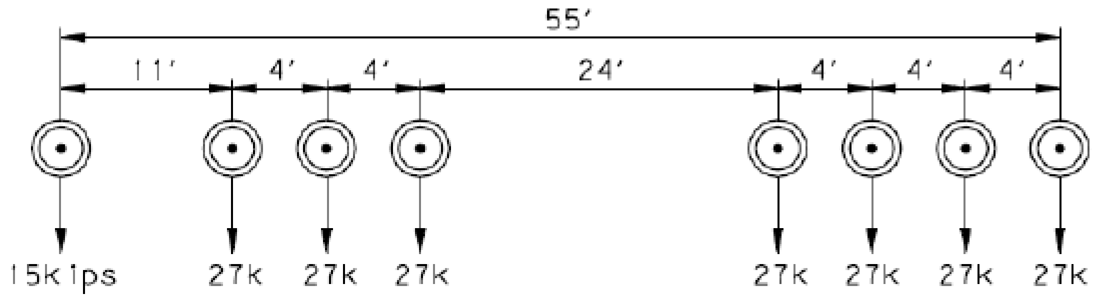
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 21.6\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.783 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2231 \cdot in^4$$ Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 14.783 \cdot in$$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.414 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1557 \cdot in^4$$ Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 11.414 \cdot in$$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 4.668 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.212}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9192.52 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8470.7 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -5.09 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.2314}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 6889.07 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8487 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 705.9 \cdot kip \cdot ft \qquad Mn = 8470.7 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -9.35\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.511\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 14.673\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.107\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.021\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.597\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 35.014\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.61\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.606\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.194 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.895 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.044 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.277 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.224 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 458.4 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.067 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 426 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1634.54 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.136 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.895 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.194 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1493.72 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 136.392 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 150.914 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 135.484 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 478.898 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 614.382 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 147.452 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 384.593 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 396.452 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 705.893 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

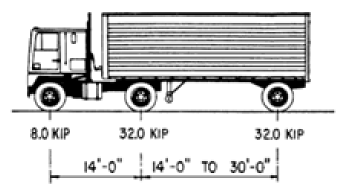
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.082 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

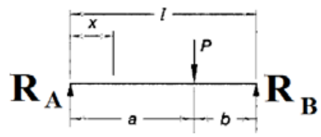
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.418 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

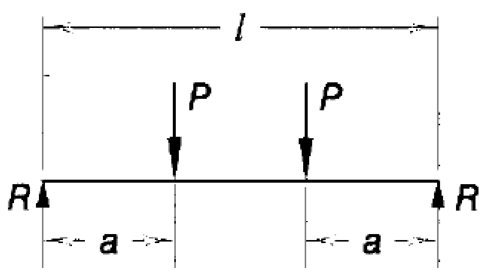
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.259 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.18 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.243 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.036 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.282 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 123.509 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.355 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.507 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 32.104 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.757 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 4.672 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.321 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.609 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 92.316 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 50.247 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 85.898 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 92.316 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements $\quad b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $\quad PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ — Compressive capacity of bearing stiffener $\quad Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

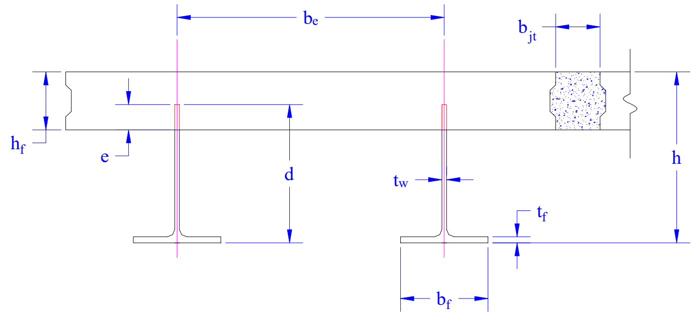
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  - AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  - PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $\quad modules := \frac{N}{2} = 7$

$L := 30ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 22in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 215.136 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 86.04 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 526 \cdot \text{in}$ $W = 43.833\text{ ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 40.458\text{ ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 4.109 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.214 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.043 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.336 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 19.104 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.313 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.356 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.087 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.511 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

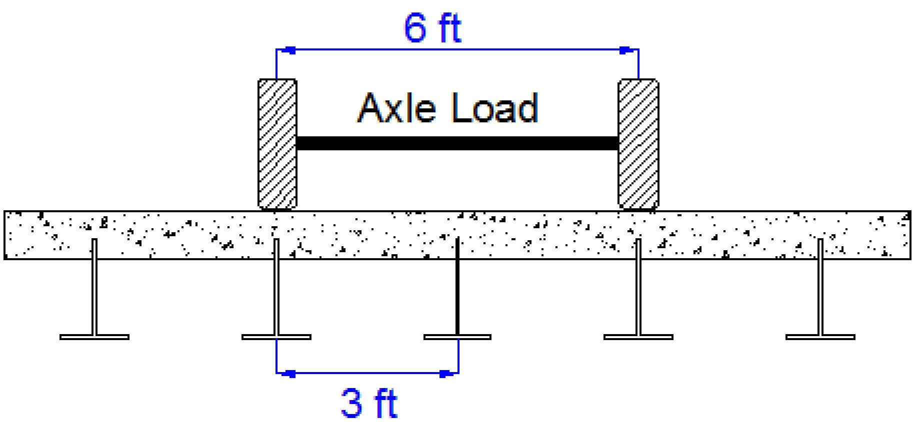
$$M_{DL} := \frac{DL \cdot L^2}{8} = 40.057 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 57.441 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.167$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 21145.34 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***<u>Per Lane</u>*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 14.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 2.895 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 2.895 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

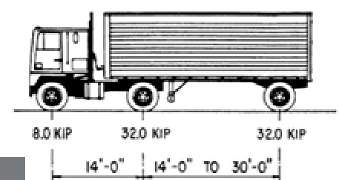
Design truck load in AASHTO (HS20 Truck) is used for this example.

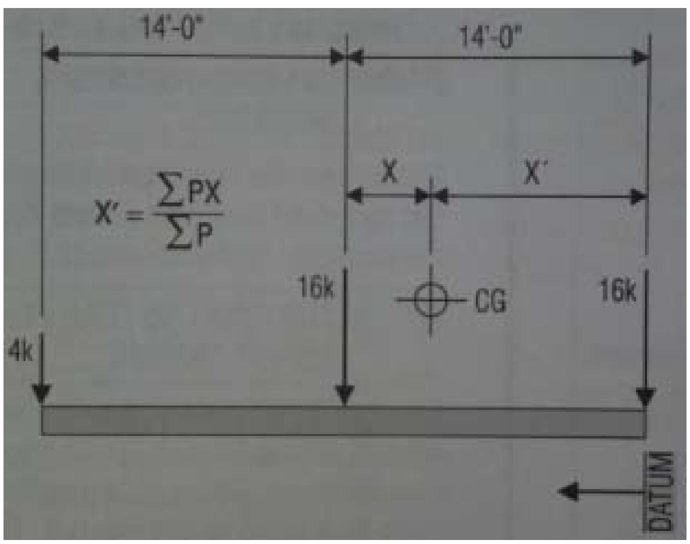
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

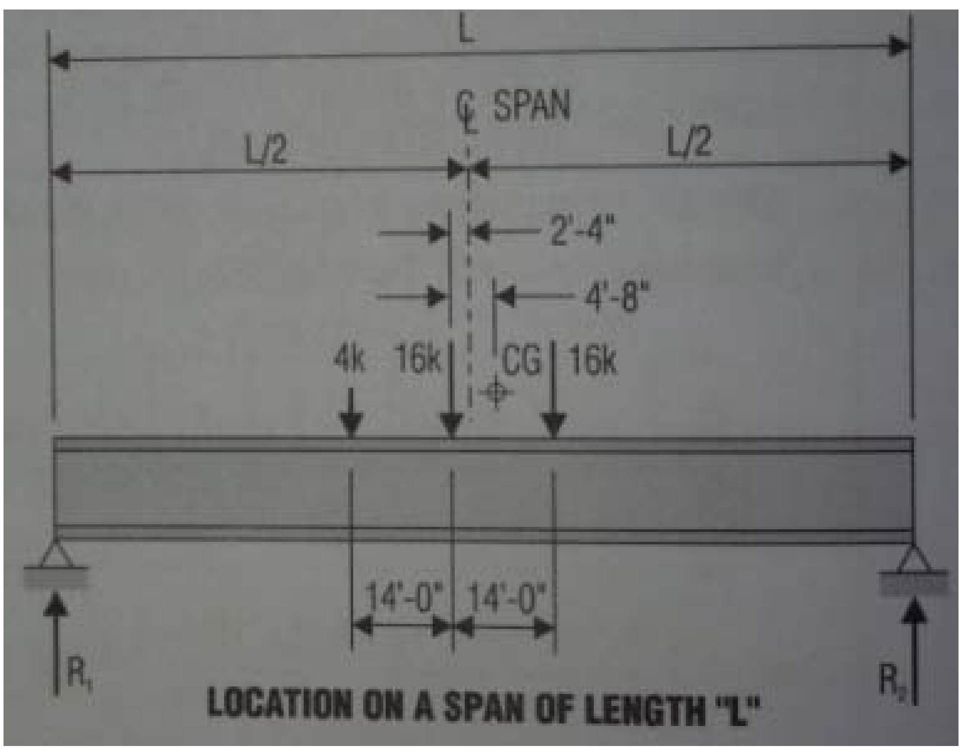
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.167\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 408.333\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333\cdot\text{kip}\cdot\text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 162.925\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

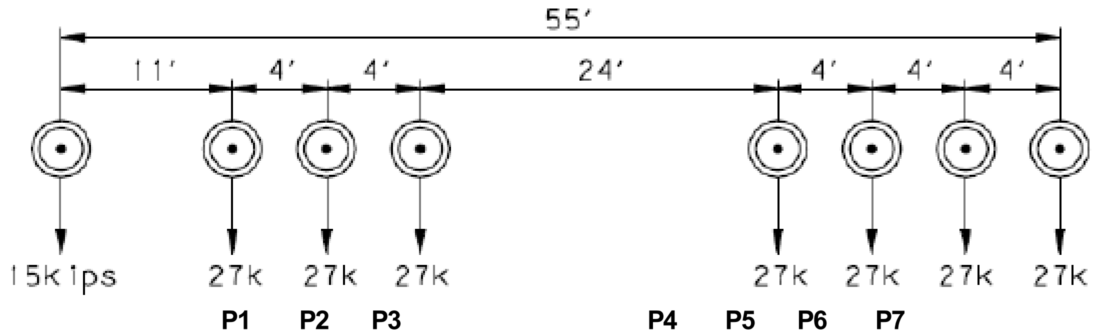
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 21.6\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.783 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2231 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 14.783 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.414 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1557 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 11.414 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 4.668 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.212}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9192.52 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8470.7 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -5.09 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.2314}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 6889.07 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8487 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 705.9 \cdot kip \cdot ft \qquad Mn = 8470.7 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -9.35\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.524\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 14.673\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.107\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.054\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.663\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 35.08\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.65\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.652\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.194 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.895 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.044 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.279 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.228 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 459.2 \cdot kip \cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.07 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 427 \cdot kip \cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1637.08 \cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.136 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.895 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.195 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1495.5 \cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 136.392 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 150.914 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 136.233 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 478.069 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 614.302 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 147.433 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 385.342 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 397.199 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 705.893 \cdot kip \cdot ft \quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: Compute Deflection

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

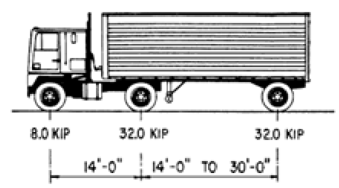
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.082 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

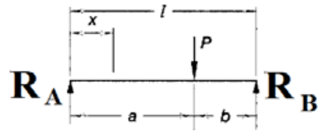
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.418 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

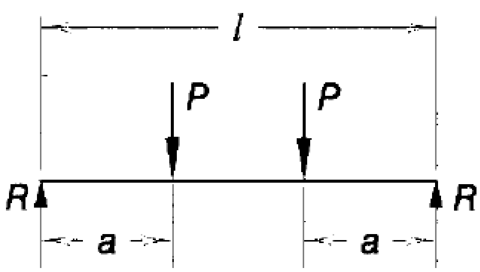
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.259 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.193 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.261 \cdot in$$

(Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.039 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.302 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 123.509 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.356 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.511 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 32.104 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.757 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 4.672 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.341 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.659 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 92.416 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 50.247 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 85.998 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 92.416 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \quad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

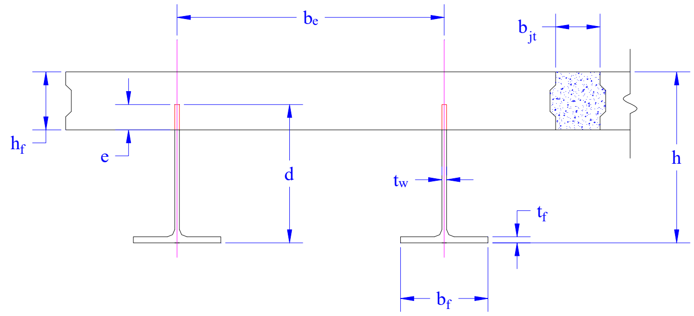
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 30ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 22in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 215.136 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 86.04 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$

Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$

MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 394 \cdot in$ $W = 32.833\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 29.458\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.078 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.884 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.043 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.351 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 20.042 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.371 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.088 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.516 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

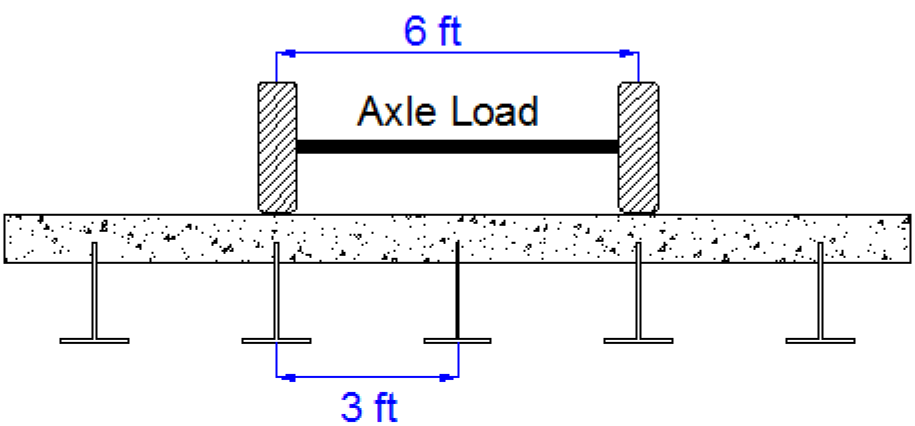
$$M_{DL} := \frac{DL \cdot L^2}{8} = 41.782 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 58.067 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 21145.34 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 14.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 2.895 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f{}^3 \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 2.895 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

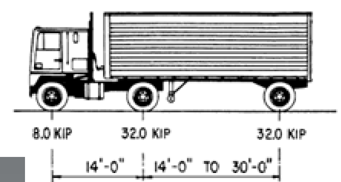
Design truck load in AASHTO (HS20 Truck) is used for this example.

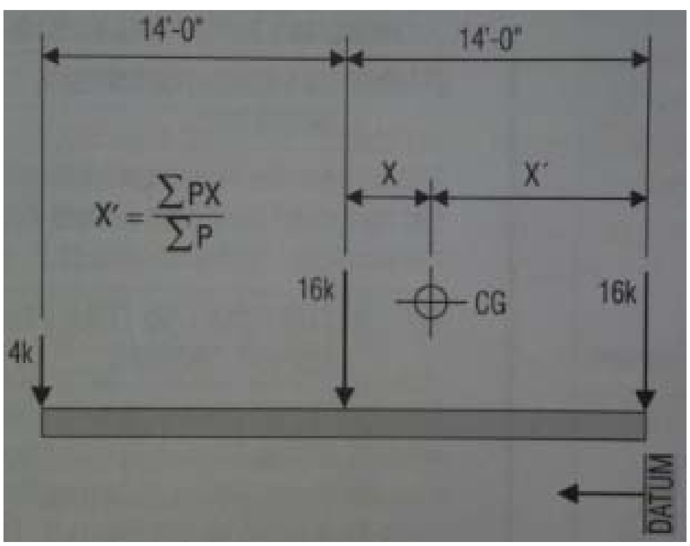
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

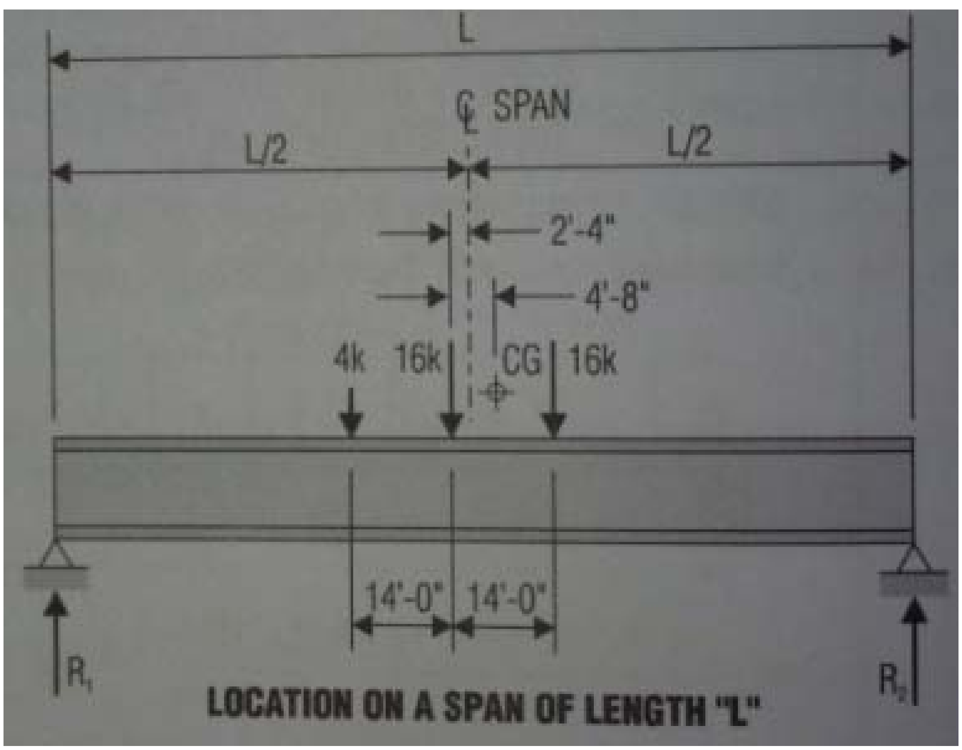
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2} + \frac{X}{2}\right) + F_T\cdot\left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.167\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2} - \frac{X}{2}\right) = 408.333\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1 + I) = 162.925\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

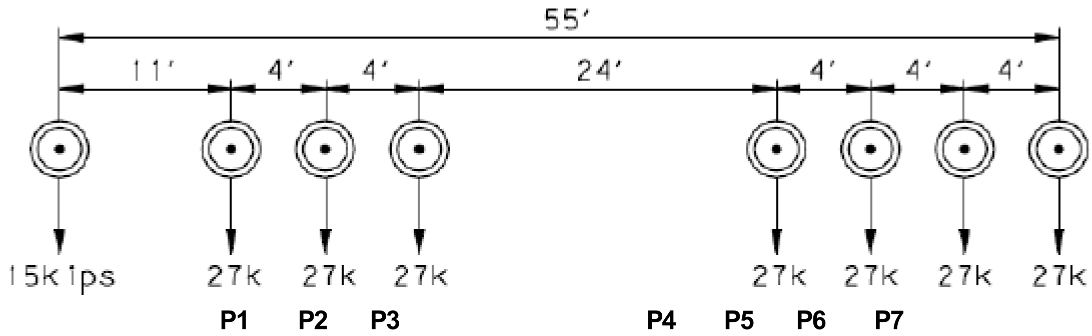
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 21.6\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2 \quad I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.913 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2261 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 14.913 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2 \qquad I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.584 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1588 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 11.584 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 4.485 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.204}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \dots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9263.7 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8590.2 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -6.45 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.2932}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \dots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 6403.55 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8166 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 715.9 \cdot kip \cdot ft \qquad Mn = 8590.2 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -10.749\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.657\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 14.602\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.024\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.083\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.749\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 35.178\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.72\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.722\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.196 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.011 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.273 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.173 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 462.2 \cdot kip \cdot ft$ $\quad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.02 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 430 \cdot kip \cdot ft$ $\quad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1597.01 \cdot psi$ This assumes short term composite section for all calculations $\quad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.137 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.19 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1455.07 \cdot psi$ $\quad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 137.097 \cdot in^3$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 151.644 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 139.328 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 477.737 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 617.065 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 148.095 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 388.437 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 400.348 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $Mn = 715.852 \cdot kip \cdot ft$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

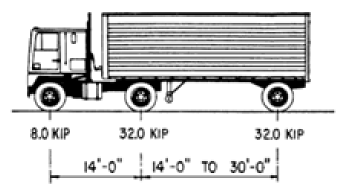
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.067 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

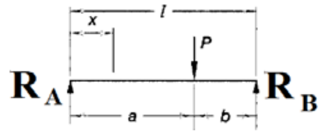
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.413 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

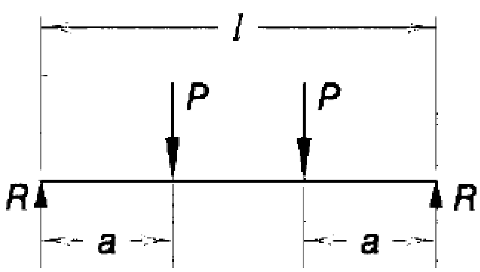
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.256 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta1 + \Delta2 + \Delta3) \cdot DF_d \cdot (1 + I) = 0.178 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.24 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.036 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\, \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.278 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 125.14 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.371 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.516 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 32.544 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 38.274 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 4.736 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.571 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.742 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1} \right) \right) = 93.845 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

<u>Strength II Max Positive Shear:</u>

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 50.936 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$\boxed{V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 87.34 \cdot kip}$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 93.845 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\ 3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

Summary of Calculations:

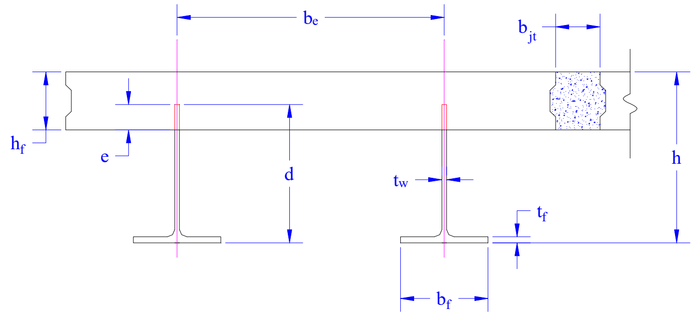
1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:

Number of beams $N := 12$ $\quad$ $modules := \frac{N}{2} = 6$



$L := 30ft$ Span length

$\boxed{b_e := 40in}$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad$ $t_f := 0.5in$ $\quad$ $b_f := 12in$

$h := 22in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$\boxed{M_{LL\_IM\_PL82} := 215.136 kip \cdot ft}$ Moment due to PL82 with distribution factor and impact factor!!

$\boxed{V_{PL82span} := 86.04kip}$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 474 \cdot \text{in}$ $W = 39.5\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 36.125\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.703 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.084 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.043 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.351 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 20.042 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.372 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.09 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.518 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

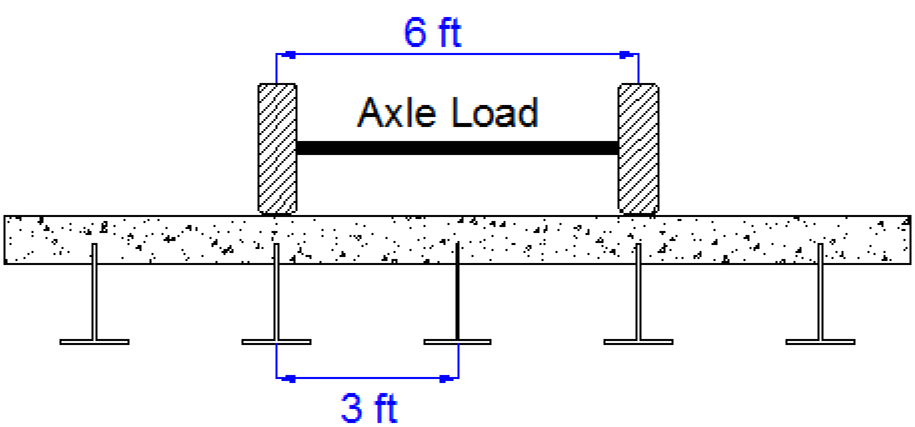
$$M_{DL} := \frac{DL \cdot L^2}{8} = 41.87 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 58.285 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 21145.34 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 14.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 2.895 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 2.895 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

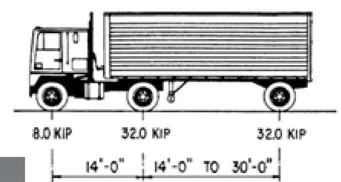
Design truck load in AASHTO (HS20 Truck) is used for this example.

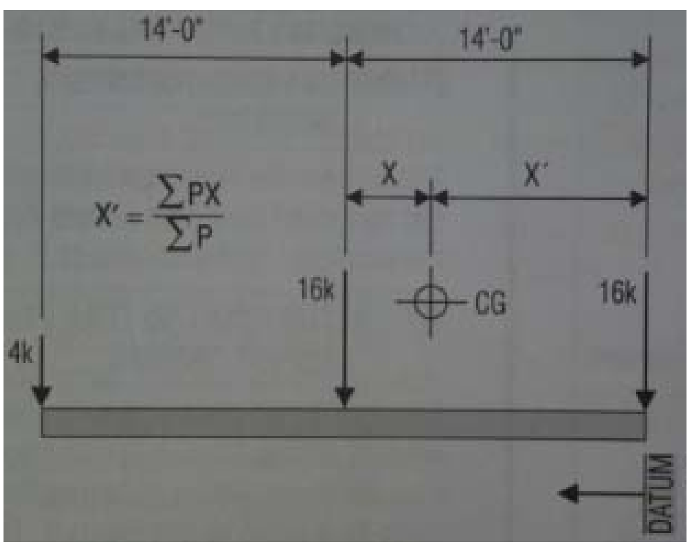
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

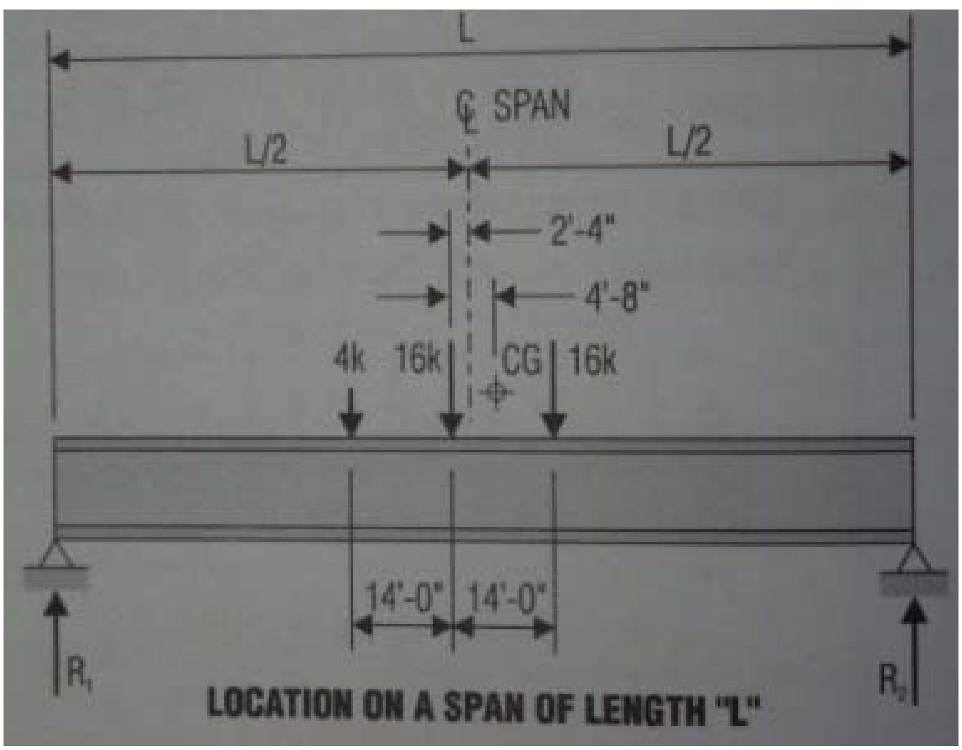
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.167 \cdot \text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 408.333 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 162.925 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

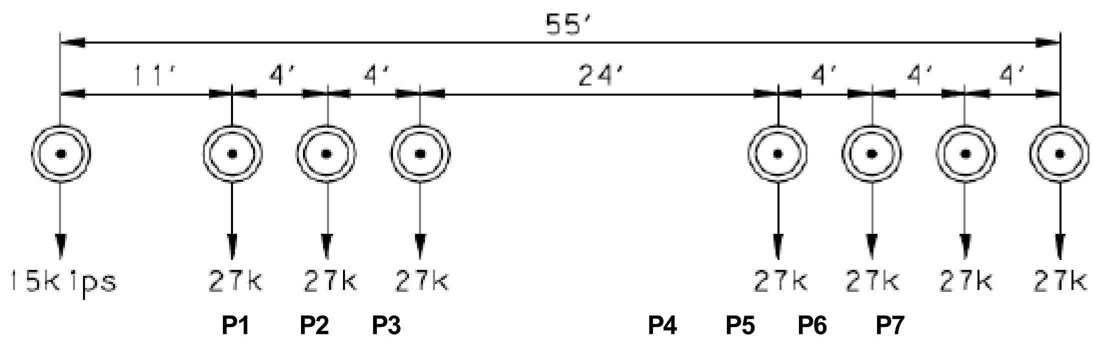
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \dfrac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\dfrac{F_{DLL} \cdot L^2}{8}\right) = 21.6 \cdot \text{kip} \cdot \text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $\quad I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.913 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2261 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 14.913 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $\quad I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.584 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1588 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 11.584 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 4.485 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.204}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9263.7 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8590.2 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -6.45 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.2932}$ must be < 0.42

$M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0 \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 6403.55 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8166 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 715.9 \cdot kip \cdot ft \qquad Mn = 8590.2 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = -10.749 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 3.665 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 14.602 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 17.024 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.102 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 37.787 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.216 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 27.75 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.749 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.197 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 1.011 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.274 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.175 \cdot ksi$$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 462.7 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.022 \cdot ksi$$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 430 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1598.45 \cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot \left(h - Y'_{c\_long}\right)}{k_3 \cdot n \cdot I_{c\_long}} = 0.137 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot \left(h - Y'_{c\_long}\right)}{k_3 \cdot n \cdot I_{c\_long}} = 0.191 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1456.07 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 137.097 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 151.644 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 139.765 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 477.253 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 617.018 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 148.084 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 388.874 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 400.783 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 715.852 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.25$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

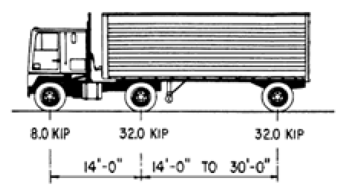
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.067 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

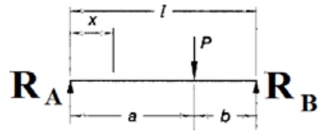
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.413 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

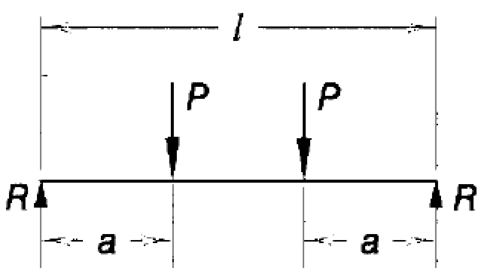
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.256 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.222 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.3 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 160 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.044 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.347 \cdot in$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 125.14 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.372 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.518 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - \left(X_{ASr} + X_{ASf}\right) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 32.544 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{\left(L - X_{AS}\right)}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.274 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot \left(DF_s\right) = 4.736 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.583 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.771 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 93.904 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 50.936 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 87.398 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 93.904 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \quad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot \left(2 \cdot b_t\right)^3}{12} = 40.131 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

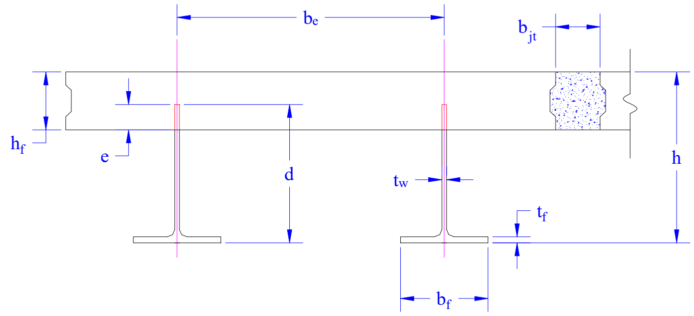
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $\quad modules := \frac{N}{2} = 7$

$L := 30ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 22in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 18 \cdot in$ depth of built up steel section

$h_w := d - t_f = 17.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 215.136kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 86.04kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 4.951 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 421.486 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 12.562 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 554 \cdot in$ $W = 46.167\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 42.792\,ft$

$D_c := (w_c \cdot h_f \cdot W) = 4.328 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot (W - 2 \cdot b_{par}) = 1.284 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.043 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{(D_c)}{N} + D_s = 0.352 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[(2b_e - b_{jt})(h_f \cdot w_c)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 20.042 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.33 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.043 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.373 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.092 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.519 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

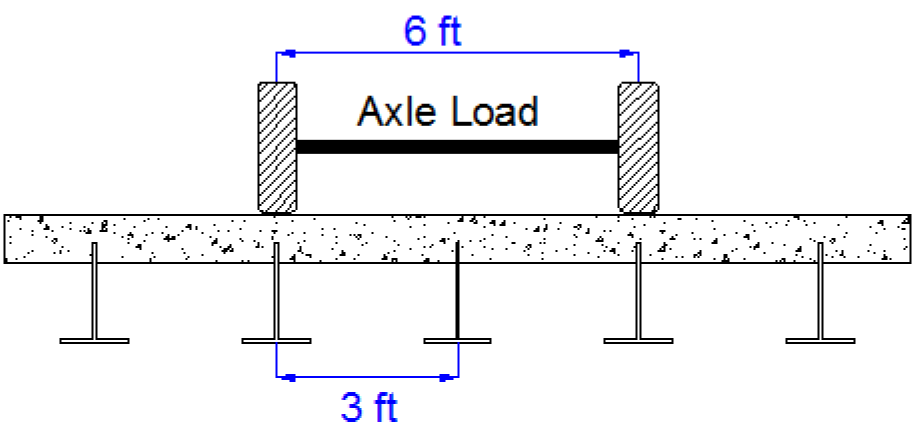
$$M_{DL} := \frac{DL \cdot L^2}{8} = 41.932 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 58.441 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.333$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 13.299 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 21145.34 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***<u>Per Lane</u>*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 14.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 2.895 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f{}^3 \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 2.895 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

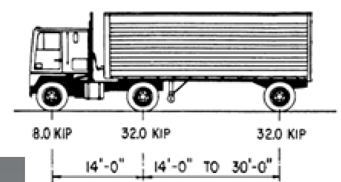
Design truck load in AASHTO (HS20 Truck) is used for this example.

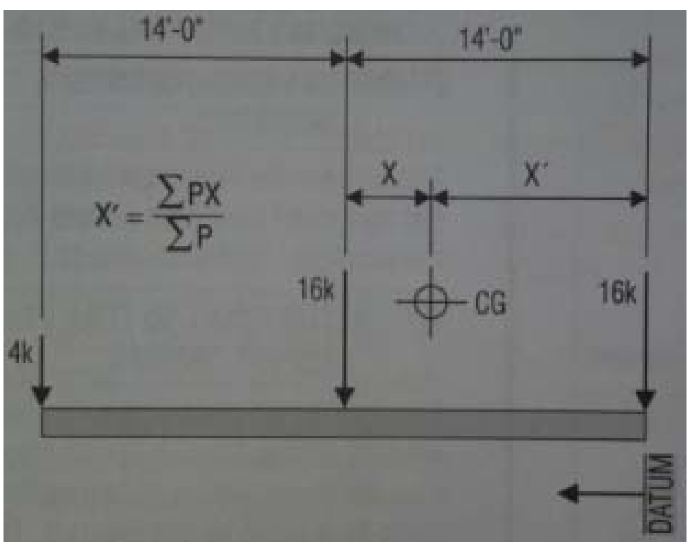
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

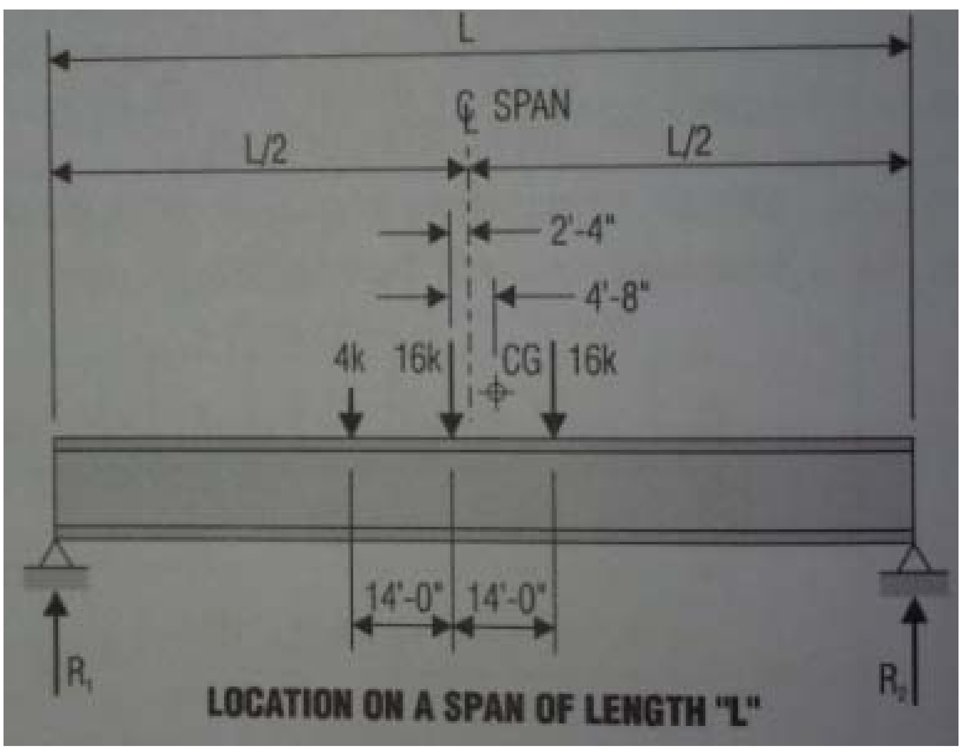
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 30.4 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 273.067 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.167 \cdot \text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$M_{LL\_tandem} := R_A \cdot \left(\dfrac{L}{2} - \dfrac{X}{2}\right) = 408.333 \cdot \text{kip} \cdot \text{ft}$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 408.333 \cdot \text{kip} \cdot \text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 162.925 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

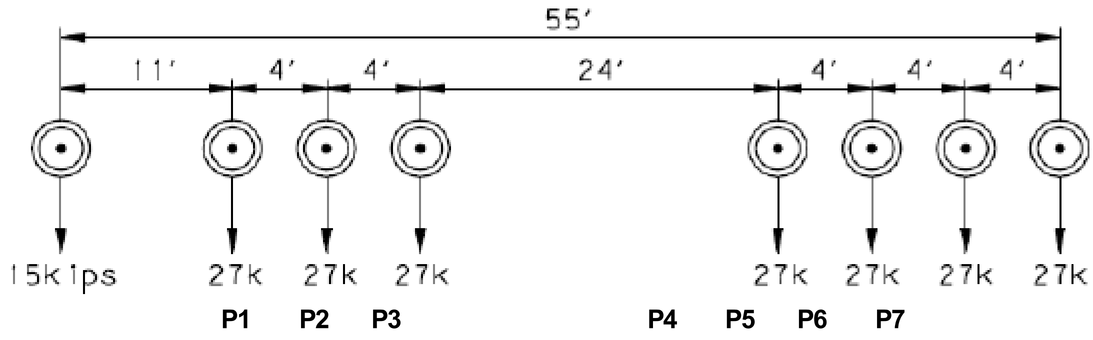
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \dfrac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\dfrac{F_{DLL} \cdot L^2}{8}\right) = 21.6 \cdot \text{kip} \cdot \text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 184.525 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 421.486 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $\quad I_{tr1} := \frac{b_{tr1} \cdot h_f^3}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 14.913 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2261 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 14.913 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $\quad I_{tr3} := \frac{b_{tr3} \cdot h_f^3}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.584 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1588 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 11.584 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 9.24 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 4.485 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.204}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 9263.7 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 8590.2 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -6.45 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.2932}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 6403.55 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 8166 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 715.9 \cdot kip \cdot ft \qquad Mn = 8590.2 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = -10.749 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 3.67 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 14.602 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 17.024 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.115 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 37.814 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.244 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 27.77 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.768 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.197 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.011 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.275 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.176 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 463 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.024 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 431 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1599.47 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.138 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.192 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1456.79 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 137.097 \cdot in^3 \qquad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 151.644 \cdot in^3$$

short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 140.077 \cdot kip \cdot ft$$

From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 476.908 \cdot kip \cdot ft$$

Additional moment to cause nominal yielding of steel flange

$$My := M_{D2} + M_{AD} = 616.985 \cdot kip \cdot ft$$

D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$$S_{xt} := \frac{My}{F_y} = 148.076 \cdot in^3$$

Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 389.186 \cdot kip \cdot ft$$

Moment due to Strength V limit state

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 401.095 \cdot kip \cdot ft$$

Equation 6.10.7.1.7-1

$$Mn = 715.852 \cdot kip \cdot ft$$

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

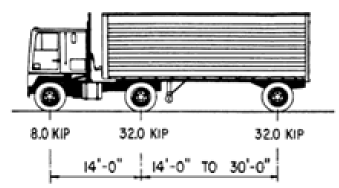
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$X_{ASr} := 14ft$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.067 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -3.667 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

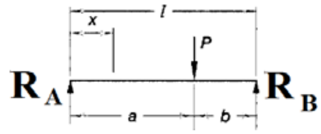
$b2 := \frac{L}{2} - (X_{ASr} - X') = 10.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 5.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.413 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

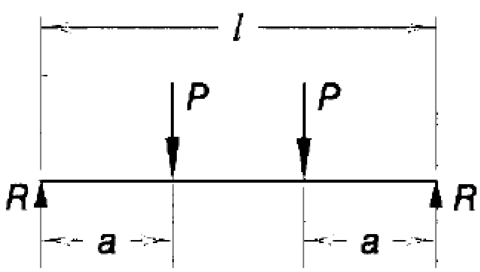
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.256 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.191 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.257 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.038 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.298 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.45 \cdot in$ $\frac{L}{1000} = 0.36 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For ONE design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 125.14 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.373 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.519 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 2 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 16 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 30 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 49.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 32.544 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 58.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.274 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 9.6 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 4.736 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.591 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 7.792 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 93.945 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 50.936 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$\boxed{V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 87.44 \cdot kip}$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 93.945 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 18 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 195.75 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 48$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 195.75 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 195.75 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 13.4 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 1.137 \times 10^5 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 323.053 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.9 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

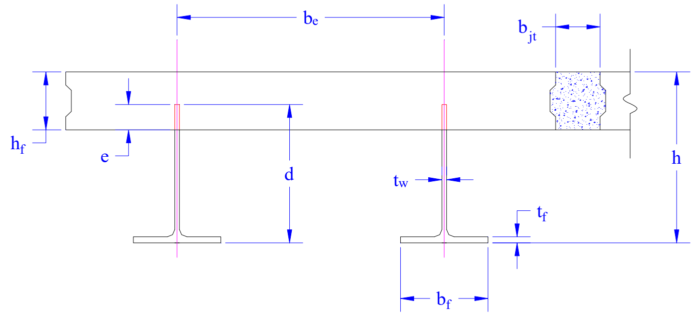
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 40ft$ Span length

$b_e := 34in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21\cdot in$ depth of built up steel section

$h_w := d - t_f = 20.5\cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012kip\cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2\cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1\cdot t_w = 1$

$$y\_bar := \frac{t_f\cdot b_f\cdot\frac{t_f}{2} + h_w\cdot t_w\cdot\left(t_f + \frac{h_w}{2}\right)}{t_f\cdot b_f + h_w\cdot t_w} = 6.147\cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f\cdot t_f^3}{12} + b_f\cdot t_f\cdot\left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w\cdot h_w^3}{12} + t_w\cdot h_w\cdot\left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 640.875\cdot in^4$$ MOI of steel

$A_{stl} := t_f\cdot b_f + h_w\cdot t_w = 13.687\cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \dfrac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 334 \cdot \text{in}$ $W = 27.833\ \text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 24.458\ \text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.609 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700 \dfrac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \dfrac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.734 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \dfrac{\text{lbf}}{\text{ft}^3} = 0.047 \cdot \dfrac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.308 \cdot \dfrac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \dfrac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 23.234 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\dfrac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.279 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.047 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.325 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.073 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.066 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.489 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.25 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.417 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

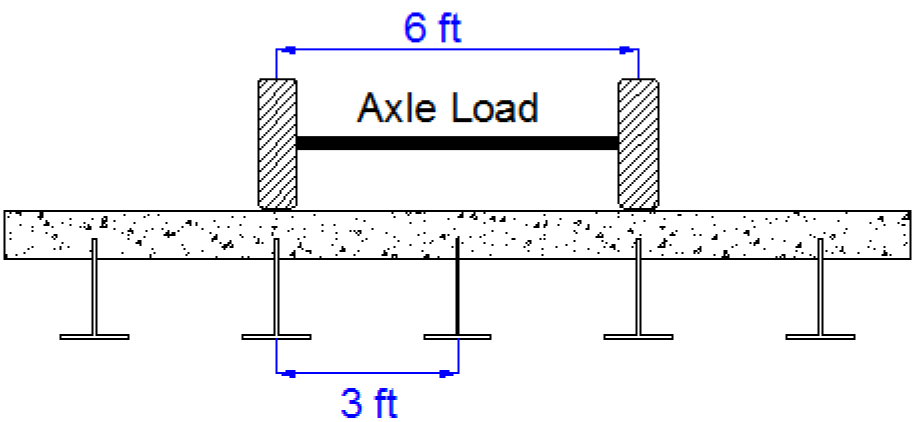
$$M_{DL} := \frac{DL \cdot L^2}{8} = 65.044 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 97.897 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 15.103 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 30103.16 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{{b_f}^3 \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 6.212 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

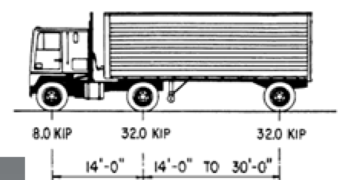
Design truck load in AASHTO (HS20 Truck) is used for this example.

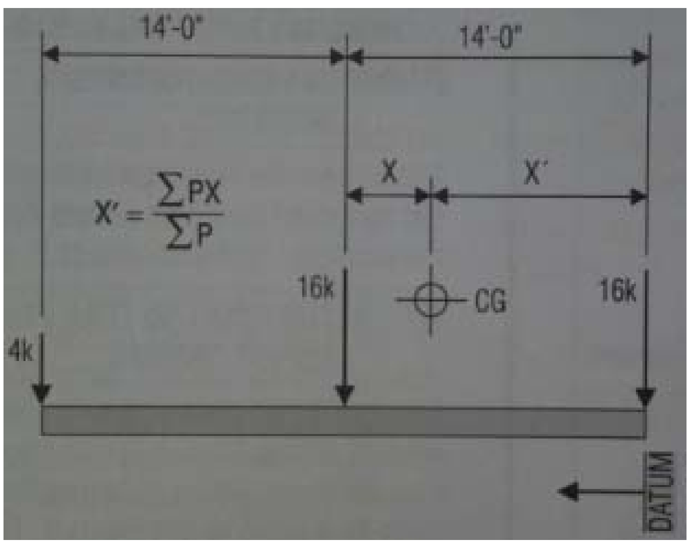
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

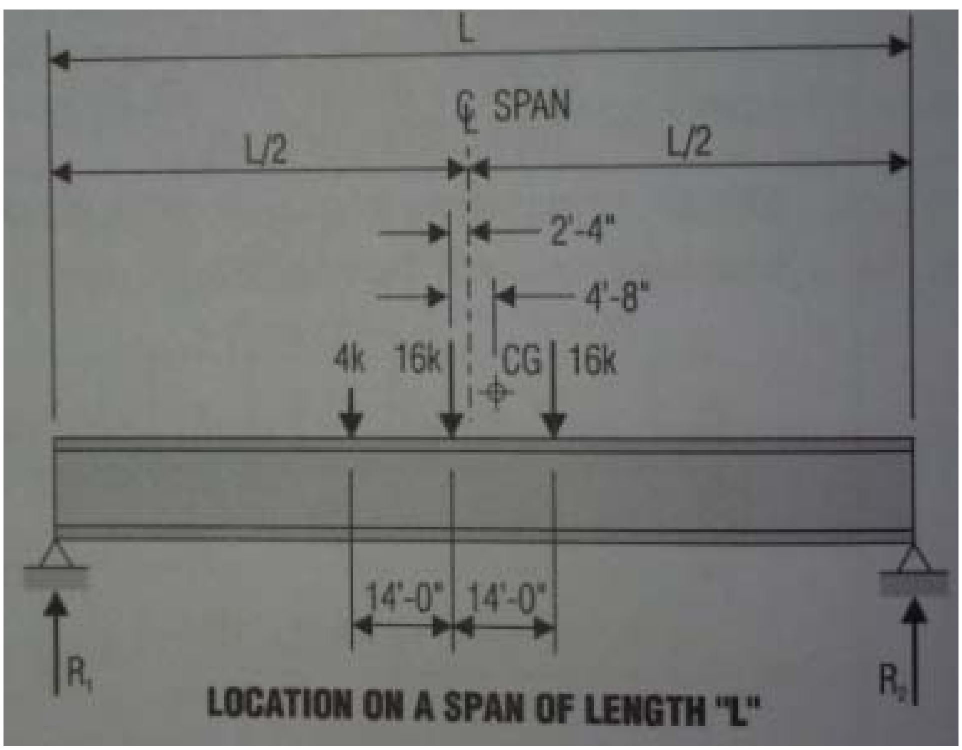
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2} + \frac{X}{2}\right) + F_T\cdot\left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.688\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2} - \frac{X}{2}\right) = 564.063\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1 + I) = 225.061\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

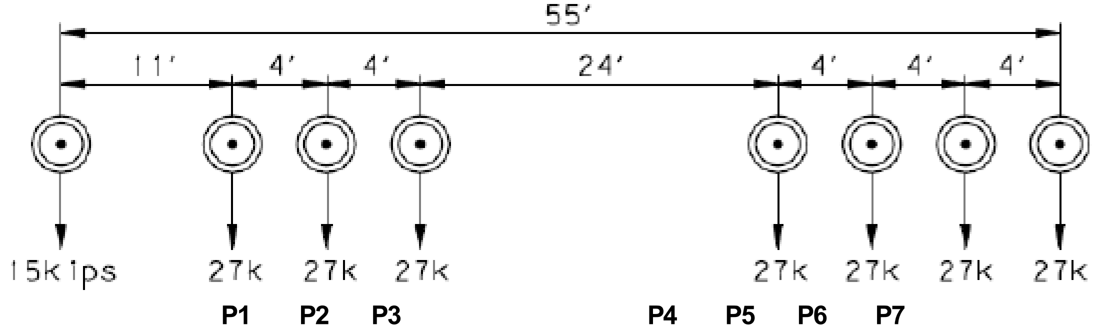
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 38.4\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 640.875 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 31.875 \cdot$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^3}{12} = 149.414 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.713 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2974 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 16.713 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 10.625 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^3}{12} = 49.805 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.747 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2055 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 12.747 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.45 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.218}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 10928.73 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 10026.1 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -0.87 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.0348}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 9936.9 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 10874.5 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 835.5 \cdot kip \cdot ft \qquad Mn = 10026.1 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = -4.989 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.842 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 17.764 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 21.038 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 7.287 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 48.07 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 45.384 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 35.22 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 35.222 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.272 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.101 \cdot ksi$$

$$f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.304 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.409 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.88 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 689.2 \cdot kip \cdot ft$$

$$M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.714 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 649 \cdot kip \cdot ft$$

$$M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400 \, psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2112.32 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.194 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.101 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.292 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1917.1 \cdot psi$$

$$f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 161.214 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 177.971 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 228.151 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 489.679 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 717.83 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 172.279 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 583.823 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 613.552 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 835.504 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

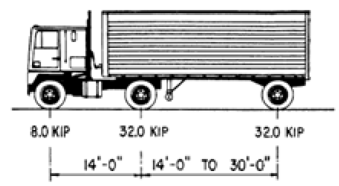
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.923 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

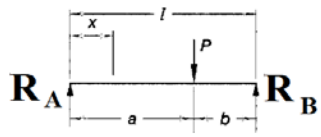
$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 10.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.021 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.79 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

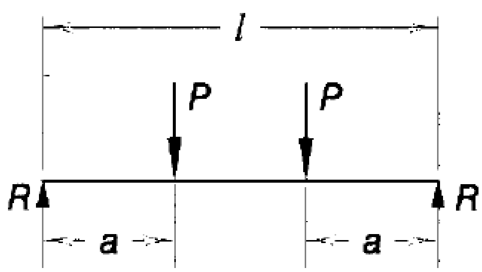
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.619 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.381 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.438 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.085 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.595 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.473}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 144.618 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.325 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.489 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 ***(at left support)*** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 34.75 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.379 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.059 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 6.504 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 9.79 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 98.83 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 53.484 \cdot kip$$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 95.019 \cdot kip$$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 98.83 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$$\frac{D}{t_w} = 56 \qquad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$$

Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.4 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.592 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.861 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.718 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

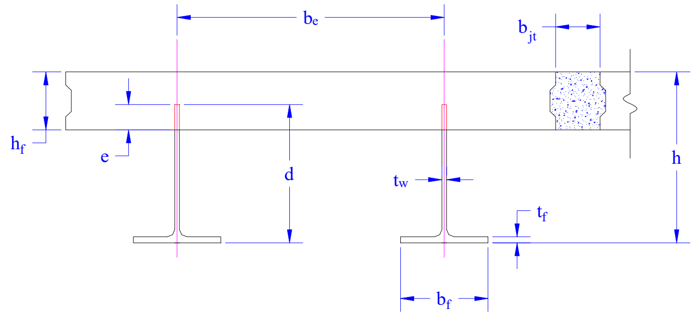
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 40ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.147 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 640.875 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 13.687 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 354 \cdot \text{in}$ $W = 29.5\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 26.125\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.766 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.784 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.047 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.323 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 24.484 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.295 \cdot \frac{kip}{ft}$ Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.047 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.342 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.078 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.498 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

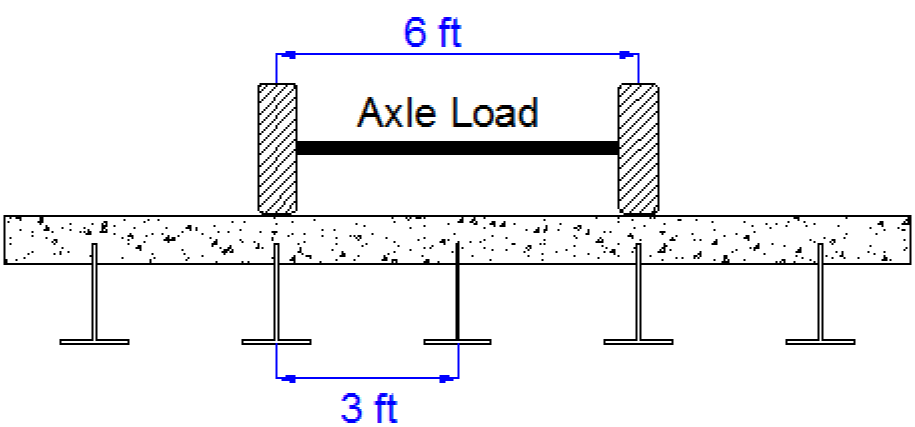
$$M_{DL} := \frac{DL \cdot L^2}{8} = 68.378 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 99.675 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 15.103 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{2}\right) = 30103.16 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 6.212 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

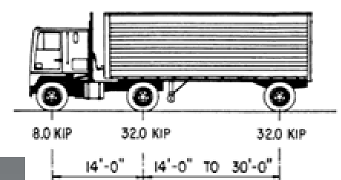
Design truck load in AASHTO (HS20 Truck) is used for this example.

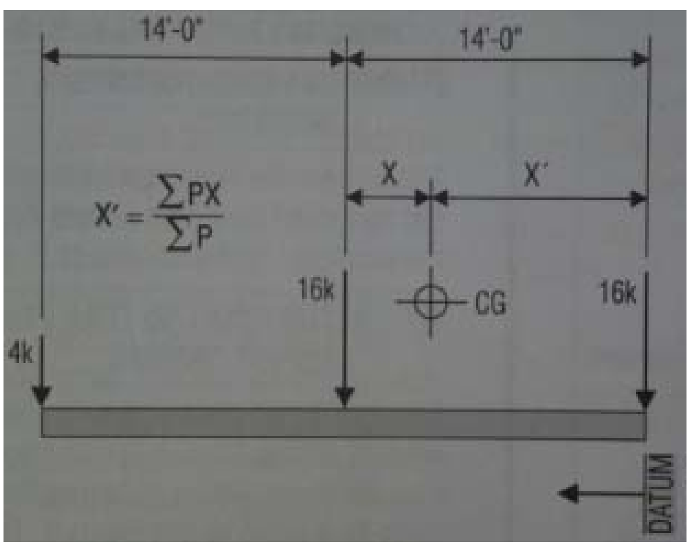
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

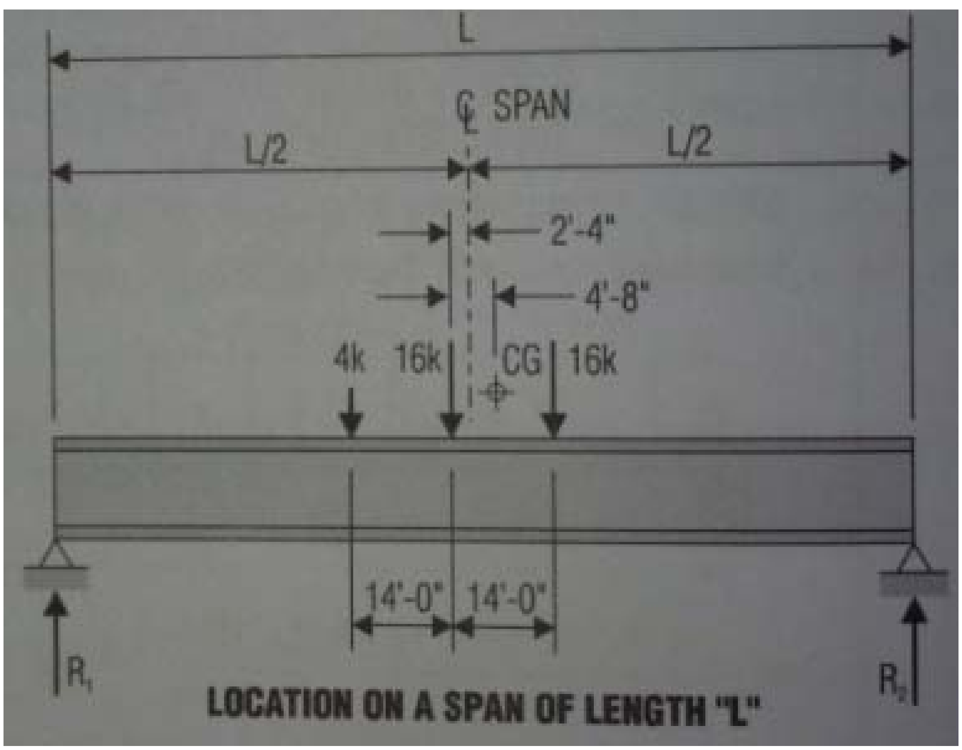
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.688 \cdot \text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 564.063 \cdot \text{kip} \cdot \text{ft}$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063 \cdot \text{kip} \cdot \text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 225.061 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

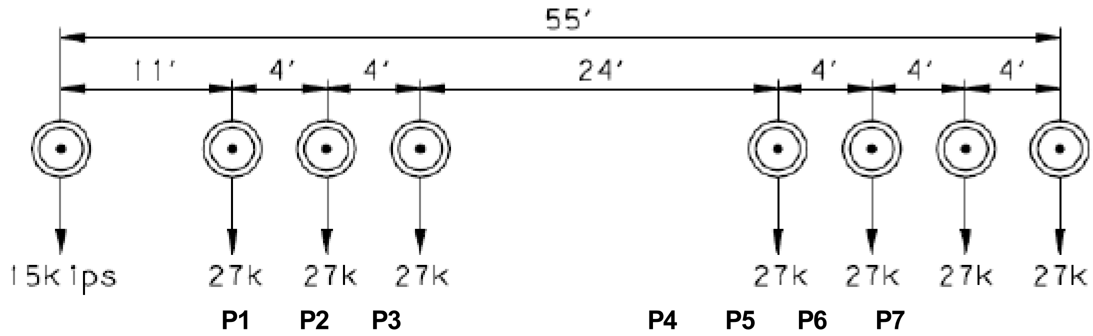
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \frac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 38.4 \cdot \text{kip} \cdot \text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 640.875 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot in^2$ $\quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 158.203 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.892 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3020 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 16.892 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$ $\quad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 52.734 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.961 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2102 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 12.961 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.218 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.209}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h - d)]] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 11025.42 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 10186.3 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -2.23 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.0892}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h - d)]] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 9666.6 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 10946.8 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 848.9 \cdot kip \cdot ft \qquad Mn = 10186.3 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -6.382\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.059\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.682\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 20.941\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 7.375\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 48.33\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 45.656\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 35.42\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 35.421\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.275\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 1.061\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 1.256\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.401\cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.803\cdot ksi$

$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 696\cdot kip\cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.642\cdot ksi$

$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 656\cdot kip\cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6\cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 2055.81\cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.196\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 1.061\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.285\cdot ksi$$

$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1860.38\cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 162.188 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 178.796 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 234.985 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 485.937 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 720.921 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 173.021 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 590.657 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 620.514 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 848.859 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

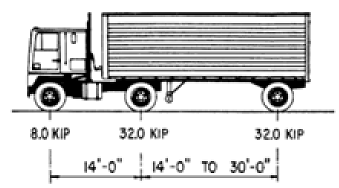
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.894 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

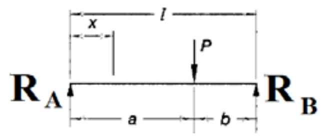
$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 10.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.021 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.778 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

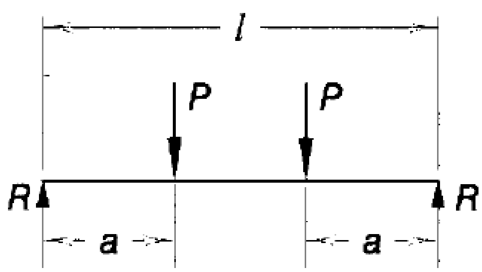
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.61 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.375 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\, EI}\left(3 l^2 - 4 a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.431 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.084 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.586 \cdot in$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.48}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 147.073 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.342 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.498 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 35.24 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.905 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.144 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 6.838 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 9.967 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 100.584 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 54.238 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 96.719 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 100.584 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.4 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\ 3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.592 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.861 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.718 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

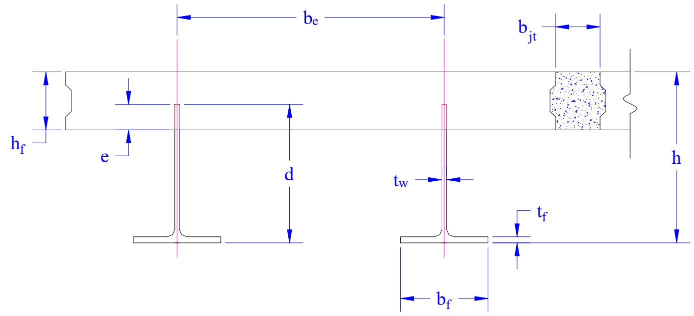
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $modules := \frac{N}{2} = 6$

$L := 40ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $t_f := 0.5in$ $b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.147 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 640.875 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 13.687 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 426 \cdot in$ $W = 35.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 32.125\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.328 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.964 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.047 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.324 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 24.484 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.296 \cdot \frac{kip}{ft}$ Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.047 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.343 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.08 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.5 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

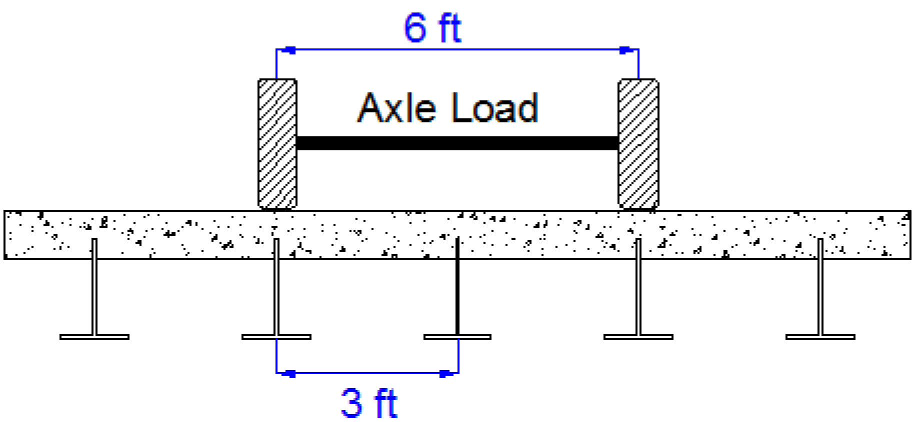
$$M_{DL} := \frac{DL \cdot L^2}{8} = 68.534 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 100.062 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 15.103 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 30103.16 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 6.212 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

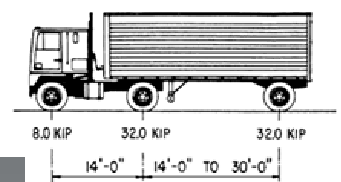
Design truck load in AASHTO (HS20 Truck) is used for this example.

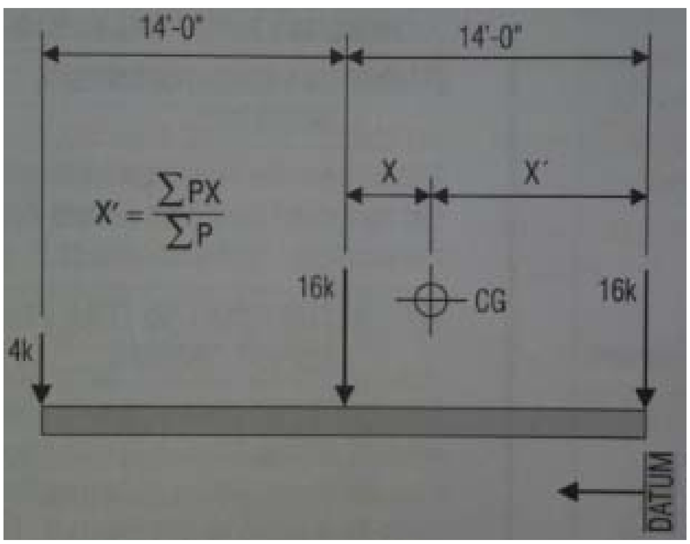
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

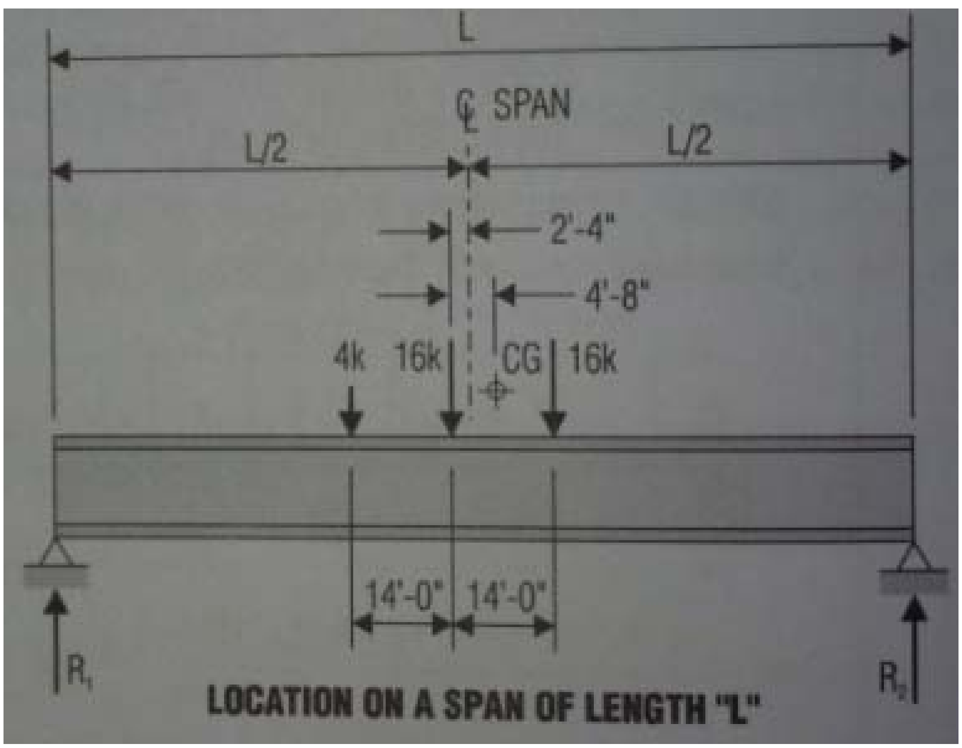
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2 \cdot ft$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.688 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 564.063 \cdot kip \cdot ft$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063 \cdot kip \cdot ft}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 225.061 \cdot kip \cdot ft$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

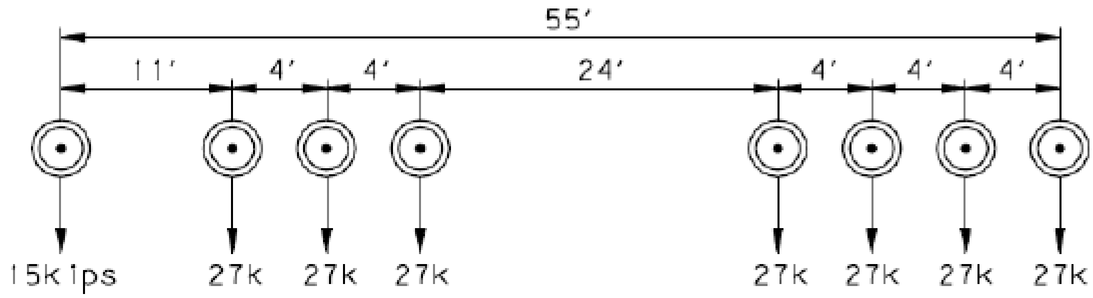
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64 \frac{kip}{ft} \cdot DF = 0.192 \cdot \frac{kip}{ft}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 38.4 \cdot kip \cdot ft$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461 \cdot kip \cdot ft}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 640.875 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot in$ $I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 158.203 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.892 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3020 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 16.892 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$ $I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 52.734 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.961 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2102 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 12.961 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.218 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.209}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 11025.42 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 10186.3 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -2.23 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.0892}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 9666.6 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 10946.8 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 848.9 \cdot kip \cdot ft \qquad Mn = 10186.3 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -6.382\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.071\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.682\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 20.941\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 7.403\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 48.388\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 45.714\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 35.46\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 35.461\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.276 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.061 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.256 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.403 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.806 \cdot ksi$$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 696.8 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.645 \cdot ksi$$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 657 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2058 \cdot psi$ $\qquad$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.196 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.061 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.287 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1861.94 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 162.188 \cdot in^3$ $\quad$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 178.796 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 235.761 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 485.081 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 720.842 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 173.002 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 591.433 \cdot kip \cdot ft$ $\quad$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 621.287 \cdot kip \cdot ft$ $\quad$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 848.859 \cdot kip \cdot ft$ $\quad$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.167$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

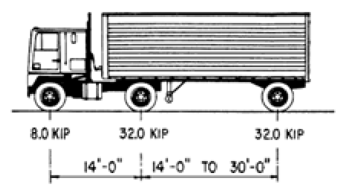
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.894 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

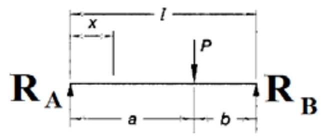
$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 10.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support



$$\Delta1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.021 \cdot in$$

Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.778 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

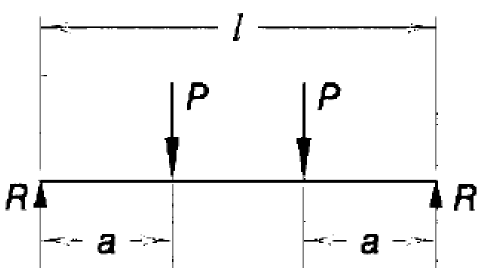
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.61 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta1 + \Delta2 + \Delta3) \cdot DF_d \cdot (1 + I) = 0.312 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.359 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 106.667 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.07 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.488 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$DF_s := 0.36 + \frac{S}{25} = 0.48$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 147.073 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.343 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.5 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$X_{ASr} := 14ft$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 35.24 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.905 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.144 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 6.853 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 10.006 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 100.662 \cdot kip}$$

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Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 54.238 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 96.797 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 100.662 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.4 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.592 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.861 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.718 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

Summary of Calculations:

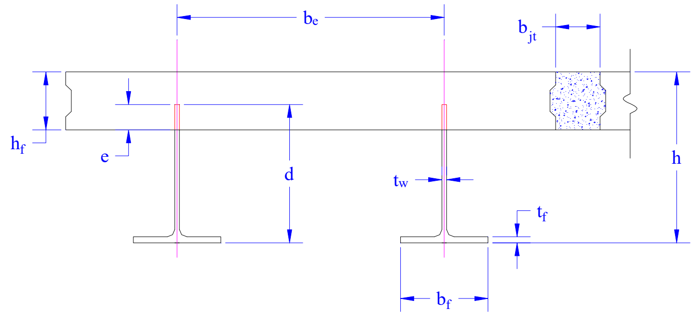
1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  - AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  - PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:

Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$



$L := 40ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.75in$ $\quad b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 5.18 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 713.995 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 16.594 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 374 \cdot in$ $W = 31.167\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 27.792\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.922 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.834 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.056 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.349 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 26.526 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \mathrm{floor}\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.312 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.056 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.368 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.083 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.507 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

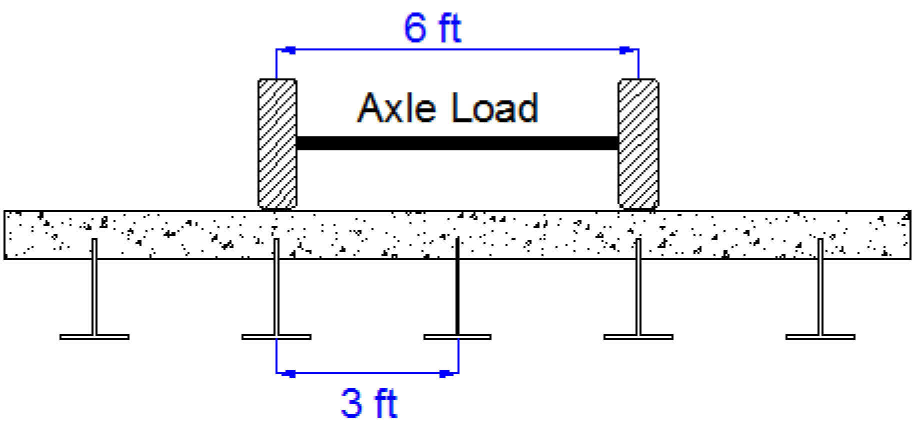
$$M_{DL} := \frac{DL \cdot L^2}{8} = 73.689 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 101.453 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.167$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 16.07 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\ 2}\right) = 39993.61 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 4.142 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

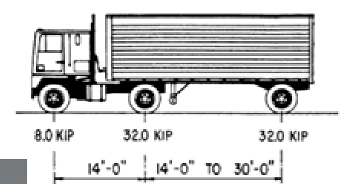
Design truck load in AASHTO (HS20 Truck) is used for this example.

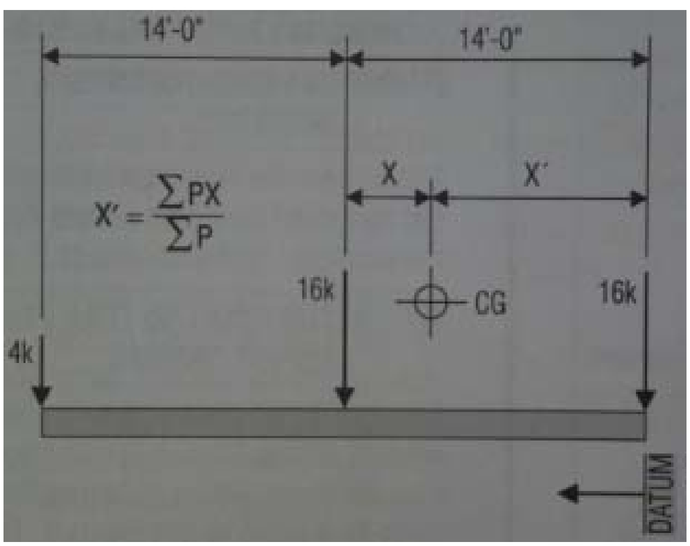
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

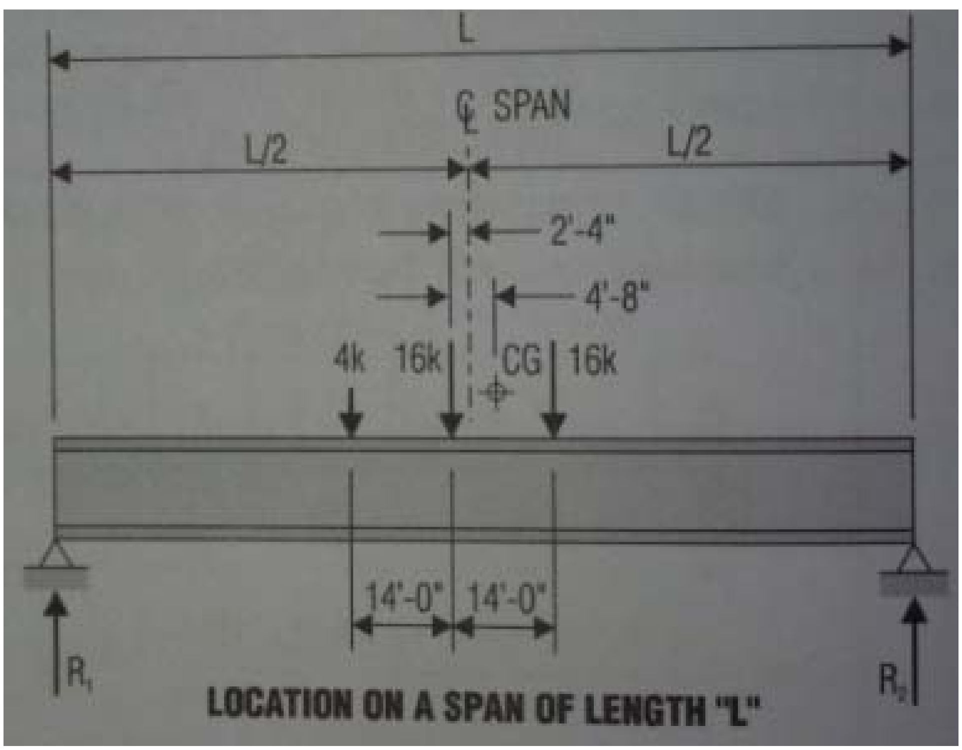
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.688 \cdot \text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 564.063 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 225.061 \cdot \text{kip} \cdot \text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

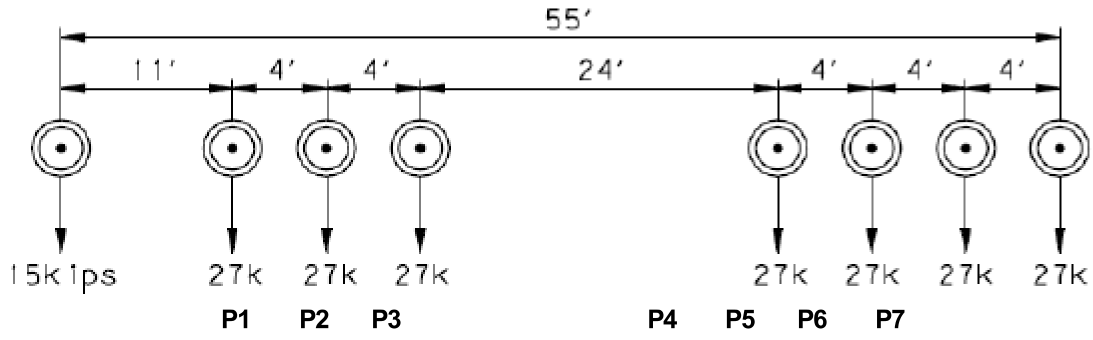
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64 \frac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 38.4 \cdot \text{kip} \cdot \text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 713.995 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.143 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3804 \cdot in^4$$ Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 16.143 \cdot in$$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.883 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2557 \cdot in^4$$ Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 11.883 \cdot in$$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$

$Dp1 := root(Fp(D_p), D_p) = 5.877 \cdot in$ Must be less than $h_f = 7.5 \cdot in$ $\quad \frac{Dp1}{h} = 0.235$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$Mp1 = 13865.55 \cdot kip \cdot in \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 12554.5 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$Dp2 := root(Fp2(D_p), D_p) = 0.285 \cdot in$ Must be more than $h_f = 7.5 \cdot in$ $\quad \frac{Dp2}{h} = 0.0114$ must be < 0.42

$M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0 \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$Mp2 = 13109.06 \cdot kip \cdot in \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 13922.1 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1046.2 \cdot kip \cdot ft \qquad Mn = 12554.5 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = -3.853 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.109 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.415 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 15.887 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.658 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 37.1 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.071 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 27.21 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.207 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.257 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.92 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.09 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.354 \cdot ksi$$

STRENGTH I

$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.463 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 705.3 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II

$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.324 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 666 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1807.56 \cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.189 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.92 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.26 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1645.17 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 215.187 \cdot in^3 \qquad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 235.667 \cdot in^3$$

short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 244.29 \cdot kip \cdot ft$$

From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 714.406 \cdot kip \cdot ft$$

Additional moment to cause nominal yielding of steel flange

$$My := M_{D2} + M_{AD} = 958.696 \cdot kip \cdot ft$$

D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$$S_{xt} := \frac{My}{F_y} = 230.087 \cdot in^3$$

Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 599.962 \cdot kip \cdot ft$$

Moment due to Strength V limit state

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 626.432 \cdot kip \cdot ft$$

Equation 6.10.7.1.7-1

$$Mn = 1.046 \times 10^3 \cdot kip \cdot ft \qquad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

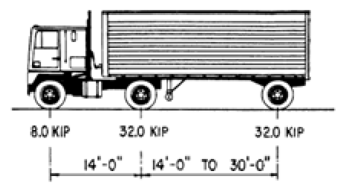
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.504 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot ft$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

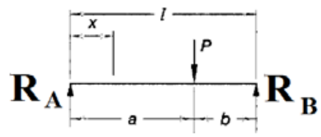
$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot ft$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 10.667 \cdot ft$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.017 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.618 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

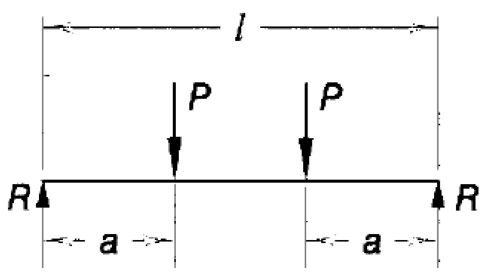
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.484 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.298 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.342 \cdot in$$

(Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.067 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.465 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 181.923 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.368 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.507 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 35.729 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.431 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.229 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 7.369 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 10.145 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 102.585 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

<u>Strength II Max Positive Shear:</u>

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 54.991 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 98.667 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 102.585 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.829 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.879 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.735 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

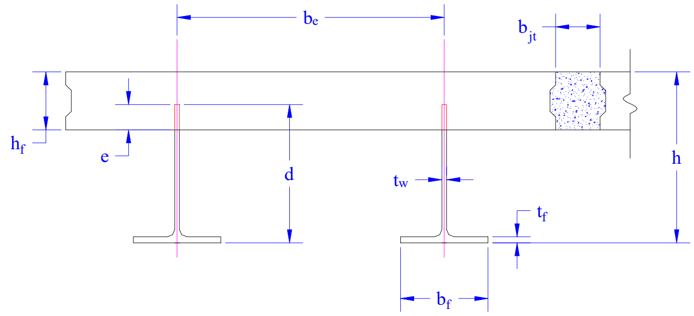
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $modules := \frac{N}{2} = 7$

$L := 40ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $t_f := 0.75in$ $b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 5.18 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 713.995 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 16.594 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\dfrac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 526 \cdot in$ $W = 43.833\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 40.458\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 4.109 \cdot \dfrac{kip}{ft}$ DL due to slab

$D_b := .700\dfrac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\dfrac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.214 \cdot \dfrac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\dfrac{lbf}{ft^3} = 0.056 \cdot \dfrac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.35 \cdot \dfrac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\dfrac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 26.526 \cdot kip}$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \mathrm{floor}\left(\dfrac{W_{lane}}{12ft}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.313 \cdot \frac{kip}{ft}$ Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.056 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.37 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.087 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.511 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*

(Based on first interior girder demands)

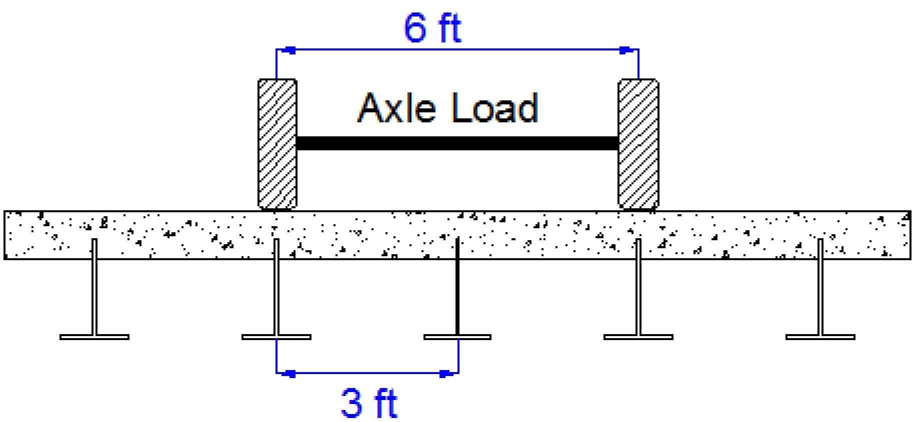
$$M_{DL} := \frac{DL \cdot L^2}{8} = 73.957 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 102.117 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 16.07 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 39993.61 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 4.142 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

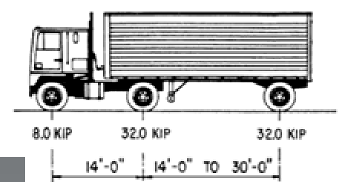
Design truck load in AASHTO (HS20 Truck) is used for this example.

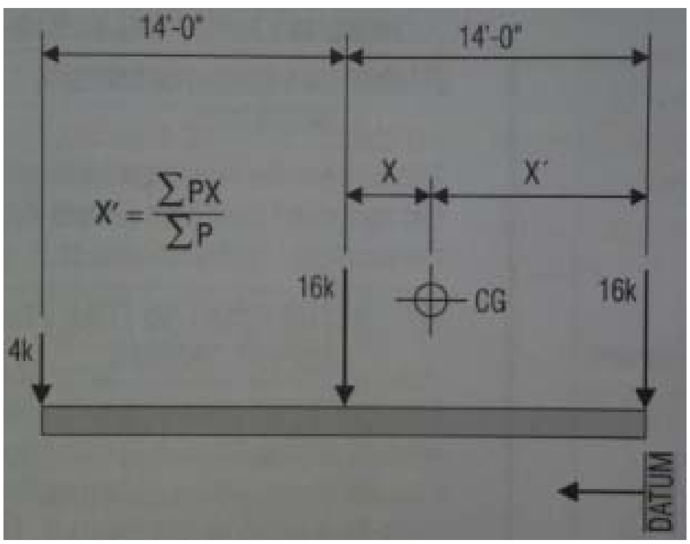
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

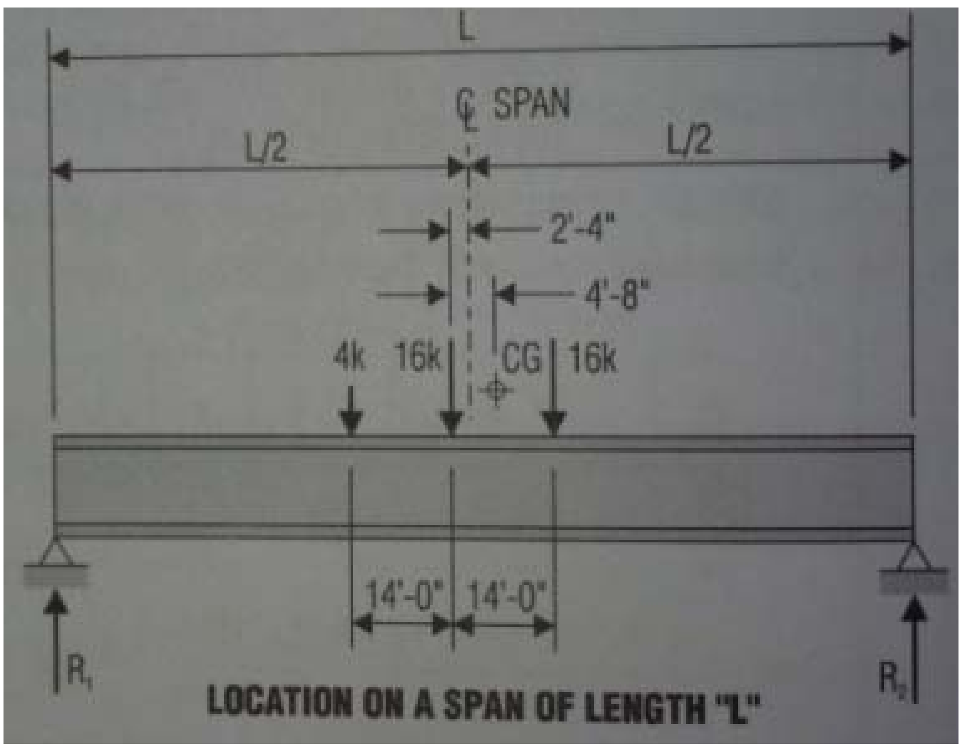
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.688 \cdot \text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 564.063 \cdot \text{kip} \cdot \text{ft}$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063 \cdot \text{kip} \cdot \text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 225.061 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

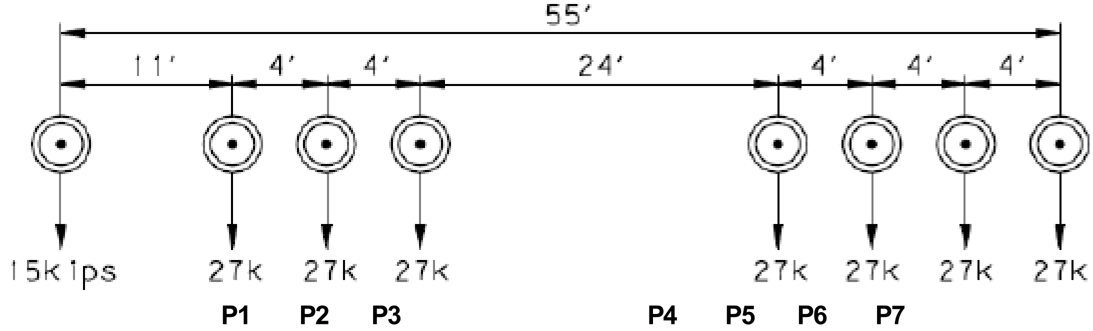
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \frac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 38.4 \cdot \text{kip} \cdot \text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 713.995 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^3}{12} = 166.992 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.143 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3804 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 16.143 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2$ $\quad I_{tr3} := \frac{b_{tr3} \cdot h_f^3}{12} = 55.664 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 11.883 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2557 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 11.883 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.877 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.235}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 13865.55 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 12554.5 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 0.285 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.0114}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 13109.06 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 13922.1 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1046.2 \cdot kip \cdot ft \qquad Mn = 12554.5 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -3.853\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 4.124\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.415\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 15.887\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.695\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.174\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 35.145\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.26\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.259\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.258 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.92 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.09 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.357 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.468 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 706.7 \cdot kip \cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.329 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 667 \cdot kip \cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1810.82 \cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.19 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.92 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.262 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1647.57 \cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 215.187 \cdot in^3$ $\quad$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 235.667 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 245.621 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 712.948 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 958.57 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 230.057 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 601.294 \cdot kip \cdot ft$ $\quad$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 627.76 \cdot kip \cdot ft$ $\quad$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 1.046 \times 10^3 \cdot kip \cdot ft$ $\quad$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

<u>Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)</u>

$$DF_d := \frac{N_l}{N} = 0.214$$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

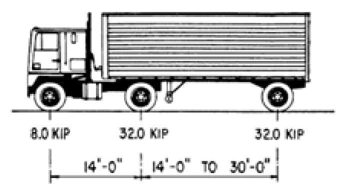
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.504 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot ft$$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

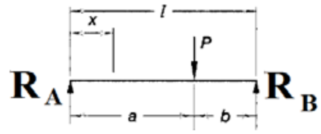
$$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot ft$$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$$b3 := \frac{L}{2} - X' = 10.667 \cdot ft$$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.017 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.618 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

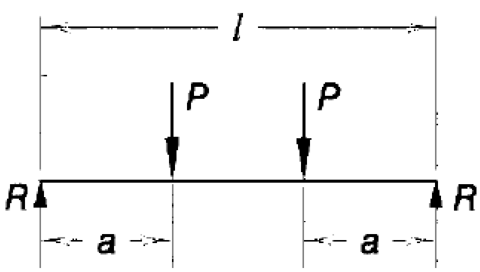
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.484 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.319 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.367 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.072 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.498 \cdot in}$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$DF_s := 0.36 + \frac{S}{25} = 0.487$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 181.923 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.37 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.511 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$X_{ASr} := 14ft$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 35.729 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.431 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.229 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 7.396 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 10.212 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 102.719 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 54.991 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 98.8 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 102.719 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.829 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.879 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.735 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

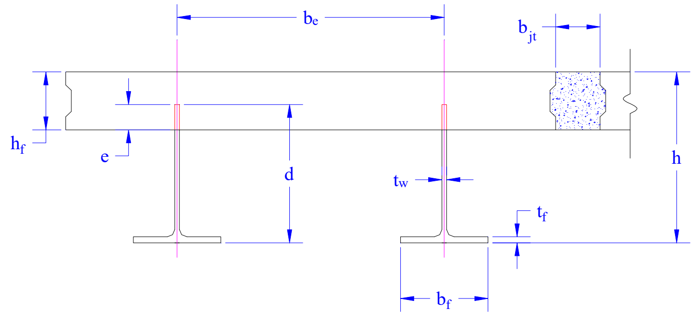
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 40ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.75in$ $\quad b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 5.18 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 713.995 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 16.594 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 394 \cdot \text{in}$ $W = 32.833\text{ ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 29.458\text{ ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.078 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.884 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.056 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.364 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 27.776 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$ Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.056 \cdot \frac{kip}{ft}$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.385 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.088 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.516 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

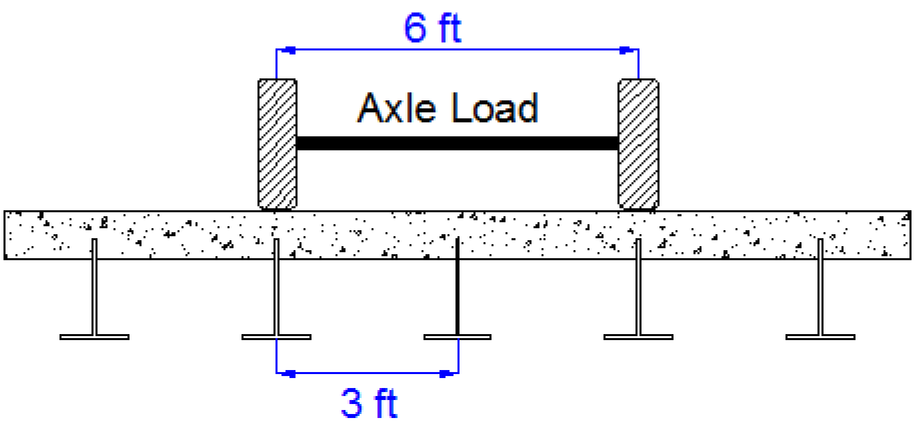
$$M_{DL} := \frac{DL \cdot L^2}{8} = 77.022 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 103.231 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 16.07 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 39993.61 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 4.142 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

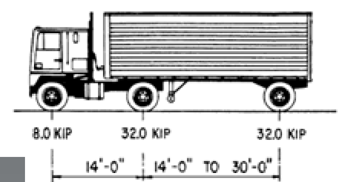
Design truck load in AASHTO (HS20 Truck) is used for this example.

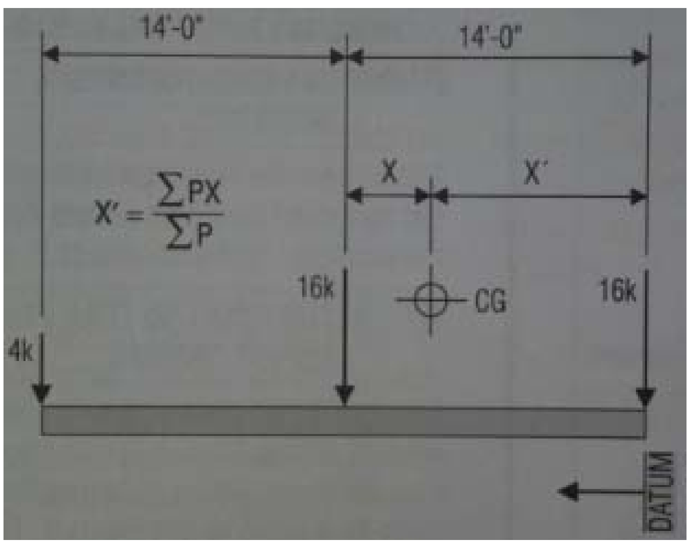
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot \left(X_{ASr} + X_{ASf}\right) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

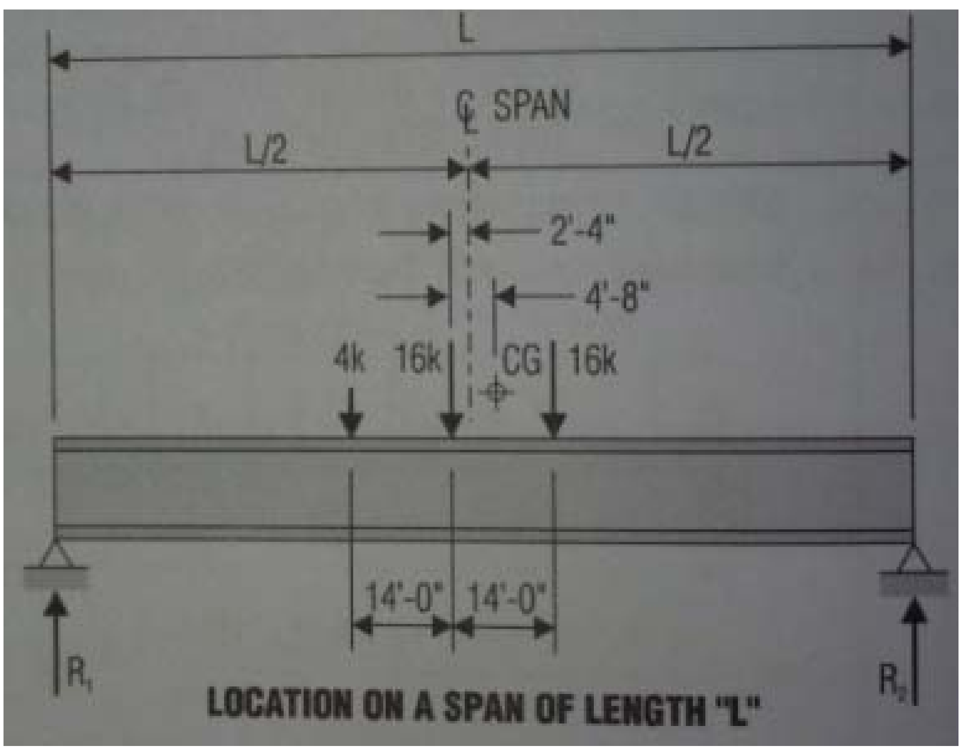
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.688\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 564.063\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 225.061\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

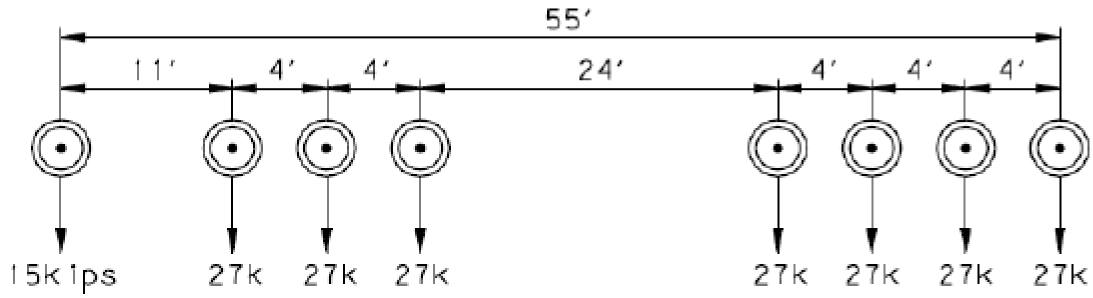
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 38.4\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $$\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461\cdot\text{kip}\cdot\text{ft}}$$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



P1 P2 P3 P4 P5 P6 P7

NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 713.995 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $\quad I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.32 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3860 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 16.32 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $\quad I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.084 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2614 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 12.084 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f

$$h - d = 4 \cdot in$$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.647 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.226}$ must be < 0.42

$$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 13978.38 \cdot kip \cdot in}$

$$M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 12746.8 \cdot kip \cdot in$$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$$D_p := 2.5in$$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -1.075 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.043}$ must be < 0.42

$$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 12897.67 \cdot kip \cdot in}$

$$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 14188.7 \cdot kip \cdot in$$

$$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1062.2 \cdot kip \cdot ft \qquad Mn = 12746.8 \cdot kip \cdot in$$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1\right) = -5.263 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.273 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.366 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 15.829 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.727 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 37.323 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.302 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 27.38 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.376 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.26 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.889 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.052 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.348 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.402 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 712.2 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.267 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 672 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1762.98 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.19 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.889 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.255 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1600.43 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 216.287 \cdot in^3$ $\quad$ $S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 236.542 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 251.124 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}} \right) \cdot S_{ST} = 710.95 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 962.073 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 230.898 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 606.796 \cdot kip \cdot ft$ $\quad$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 633.359 \cdot kip \cdot ft$ $\quad$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 1.062 \times 10^3 \cdot kip \cdot ft$ $\quad$ $M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

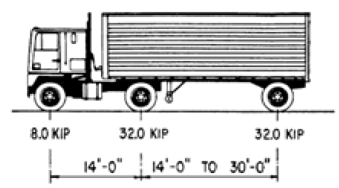
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.482 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot \text{ft}$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

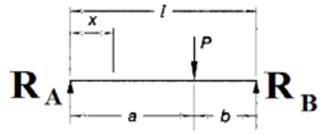
$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot \text{ft}$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 10.667 \cdot \text{ft}$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.016 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.609 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

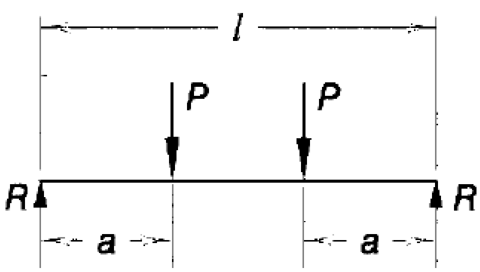
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.477 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta1 + \Delta2 + \Delta3) \cdot DF_d \cdot (1 + I) = 0.293 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.337 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.066 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.458 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 184.86 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.385 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.516 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 36.219 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.958 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.315 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 7.702 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 10.323 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 104.339 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 55.744 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 100.367 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 104.339 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56 \qquad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.829 \times 10^4 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.879 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.735 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

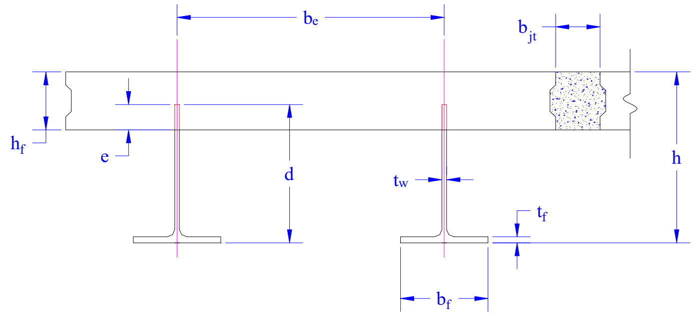
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $modules := \frac{N}{2} = 6$

$L := 40ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $t_f := 0.75in$ $b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 5.18 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 713.995 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 16.594 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 474 \cdot \text{in}$ $W = 39.5\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 36.125\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.703 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.084 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.056 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.365 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 27.776 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.056 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.386 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.09 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$

distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.518 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*

(Based on first interior girder demands)

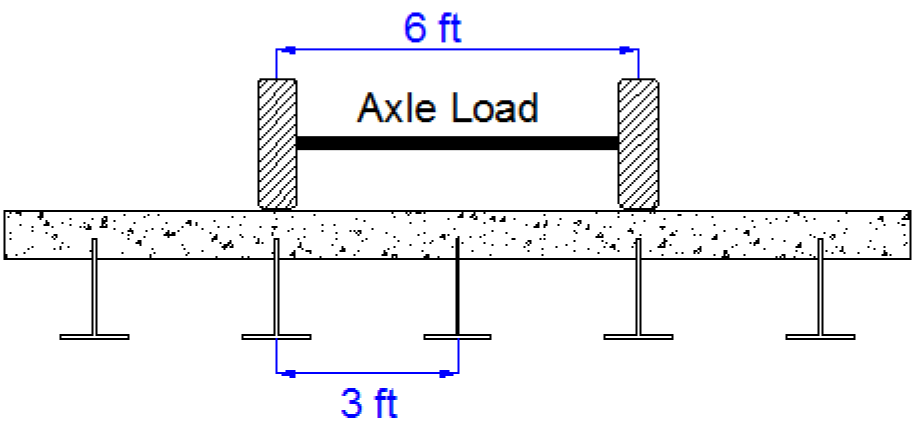
$$M_{DL} := \frac{DL \cdot L^2}{8} = 77.178 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 103.618 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 16.07 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 39993.61 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***<u>Per Lane</u>*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 17.5 \cdot in$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 4.142 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

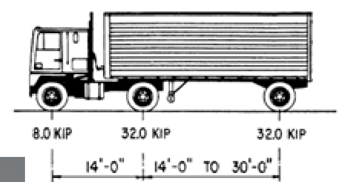
Design truck load in AASHTO (HS20 Truck) is used for this example.

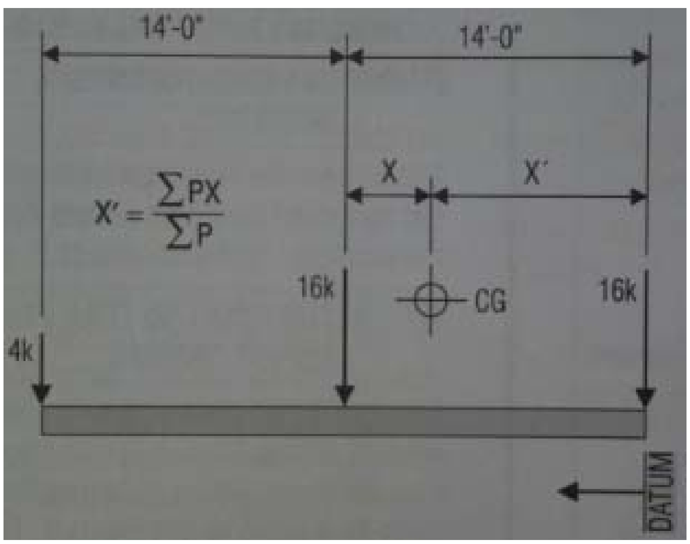
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

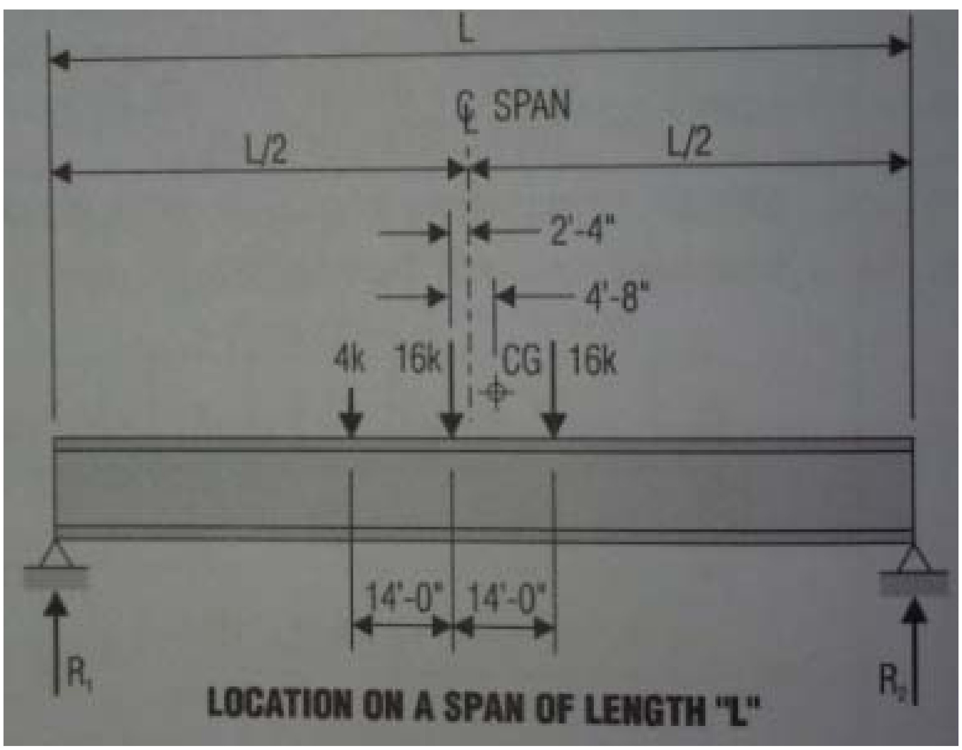
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2} + \frac{X}{2}\right) + F_T\cdot\left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.688\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2} - \frac{X}{2}\right) = 564.063\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1 + I) = 225.061\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

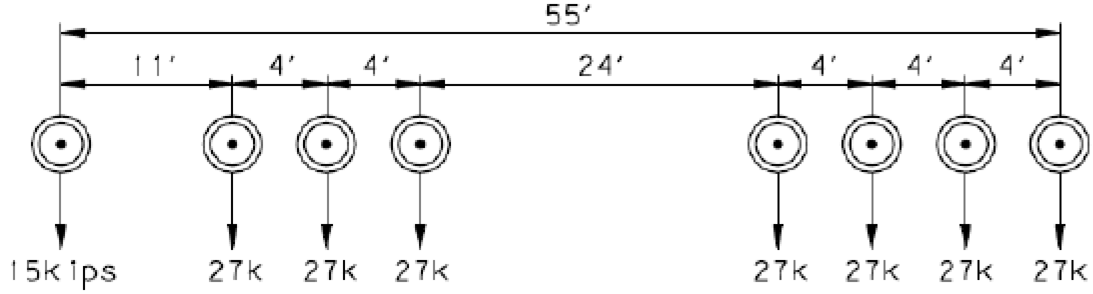
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 38.4\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



P1 **P2** **P3** **P4** **P5** **P6** **P7**

NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 713.995 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^3}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.32 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3860 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 16.32 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^3}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.084 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2614 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 12.084 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4\cdot in$

$$Fp(D_p) := 0.85\cdot f_c\cdot b_e\cdot D_p + F_y\cdot\left[t_w\cdot\left[D_p - (h-d)\right]\right] - F_y\cdot\left[A_{stl} - \left[t_w\cdot\left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42\cdot h = 10.5\cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.647\cdot in}$ Must be less than $h_f = 7.5\cdot in$ $\boxed{\frac{Dp1}{h} = 0.226}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f)\cdot[(h-d) < Dp1]\cdot(Dp1 < 0.42\cdot h) = 1 \qquad Dp1 \le 0.42\cdot h = 1$

$$Mp1 := 0.85\cdot f_c\cdot b_e\cdot Dp1\cdot\left(\frac{Dp1}{2}\right) + F_y\cdot\left[t_w\cdot[Dp1 - (h-d)]\right]\cdot\frac{[Dp1-(h-d)]}{2} \ldots$$
$$+\left[F_y\cdot(b_f\cdot t_f)\cdot\left(h - Dp1 - \frac{t_f}{2}\right) + F_y\cdot\left[t_w\cdot(h - Dp1 - t_f)\right]\cdot\frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 13978.38\cdot kip\cdot in} \qquad M_{n1} := Mp1\cdot\left(1.07 - 0.7\cdot\frac{Dp1}{h}\right) = 12746.8\cdot kip\cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85\cdot f_c\cdot b_e\cdot h_f + F_y\cdot\left[t_w\cdot\left[D_p - (h-d)\right]\right] - F_y\cdot\left[A_{stl} - \left[t_w\cdot\left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -1.075\cdot in}$ Must be more than $h_f = 7.5\cdot in$ $\boxed{\frac{Dp2}{h} = -0.043}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f)\cdot(Dp2 < h)\cdot(Dp2 < 0.42\cdot h) = 0} \qquad Dp2 \le 0.42\cdot h = 1$

$$Mp2 := 0.85\cdot f_c\cdot b_e\cdot h_f\cdot\left(Dp2 - \frac{h_f}{2}\right) + F_y\cdot\left[t_w\cdot[Dp2 - (h-d)]\right]\cdot\frac{[Dp2-(h-d)]}{2} \ldots$$
$$+F_y\cdot(b_f\cdot t_f)\cdot\left(h - Dp2 - \frac{t_f}{2}\right) + F_y\cdot\left[t_w\cdot(h - Dp2 - t_f)\right]\cdot\frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 12897.67\cdot kip\cdot in}$

$M_{n2} := Mp2\cdot\left(1.07 - 0.7\cdot\frac{Dp2}{h}\right) = 14188.7\cdot kip\cdot in$

$Mn := max(M_{n1}\cdot M_{n1check}, M_{n2}\cdot M_{n2check}) = 1062.2\cdot kip\cdot ft \qquad Mn = 12746.8\cdot kip\cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -5.263\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 4.282\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.366\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 15.829\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.749\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.366\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 35.345\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.41\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.406\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.26 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.889 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.052 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.349 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.404 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 713 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.27 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 673 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1764.81 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.191 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.889 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.256 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1601.78 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 216.287 \cdot in^3$ $\quad$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 236.542 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 251.9 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 710.1 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 962 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 230.88 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 607.572 \cdot kip \cdot ft$ $\quad$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 634.133 \cdot kip \cdot ft$ $\quad$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 1.062 \times 10^3 \cdot kip \cdot ft$ $\quad$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.25$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

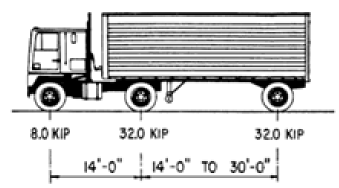
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.482 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

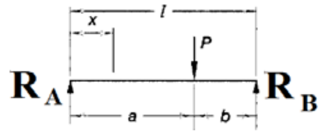
$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 10.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.016 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.609 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

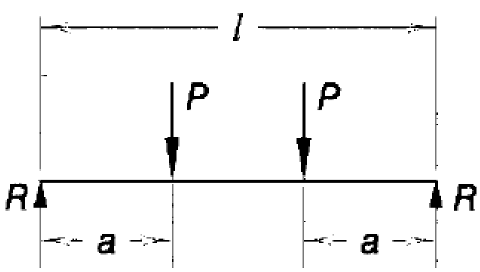
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.477 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.367 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.421 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 160 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.082 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.573 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 184.86 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.386 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.518 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 36.219 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 38.958 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.315 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 7.718 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 10.362 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1} \right) \right) = 104.417 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 55.744 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 100.445 \cdot kip$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 104.417 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.829 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.879 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.735 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

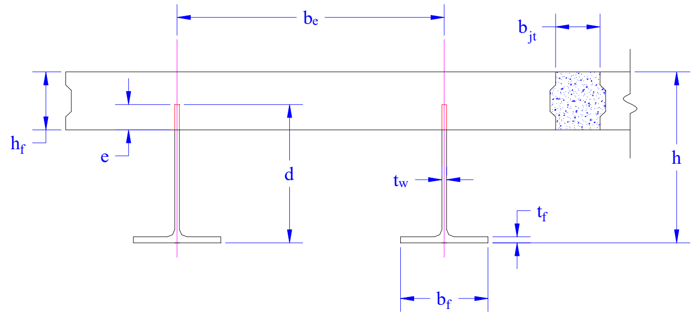
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $\quad modules := \frac{N}{2} = 7$

$L := 40ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.75in$ $\quad b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 312.012 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 94.1625 kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 5.18 \cdot in$$

Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 713.995 \cdot in^4$$

MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 16.594 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel	$E_s := 29000\text{ksi}$	
Yield Strength of Steel	$F_y := 50\text{ksi}$	
Yield Strength of Reinforcement	$f_y := 60\text{ksi}$	
Compressive Strength of Concrete	$f_c := 4000\text{psi}$	AAAP PennDOT Concrete
Concrete Modular Ratio	$n := 8$	modular ratio for 4ksi concrete per AASHTO
Unit weight of concrete slab	$w_c := 0.150 \dfrac{\text{kip}}{\text{ft}^3}$	Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 554 \cdot \text{in}$ $W = 46.167\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 42.792\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 4.328 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700 \dfrac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \dfrac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.284 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \dfrac{\text{lbf}}{\text{ft}^3} = 0.056 \cdot \dfrac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.366 \cdot \dfrac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \dfrac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 27.776 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\dfrac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.33 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.056 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.386 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.092 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.519 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

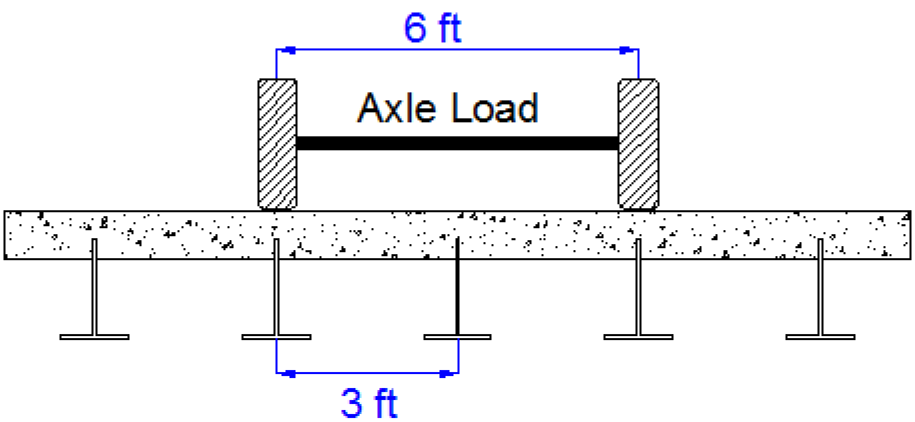
$$M_{DL} := \frac{DL \cdot L^2}{8} = 77.29 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 103.895 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 16.07 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 39993.61 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 6.212 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 4.142 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

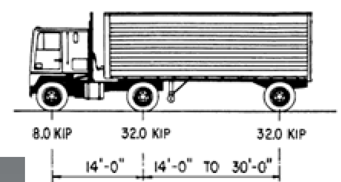
Design truck load in AASHTO (HS20 Truck) is used for this example.

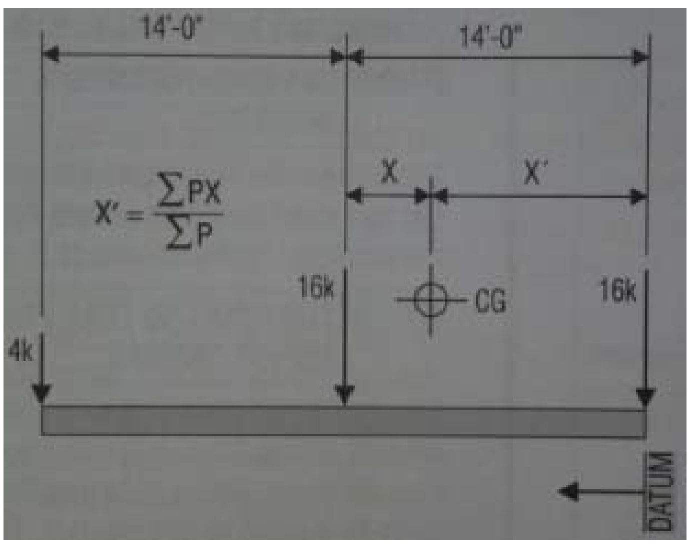
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

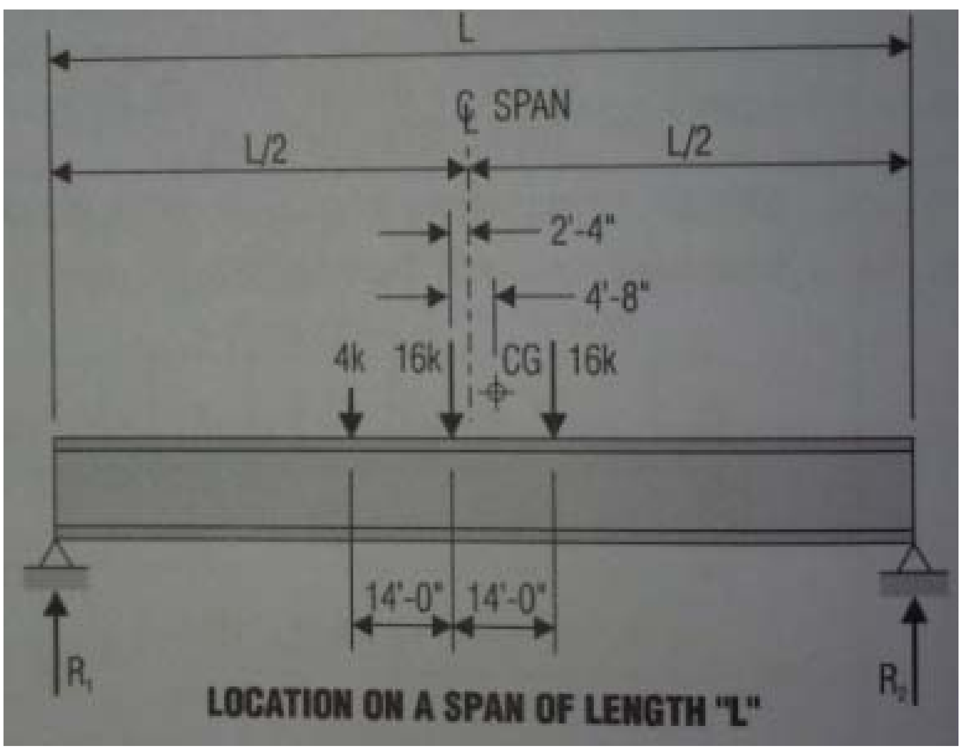
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.8 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 449.8 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right) + F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.688\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 564.063\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 564.063\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 225.061\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

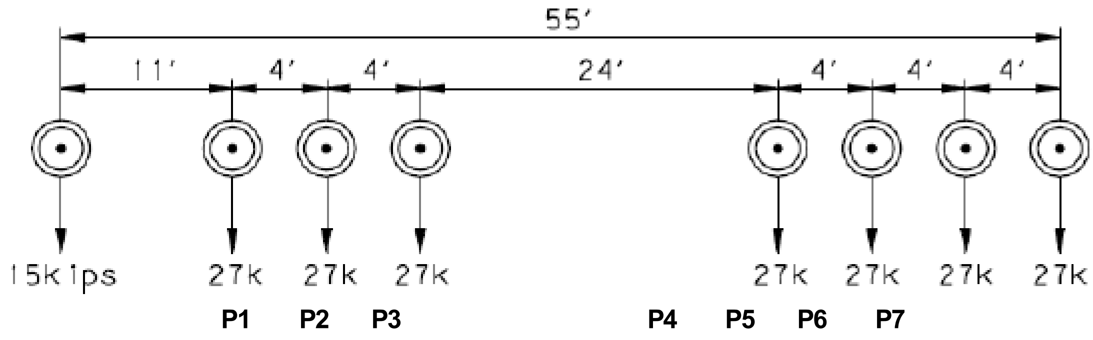
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\,\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 38.4\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 263.461\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 713.995 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2 \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.32 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3860 \cdot in^4$$

Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 16.32 \cdot in$$

N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.084 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2614 \cdot in^4$$

Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 12.084 \cdot in$$

N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.647 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.226}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 13978.38 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 12746.8 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = -1.075 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = -0.043}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 12897.67 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 14188.7 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1062.2 \cdot kip \cdot ft \qquad Mn = 12746.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = -5.263 \cdot in$ $D_{cp0} := D_{cp} > 0 = 0$

$D_{cp} := D_{cp} \cdot D_{cp0} = 0 \cdot in$ $\frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$ Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.288 \cdot ksi$

$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.366 \cdot ksi$ $f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 15.829 \cdot ksi$

$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.764 \cdot ksi$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 37.397 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.375 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 27.43 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$f_f + \frac{f_{lSII}}{2} = 27.428 \cdot ksi$ $0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi$ $f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.261 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.889 \cdot ksi$$

$$f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.052 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.35 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.406 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 713.5 \cdot kip \cdot ft$$

$$M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.272 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 674 \cdot kip \cdot ft$$

$$M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1766.12 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.191 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.889 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.257 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1602.74 \cdot psi$$

$$f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 216.287 \cdot in^3$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 236.542 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 252.455 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 709.494 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 961.948 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 230.868 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 608.127 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 634.686 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $Mn = 1.062 \times 10^3 \cdot kip \cdot ft$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

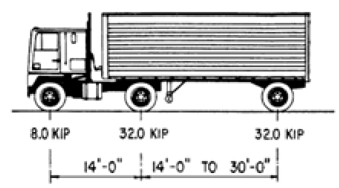
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.482 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 1.333 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

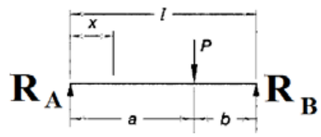
$b2 := \frac{L}{2} - (X_{ASr} - X') = 15.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 10.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.016 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.609 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

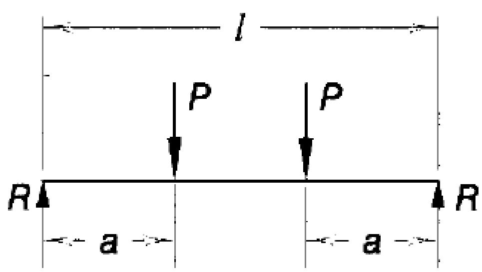
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.477 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.314 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.361 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.071 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.491 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.6 \cdot in$ $\frac{L}{1000} = 0.48 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 184.86 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.386 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.519 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support)

(Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 12 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 26 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 40 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 55.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 36.219 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.375 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.958 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 12.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 6.315 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 7.729 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 10.389 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 104.472 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 55.744 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 100.5 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 104.472 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\ 3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.829 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.879 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.735 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

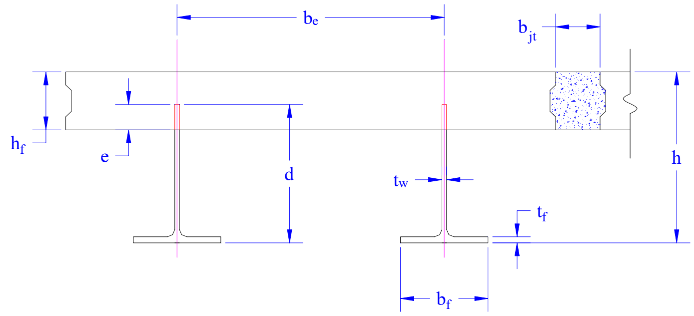
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad$ $modules := \frac{N}{2} = 5$

$L := 50ft$ Span length

$b_e := 34in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500in$ $\quad$ $t_f := 0.625in$ $\quad$ $b_f := 12in$

$h := 29in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 25 \cdot in$ depth of built up steel section

$h_w := d - t_f = 24.375 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 8.051 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.329 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 19.687 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 334 \cdot \text{in}$ $W = 27.833\text{ ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 24.458\text{ ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.609 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700 \frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.734 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{\text{lbf}}{\text{ft}^3} = 0.067 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.328 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 31.051 \cdot \text{kip}}$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.279 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.067 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.346 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.073 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$

distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.066 \cdot \frac{kip}{ft}$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.489 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.25 \cdot in$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.417 \cdot in$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

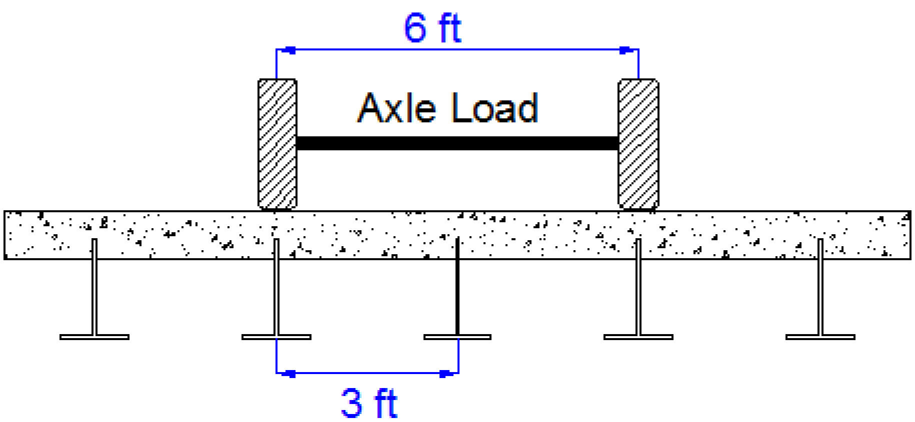
$$M_{DL} := \frac{DL \cdot L^2}{8} = 108.012 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 152.964 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 2.833$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 17.199 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 57224.49 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 21.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f{}^3 \cdot t_f}{12} = 90 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 9.54 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

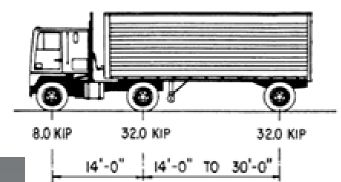
Design truck load in AASHTO (HS20 Truck) is used for this example.

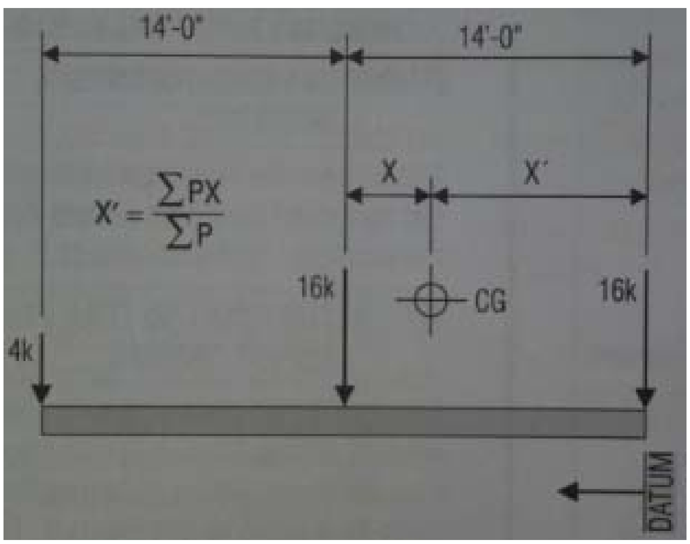
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

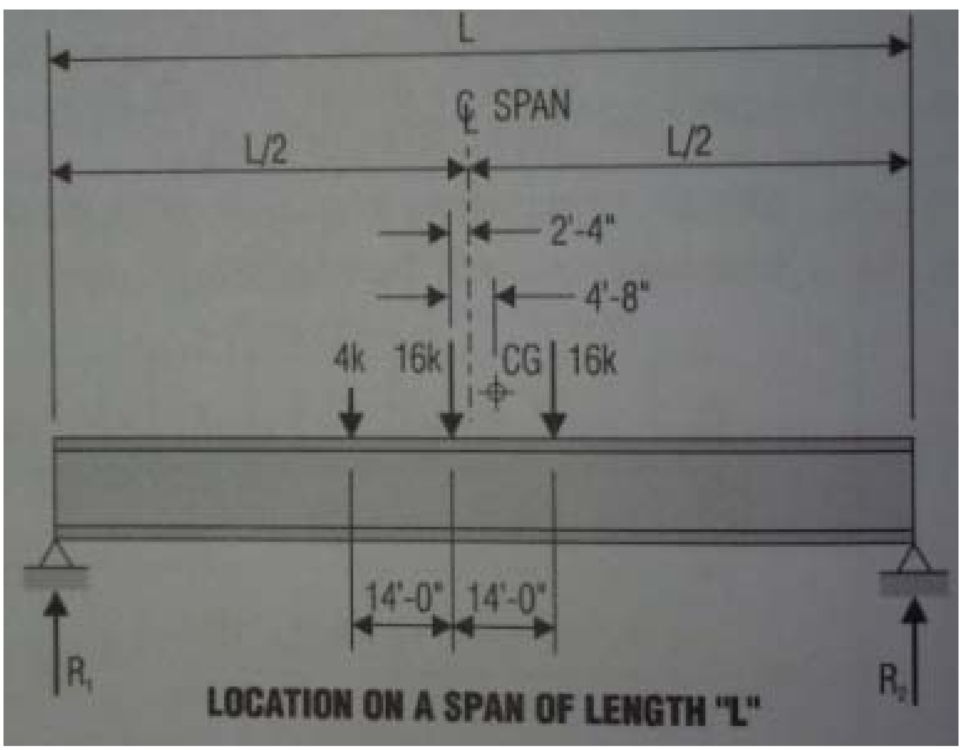
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 720\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 287.28\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

<u>Design Lane Load (distributed over a 10-ft width for each lane)</u>

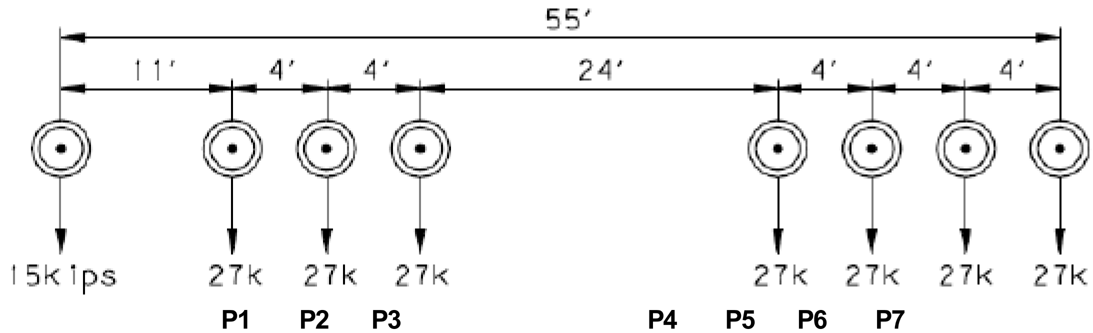
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 60\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.329 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 31.875 \cdot \; I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 149.414 \cdot in^4$

$Y' := \dfrac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 18.683 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 5079 \cdot in^4$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 18.683 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 10.625 \cdot in^2$

$I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 49.805 \cdot in^4$

$Y' := \dfrac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 14.079 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 3420 \cdot in^4$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 14.079 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 7.152 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.247}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 16786.73 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 15063.8 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 6.348 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.2189}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 16763.55 \cdot kip \cdot in}$

$$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 15368.6 \cdot kip \cdot in$$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1255.3 \cdot kip \cdot ft \qquad Mn = 15063.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 2.408\cdot in$$

$$D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 2.408\cdot in$$

$$\frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.335\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 15.33\cdot ksi$$

$$f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 18.055\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 7.556\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 44.831\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 42.378\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 32.82\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 32.821\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.329 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.058 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.246 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.466 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.962 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 972.2 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.793 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 917 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2170.88 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.236 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.058 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.334 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1944.9 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 242.932 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 271.841 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 364.461 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 724.838 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.089 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 261.432 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 833.289 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 902.571 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 1.255 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

<u>Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)</u>

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

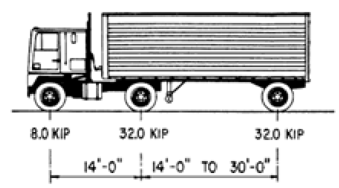
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip \quad P_2 := 32kip \quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.2 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

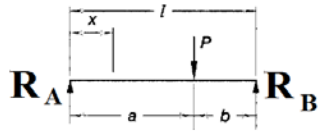
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.091 \cdot in$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.93 \cdot in$

Deflection of the beam at midspan due to P_3: $P := P_3$

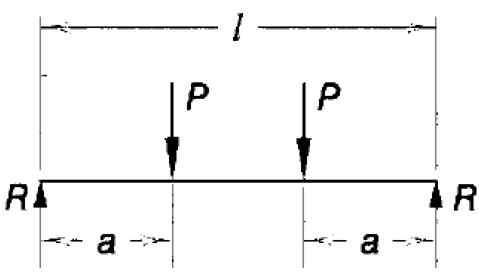
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.799 \cdot in$

Total Deflection at midspan:

$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.484 \cdot in}$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.503 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.122 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.756 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 0$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.473}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 209.325 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.346 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.489 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 36.865 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.772 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 7.573 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 8.641 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.237 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 108.511 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 62.97 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 114.166 \cdot kip$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 114.166 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 50 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 20.275 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\ 3} + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 1.881 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.763 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.858 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.965 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

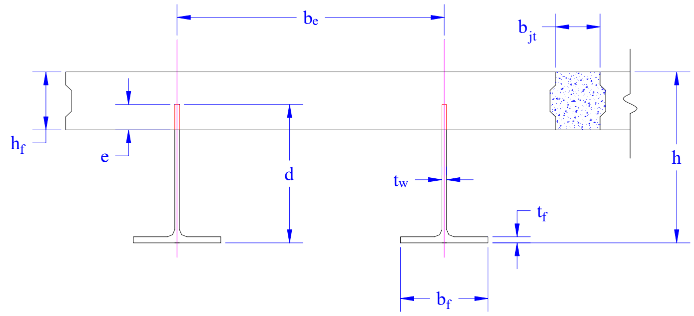
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\text{modules} := \frac{N}{2} = 5$

$L := 50\text{ft}$ Span length

$b_e := 36\text{in}$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500\text{in}$ $t_f := 0.625\text{in}$ $b_f := 12\text{in}$

$h := 29\text{in}$ Overall Height of section

$h_f := 7.5\text{in}$ Total thickness of deck

$e := 3.5\text{in}$ Deck/Steel overlap

$d := h - h_f + e = 25\cdot\text{in}$ depth of built up steel section

$h_w := d - t_f = 24.375\cdot\text{in}$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176\text{kip}\cdot\text{ft}$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862\text{kip}$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2\cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1\cdot t_w = 1$

$$y\_bar := \frac{t_f\cdot b_f\cdot\frac{t_f}{2} + h_w\cdot t_w\cdot\left(t_f + \frac{h_w}{2}\right)}{t_f\cdot b_f + h_w\cdot t_w} = 8.051\cdot\text{in}$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f\cdot t_f^{\,3}}{12} + b_f\cdot t_f\cdot\left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w\cdot h_w^{\,3}}{12} + t_w\cdot h_w\cdot\left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.329\times 10^3\cdot\text{in}^4$$ MOI of steel

$A_{stl} := t_f\cdot b_f + h_w\cdot t_w = 19.687\cdot\text{in}^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 354 \cdot in$ $W = 29.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 26.125\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.766 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700\frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.784 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\frac{lbf}{ft^3} = 0.067 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.344 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 32.614 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.295 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.067 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.362 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.078 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.498 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

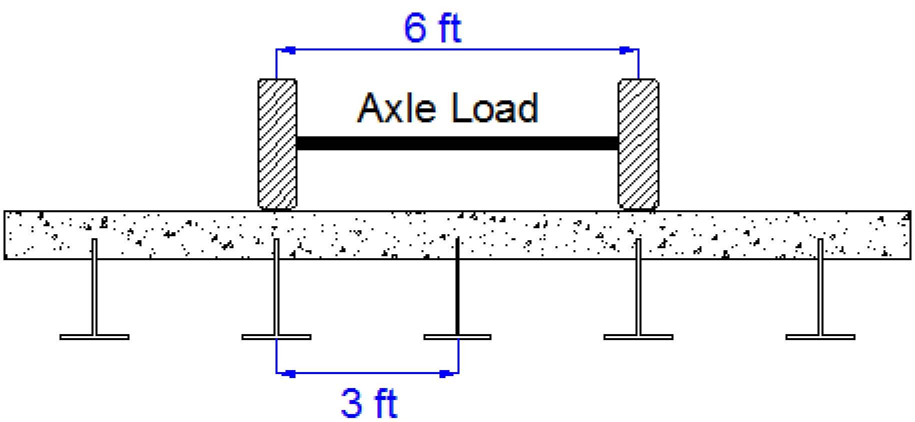
$$M_{DL} := \frac{DL \cdot L^2}{8} = 113.22 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 155.742 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 17.199 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 57224.49 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 21.5 \cdot in$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 90 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 9.54 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

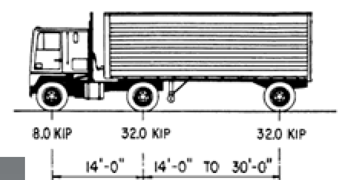
Design truck load in AASHTO (HS20 Truck) is used for this example.

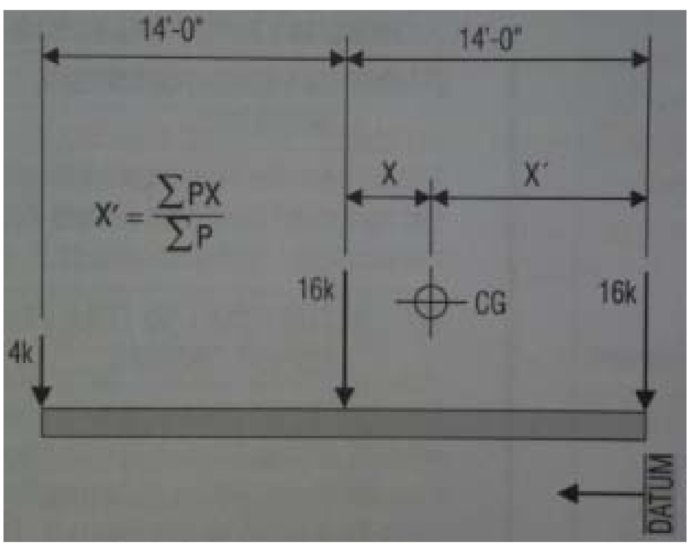
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

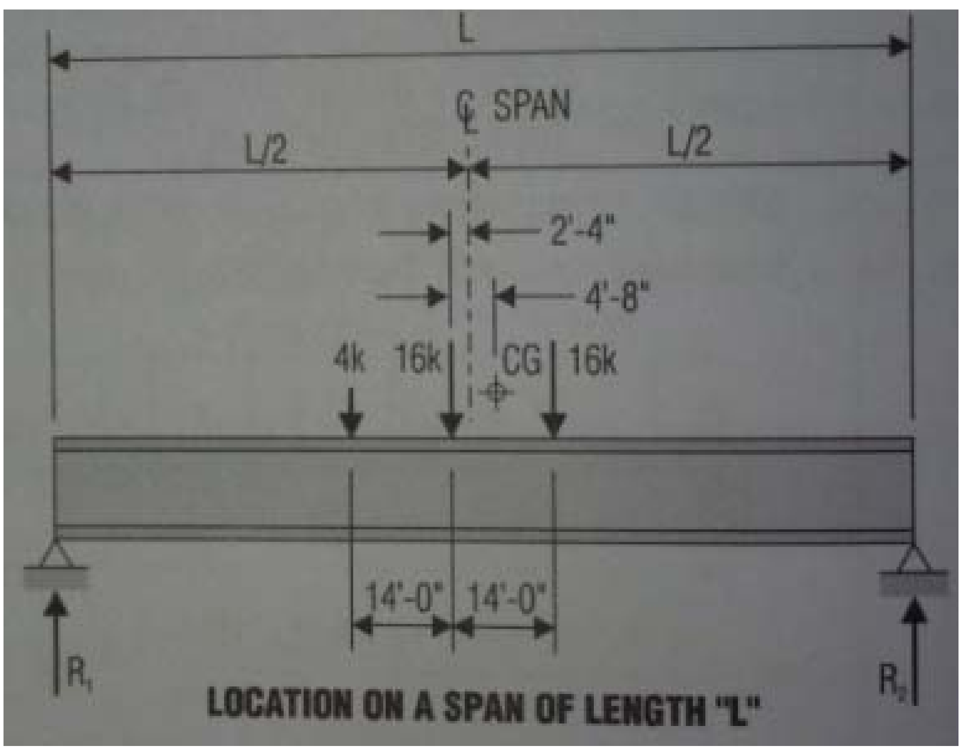
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2} + \frac{X}{2}\right) + F_T\cdot\left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2} - \frac{X}{2}\right) = 720\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1 + I) = 287.28\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

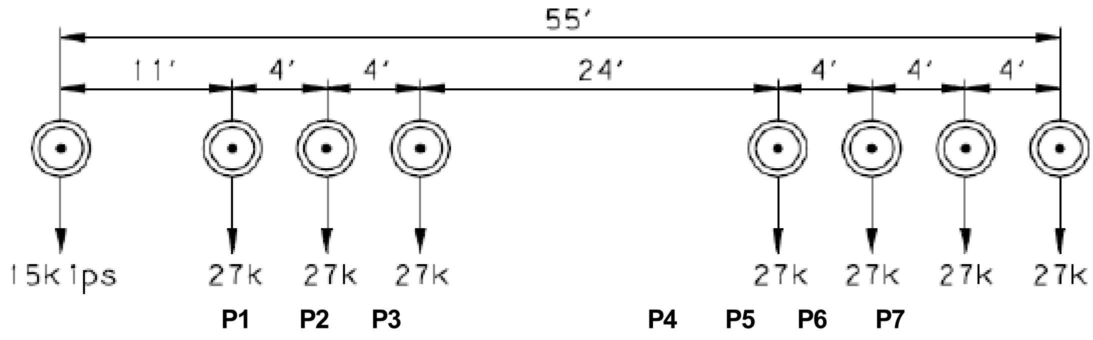
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 60\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.329 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot in \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\ 3}}{12} = 158.203 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 18.913 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 5166 \cdot in^4$$ Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 18.913 \cdot in$$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\ 3}}{12} = 52.734 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 14.305 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3500 \cdot in^4$$ Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 14.305 \cdot in$$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 6.87 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.237}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 16953.79 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 15329.2 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 5.328 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.1837}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 16870.01 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 15881.5 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1277.4 \cdot kip \cdot ft \qquad Mn = 15329.2 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 1.362 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 1.362 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.553 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 15.258 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 17.971 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 7.639 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 45.103 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 42.662 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 33.03 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 33.029 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.332\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.017\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.198\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.456\cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.879\cdot ksi$

$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 982.9\cdot kip\cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.716\cdot ksi$

$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 927\cdot kip\cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6\cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2110.11\cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.238\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.017\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.327\cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1887.02\cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 244.646 \cdot in^3 \qquad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 273.119 \cdot in^3$$

short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 375.139 \cdot kip \cdot ft$$

From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 719.197 \cdot kip \cdot ft$$

Additional moment to cause nominal yielding of steel flange

$$My := M_{D2} + M_{AD} = 1.094 \times 10^3 \cdot kip \cdot ft$$

D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$$S_{xt} := \frac{My}{F_y} = 262.641 \cdot in^3$$

Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 843.967 \cdot kip \cdot ft$$

Moment due to Strength V limit state

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 913.568 \cdot kip \cdot ft$$

Equation 6.10.7.1.7-1

$$Mn = 1.277 \times 10^3 \cdot kip \cdot ft$$

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

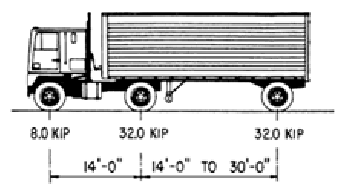
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.163 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

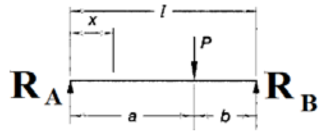
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.089 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.914 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

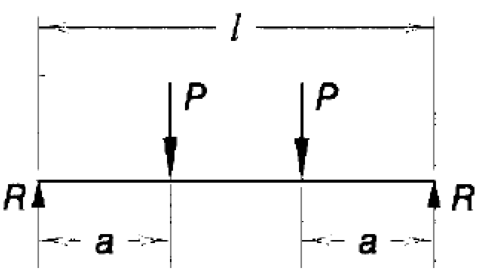
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.785 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.476 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.495 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.12 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.743 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.48}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 213.861 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.362 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.498 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support)

(Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.385 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.304 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 7.68 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 9.058 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.459 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max(V_{truck1}, V_{tandem1})\right) = 110.483 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 63.857 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 116.217 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 116.217 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 50$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 20.275 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.881 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.763 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.858 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.965 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

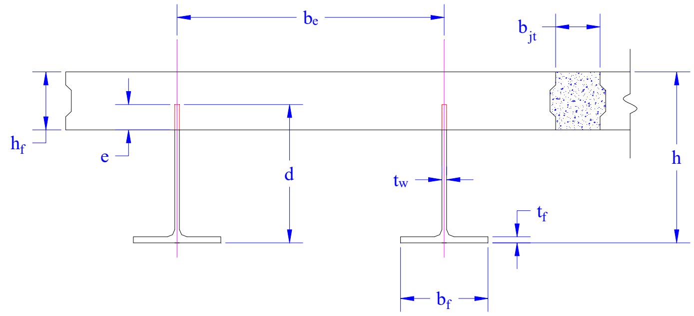
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  - AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  - PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $\qquad$ $modules := \frac{N}{2} = 6$

$L := 50ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500in$ $\qquad$ $t_f := 0.625in$ $\qquad$ $b_f := 12in$

$h := 29in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 25 \cdot in$ depth of built up steel section

$h_w := d - t_f = 24.375 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\qquad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\qquad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 8.051 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.329 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 19.687 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 426 \cdot in$ $W = 35.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 32.125\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.328 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.964 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.067 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.344 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 32.614 \cdot kip}$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.296 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.067 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.363 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.08 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$

distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.5 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

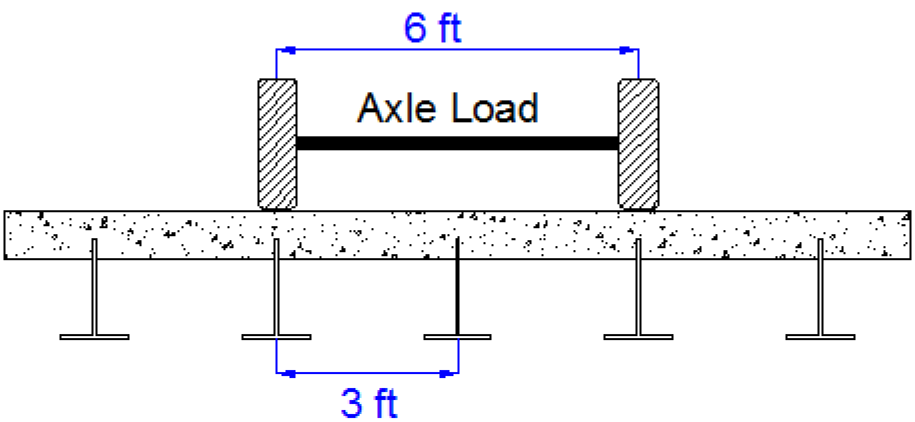
$$M_{DL} := \frac{DL \cdot L^2}{8} = 113.464 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 156.348 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 17.199 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\ 2}\right) = 57224.49 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 21.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\ 3} \cdot t_f}{12} = 90 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 9.54 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

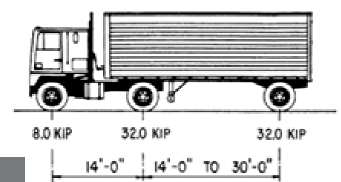
Design truck load in AASHTO (HS20 Truck) is used for this example.

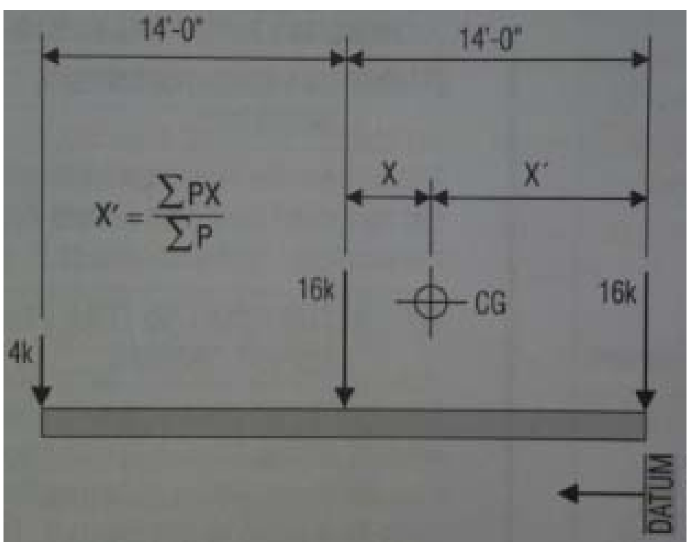
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ $\quad$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ $\quad$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ $\quad$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

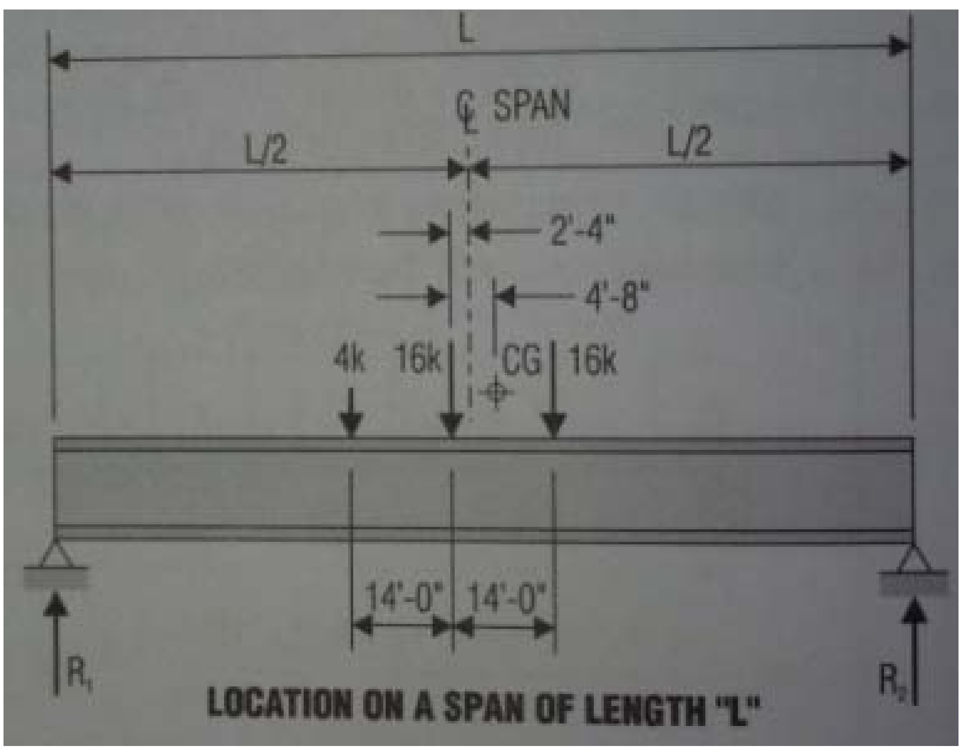
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30 \cdot \text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 720 \cdot \text{kip} \cdot \text{ft}$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720 \cdot \text{kip} \cdot \text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 287.28 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

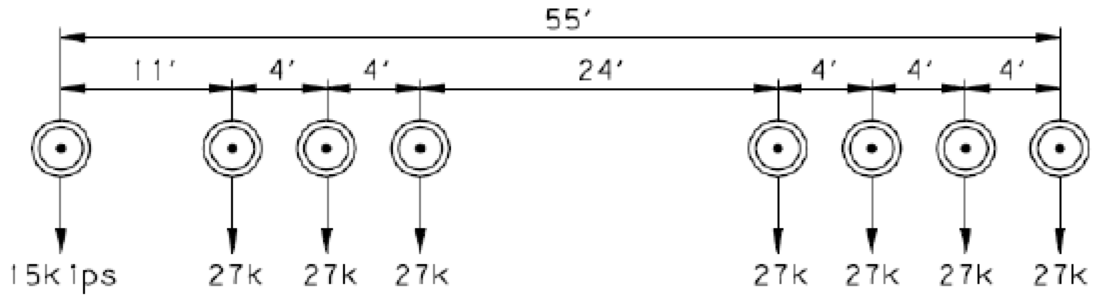
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \frac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 60 \cdot \text{kip} \cdot \text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



P1 P2 P3 P4 P5 P6 P7

NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.329 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot i$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\ 3}}{12} = 158.203 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 18.913 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 5166 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 18.913 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\ 3}}{12} = 52.734 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 14.305 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 3500 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 14.305 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 6.87 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\quad \frac{Dp1}{h} = 0.237$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 16953.79 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 15329.2 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 5.328 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\quad \frac{Dp2}{h} = 0.1837$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 16870.01 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 15881.5 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1277.4 \cdot kip \cdot ft \qquad Mn = 15329.2 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 1.362\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 1.362\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.565\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 15.258\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.971\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 7.669\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 45.162\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 42.721\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 33.07\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 33.07\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.332 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.017 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.198 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.458 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.882 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 984.1 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.72 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 929 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2112.6 \cdot psi$ $\qquad$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.238 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.017 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.328 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1888.8 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 244.646 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 273.119 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 376.352 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 717.843 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.094 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 262.607 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 845.18 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 914.773 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 1.277 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.167$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

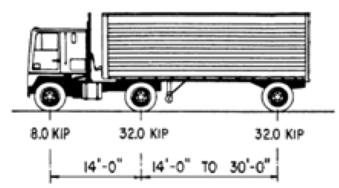
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.163 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

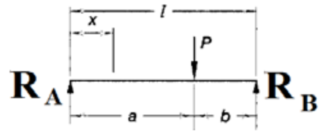
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.089 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.914 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

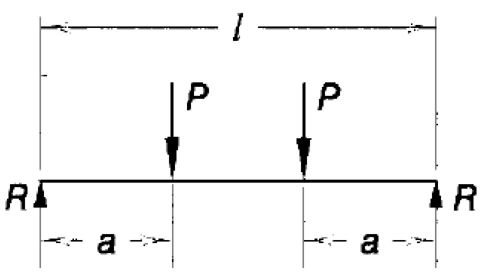
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.785 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.397 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.412 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 106.667 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.1 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\,max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.620 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$DF_s := 0.36 + \frac{S}{25} = 0.48$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 213.861 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.363 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.5 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$X_{ASr} := 14ft$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.385 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.304 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 7.68 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 9.077 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.508 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 110.58 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 63.857 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 116.314 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 116.314 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 50 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 20.275 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.881 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.763 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.858 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.965 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

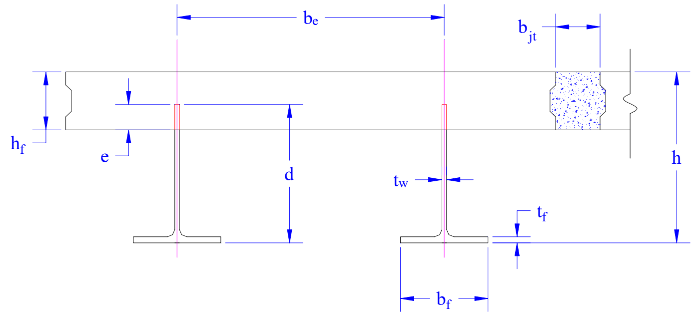
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 50ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500in$ $\quad t_f := 1.00in$ $\quad b_f := 12in$

$h := 29in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 25 \cdot in$ depth of built up steel section

$h_w := d - t_f = 24 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.75 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.514 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 24 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\dfrac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 374 \cdot \text{in}$ $W = 31.167\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 27.792\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.922 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\dfrac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\dfrac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.834 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\dfrac{\text{lbf}}{\text{ft}^3} = 0.082 \cdot \dfrac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.374 \cdot \dfrac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\dfrac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 35.644 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\dfrac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.312 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.082 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.394 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.083 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.507 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

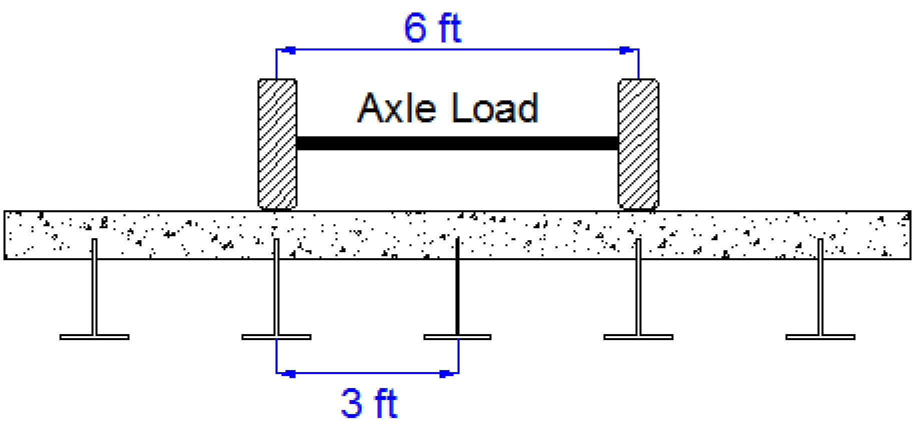
$$M_{DL} := \frac{DL \cdot L^2}{8} = 123.014 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 158.52 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \dfrac{b_e}{12in} = 3.167$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \dfrac{h_f}{2} = 18.5 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 77828 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 21.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 144 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 5.963 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

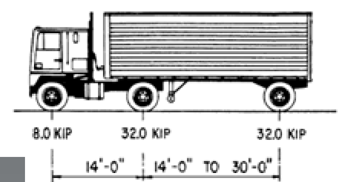
Design truck load in AASHTO (HS20 Truck) is used for this example.

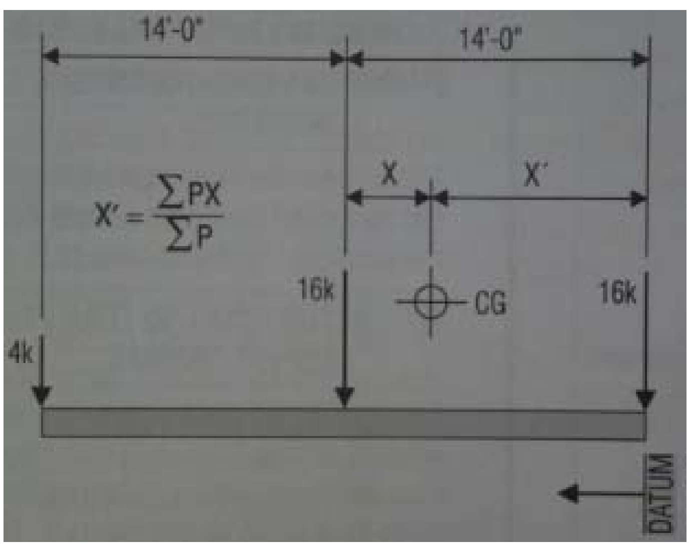
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

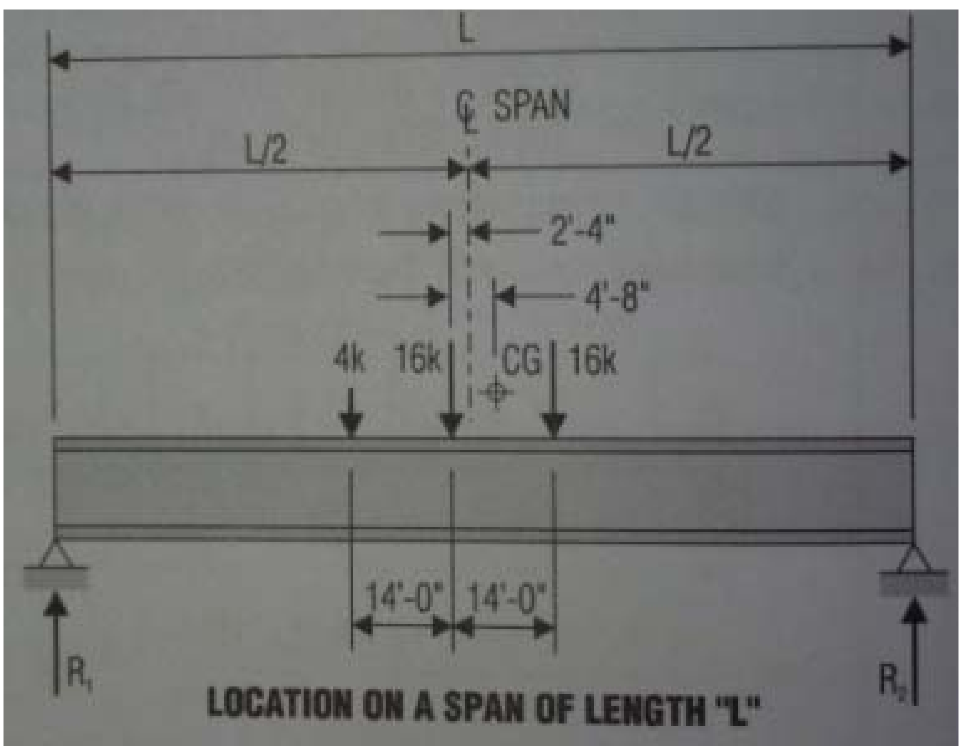
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2 \cdot \text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30 \cdot \text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 720 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 287.28 \cdot \text{kip} \cdot \text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

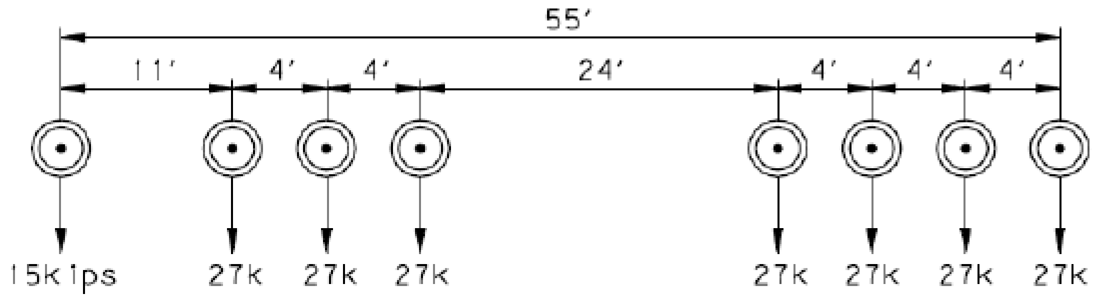
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64 \frac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 60 \cdot \text{kip} \cdot \text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.514 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot \; I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 17.803 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6589 \cdot in^4$$ Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 17.803 \cdot in$$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.874 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 4289 \cdot in^4$$ Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 12.874 \cdot in$$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 7.813 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.269}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 21631.25 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 19066.3 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 8.62 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.2972}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 21608.64 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 18625.2 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1552.1 \cdot kip \cdot ft \qquad Mn = 18625.2 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 4.813 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 4.813 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.431 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 11.26 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.262 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.71 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 33.807 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 32.006 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 24.78 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 24.778 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.314\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.885\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 1.043\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.404\cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.547\cdot ksi$

$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 999.3\cdot kip\cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.405\cdot ksi$

$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 944\cdot kip\cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6\cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1868.29\cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.231\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.885\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.298\cdot ksi$$

$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1679.97\cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 333.167 \cdot in^3$ $S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 370.109 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 391.548 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.107 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.499 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 359.689 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 860.376 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 919.951 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $Mn = 1.552 \times 10^3 \cdot kip \cdot ft$ $M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

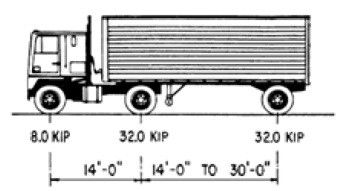
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.696 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

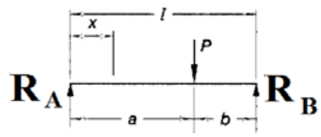
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.07 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.717 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

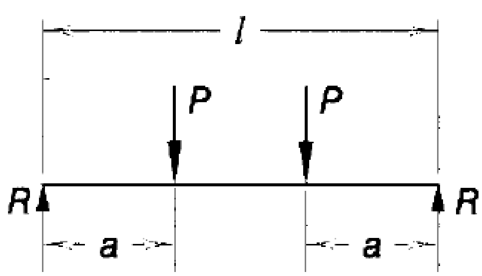
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.616 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.373 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.388 \cdot in$$

(Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.094 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.583 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 265.283 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.394 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.507 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 ***(at left support)*** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 37.904 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 38.836 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 7.787 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 9.841 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.682 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max(V_{truck1}, V_{tandem1})\right) = 112.913 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 64.743 \cdot kip$$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 118.727 \cdot kip$$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 118.727 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$$\frac{D}{t_w} = 50 \qquad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$$

Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \dfrac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 19.9 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$b_t \leq 0.48 \cdot t_p \cdot \sqrt{\dfrac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \geq Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \dfrac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\dfrac{I_s}{A_g}} = 1.881 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \dfrac{\pi^2 \cdot E_s}{\left(\dfrac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.944 \times 10^4 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$PePo := \dfrac{Pe}{Po} \geq 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.891 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.996 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \geq Vu = 1$

Flexbeam Example Configuration Calculations

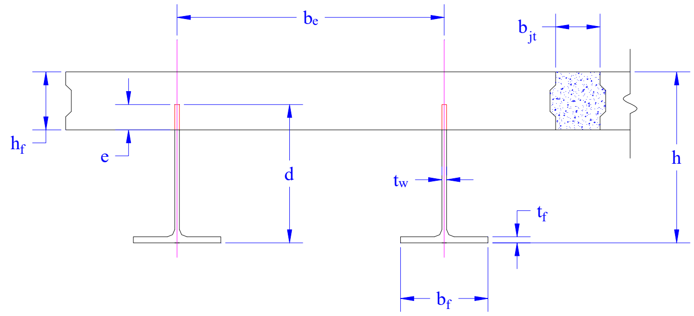
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  - AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  - PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $modules := \frac{N}{2} = 7$

$L := 50ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500in$ $t_f := 1.00in$ $b_f := 12in$

$h := 29in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 25 \cdot in$ depth of built up steel section

$h_w := d - t_f = 24 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.75 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.514 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 24 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 526\cdot\text{in}$ $W = 43.833\text{ ft}$

Clear width of roadway $W_{lane} := W - 2\cdot b_{par} = 40.458\text{ ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 4.109\cdot\frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2}\cdot\left(W - 2\cdot b_{par}\right) = 1.214\cdot\frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl}\cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.082\cdot\frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.375\cdot\frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}}\cdot 32\text{in} = 0.067\cdot\text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f\cdot w_c\right)\right] + 2D_s\right]\cdot L + 2\cdot W_{end\_diaphragms} = 35.644\cdot\text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.313 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.082 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.395 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.087 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.511 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

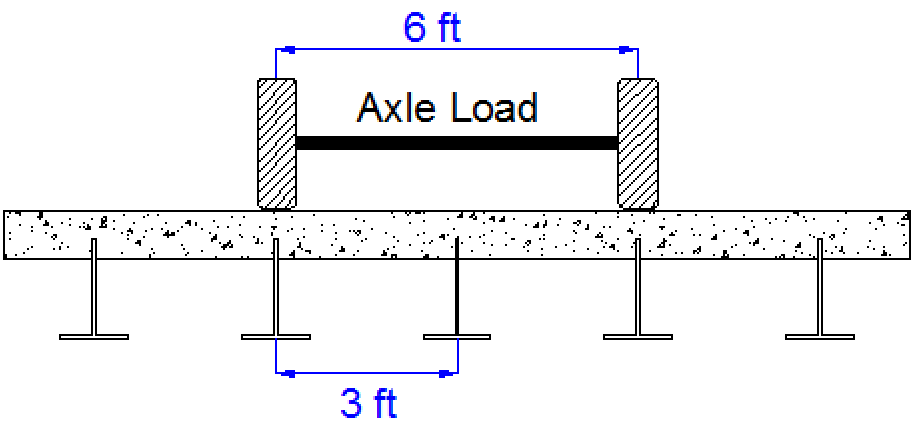
$$M_{DL} := \frac{DL \cdot L^2}{8} = 123.433 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 159.558 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 18.5 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 77828 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 21.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{{b_f}^3 \cdot t_f}{12} = 144 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 5.963 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

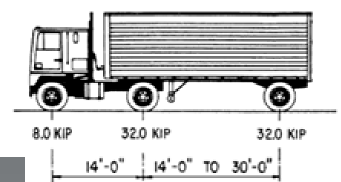
Design truck load in AASHTO (HS20 Truck) is used for this example.

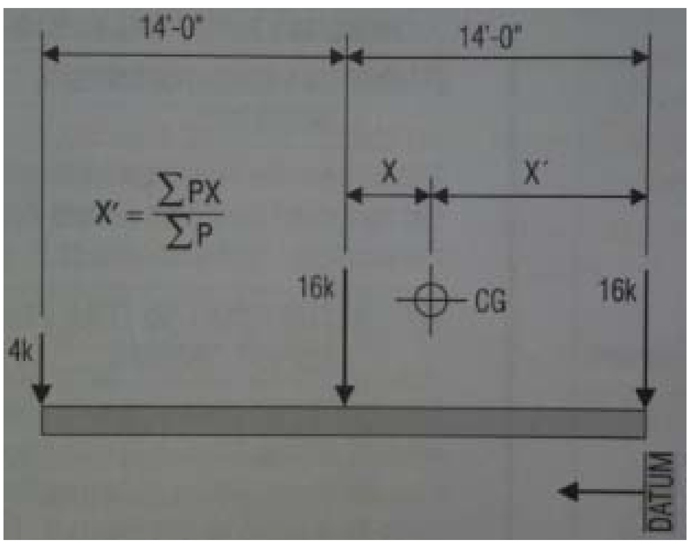
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

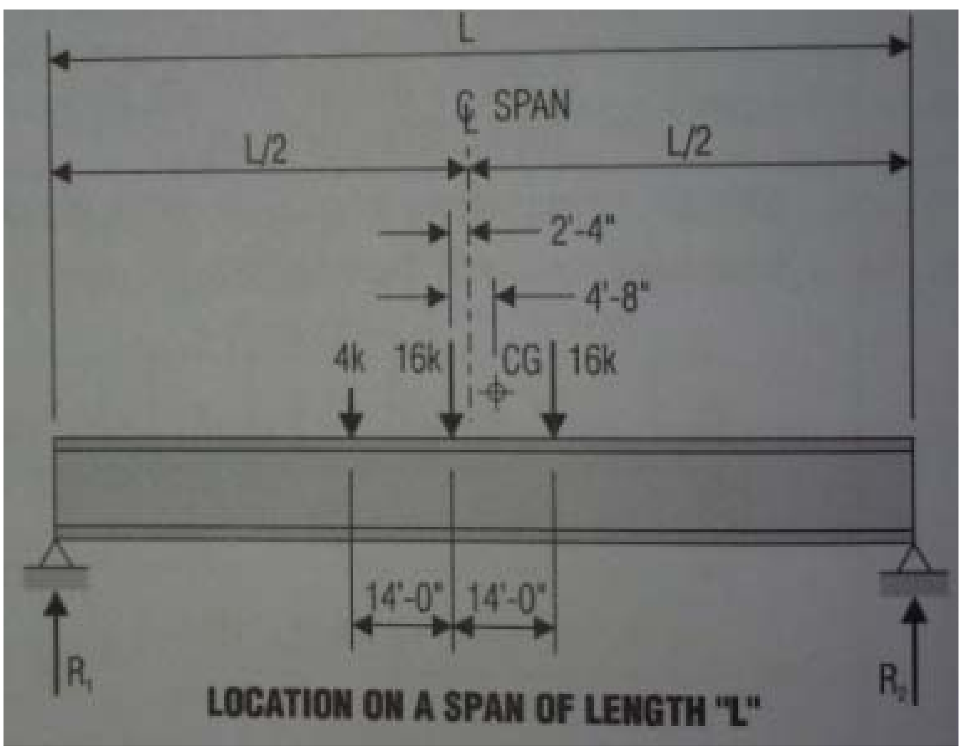
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2 \cdot \text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30 \cdot \text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 720 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 287.28 \cdot \text{kip} \cdot \text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

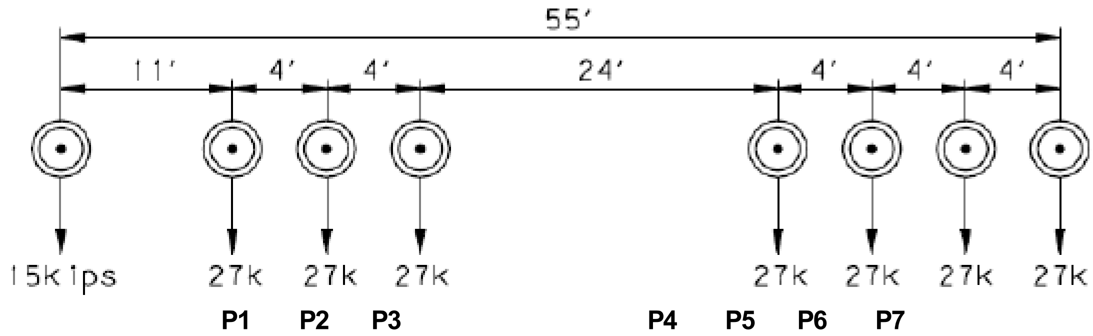
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64 \frac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 60 \cdot \text{kip} \cdot \text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.514 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\ 3}}{12} = 166.992 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 17.803 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6589 \cdot in^4$$

Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 17.803 \cdot in$$

N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\ 3}}{12} = 55.664 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.874 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 4289 \cdot in^4$$

Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 12.874 \cdot in$$

N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f

$$h - d = 4 \cdot in$$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 7.5in$ $\qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 7.813 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.269}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0$ $\qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h - d)]] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 21631.25 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 19066.3 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 8.62 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.2972}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1}$ $\qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h - d)]] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 21608.64 \cdot kip \cdot in}$

$$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 18625.2 \cdot kip \cdot in$$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1552.1 \cdot kip \cdot ft$ $\qquad Mn = 18625.2 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 4.813\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 4.813\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 4.446\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 11.26\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.262\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.747\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 33.882\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 32.081\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 24.83\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 24.831\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.315 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.885 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 1.043 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.407 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.552 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1001.4 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.411 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 946 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1872 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot \left(h - Y'_{c\_long}\right)}{k_3 \cdot n \cdot I_{c\_long}} = 0.232 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.885 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot \left(h - Y'_{c\_long}\right)}{k_3 \cdot n \cdot I_{c\_long}} = 0.3 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1682.71 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 333.167 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 370.109 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 393.628 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.105 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.498 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 359.634 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 862.456 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 922.022 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 1.552 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

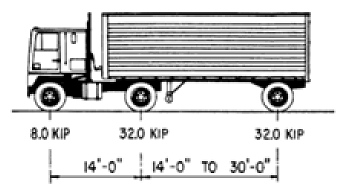
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.696 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot ft$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

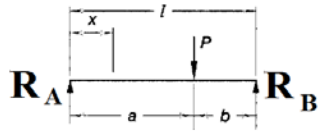
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot ft$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot ft$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.07 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.717 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

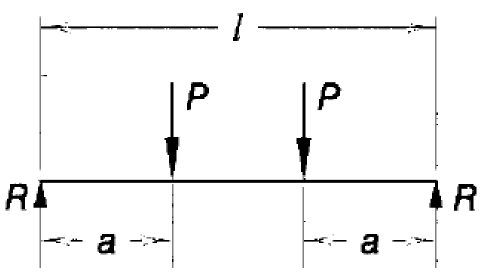
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.616 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.4 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.416 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.101 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\,max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.624 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 265.283 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.395 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.511 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 ***(at left support)*** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 37.904 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.836 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 7.787 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 9.875 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.765 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 113.08 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 64.743 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$\boxed{V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 118.894 \cdot kip}$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 118.894 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 50 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \quad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 19.9 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.881 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.944 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.891 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.996 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

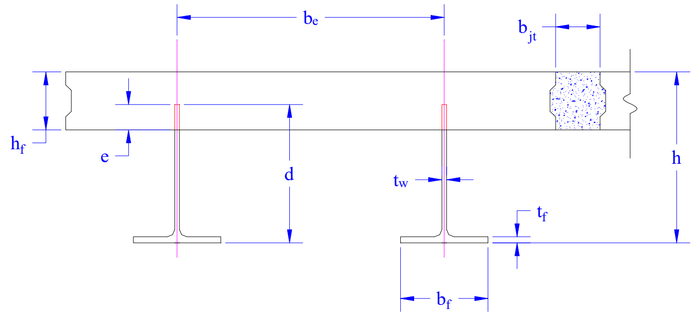
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad$ $modules := \frac{N}{2} = 5$

$L := 50ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500in$ $\quad$ $t_f := 1.00in$ $\quad$ $b_f := 12in$

$h := 29in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 25 \cdot in$ depth of built up steel section

$h_w := d - t_f = 24 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.75 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.514 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 24 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \dfrac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 394 \cdot \text{in}$ $W = 32.833\text{ ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 29.458\text{ ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.078 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700 \dfrac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \dfrac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.884 \cdot \dfrac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \dfrac{\text{lbf}}{\text{ft}^3} = 0.082 \cdot \dfrac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.389 \cdot \dfrac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \dfrac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 37.206 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\dfrac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.082 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.41 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.088 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.516 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

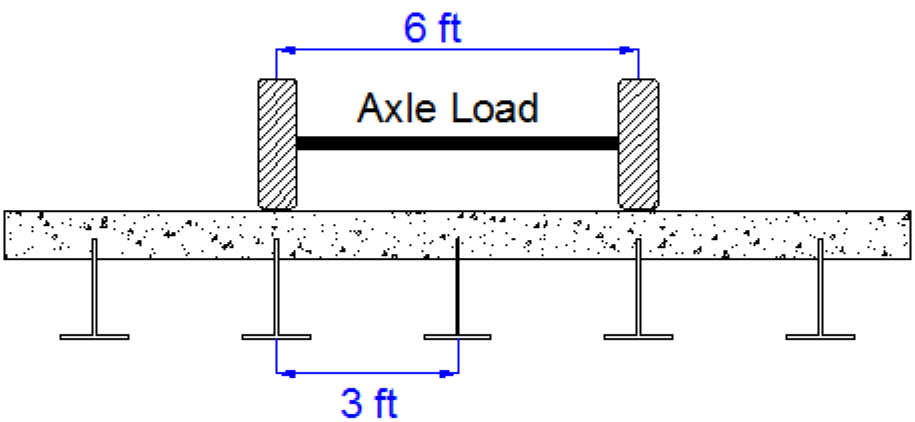
$$M_{DL} := \frac{DL \cdot L^2}{8} = 128.223 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 161.298 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 18.5 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^2\right) = 77828 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 21.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 144 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 5.963 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

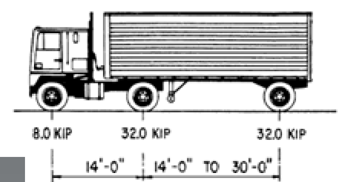
Design truck load in AASHTO (HS20 Truck) is used for this example.

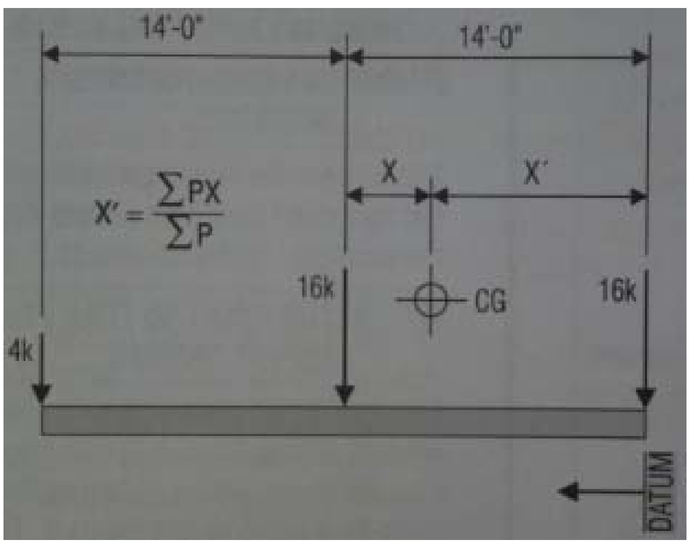
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

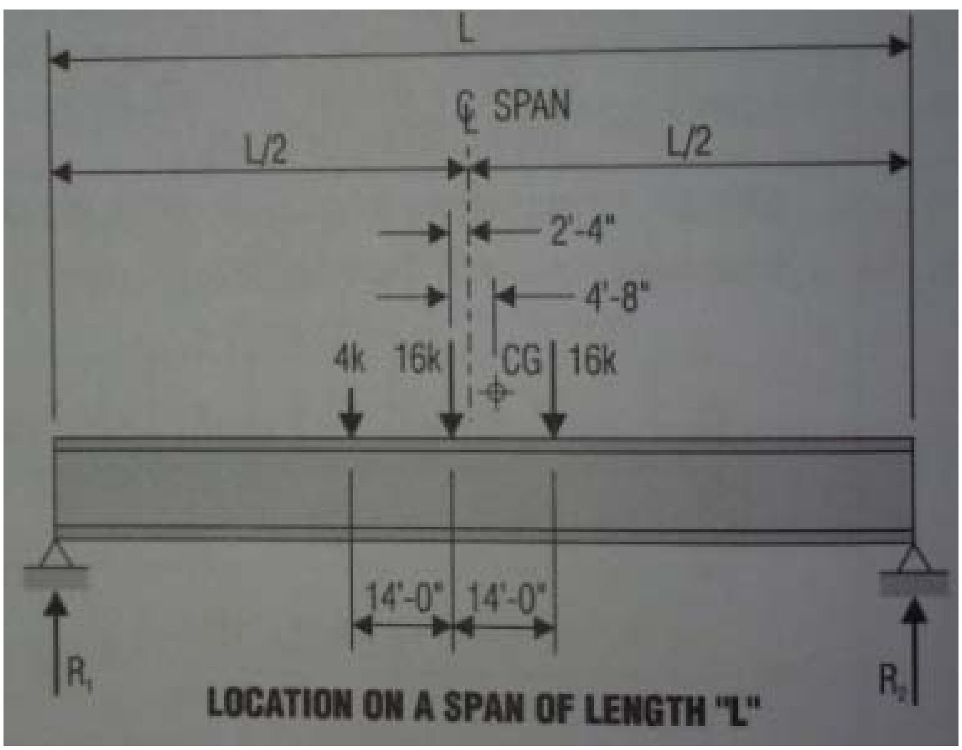
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2 \cdot ft$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 720 \cdot kip \cdot ft$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720 \cdot kip \cdot ft}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 287.28 \cdot kip \cdot ft$$

Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

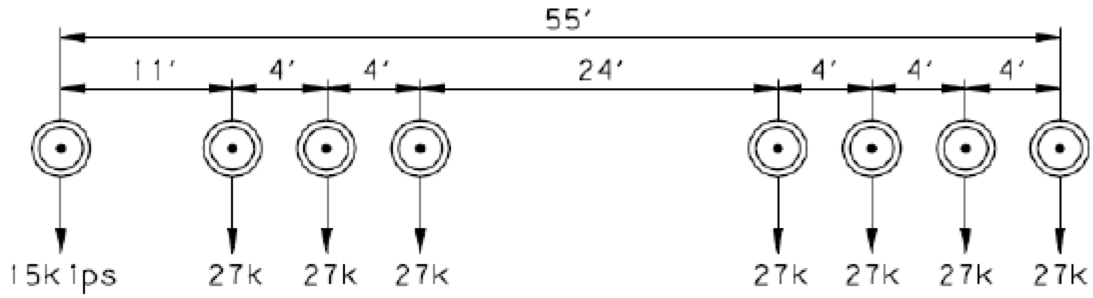
$F_{DLL} := 0.64 \frac{kip}{ft} \cdot DF = 0.192 \cdot \frac{kip}{ft}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 60 \cdot kip \cdot ft$$

Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28 \cdot kip \cdot ft}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.514 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $\quad I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\ 3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 18.03 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6699 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 18.03 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $\quad I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\ 3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 13.086 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 4386 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 13.086 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 7.527 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.26}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0$ $\qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h - d)]] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 21831.18 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 19393 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 7.6 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.2621}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1}$ $\qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h - d)]] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 21831 \cdot kip \cdot in}$

$$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 19354.3 \cdot kip \cdot in$$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1612.9 \cdot kip \cdot ft$ $\qquad Mn = 19354.3 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 3.75 \cdot in$$

$$D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 3.75 \cdot in$$

$$\frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.591 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 11.217 \cdot ksi$$

$$f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.211 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.775 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 34.03 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 32.235 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 24.95 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 24.947 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.315\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.853\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.005\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.396\cdot ksi$$

STRENGTH I

$f_{c\_str1} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.481\cdot ksi$

$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 1010\cdot kip\cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II

$f_{c\_str2} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.344\cdot ksi$

$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 954\cdot kip\cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6\cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1820.08\cdot psi$ $\qquad$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.233\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.853\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.293\cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1634.17\cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 335.185 \cdot in^3$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 371.527 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 402.225 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.102 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.504 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 361.061 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 871.053 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 930.855 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $Mn = 1.613 \times 10^3 \cdot kip \cdot ft$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

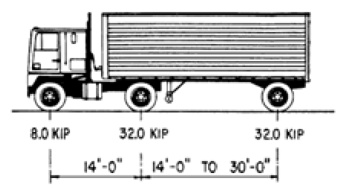
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.668 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

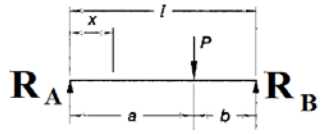
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.069 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.705 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

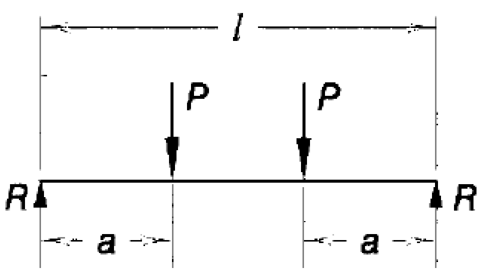
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.606 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.367 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.382 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.093 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.573 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 270.732 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.41 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.516 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.423 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.368 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 7.893 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 10.258 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.904 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 114.885 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 65.63 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 120.779 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 120.779 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 50 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 19.9 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.881 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.944 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.891 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.996 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

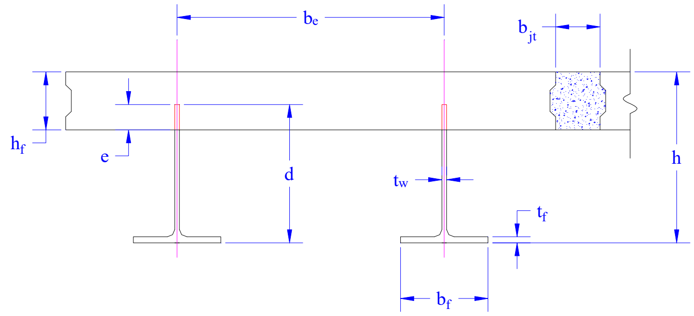
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $\quad modules := \frac{N}{2} = 6$

$L := 50ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500in$ $\quad t_f := 1.00in$ $\quad b_f := 12in$

$h := 29in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 25 \cdot in$ depth of built up steel section

$h_w := d - t_f = 24 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.75 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.514 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 24 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 474 \cdot in$ $W = 39.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 36.125\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.703 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.084 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.082 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.39 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 37.206 \cdot kip}$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12ft}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.082 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.411 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.09 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.518 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

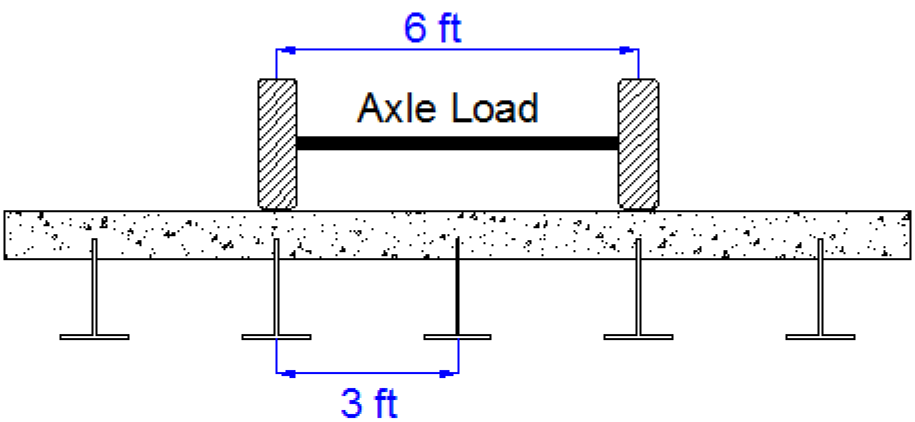
$$M_{DL} := \frac{DL \cdot L^2}{8} = 128.467 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 161.903 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.333$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 18.5 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 77828 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 21.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 144 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 5.963 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

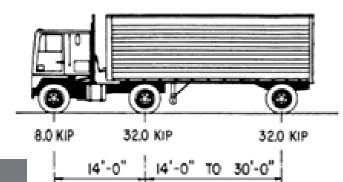
Design truck load in AASHTO (HS20 Truck) is used for this example.

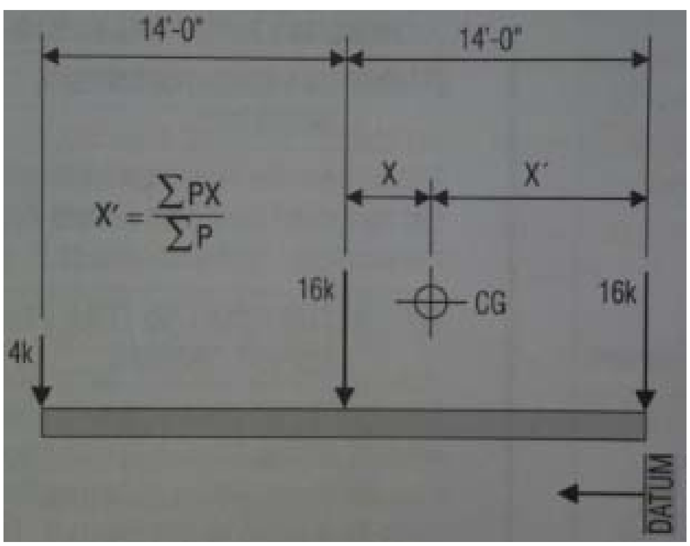
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

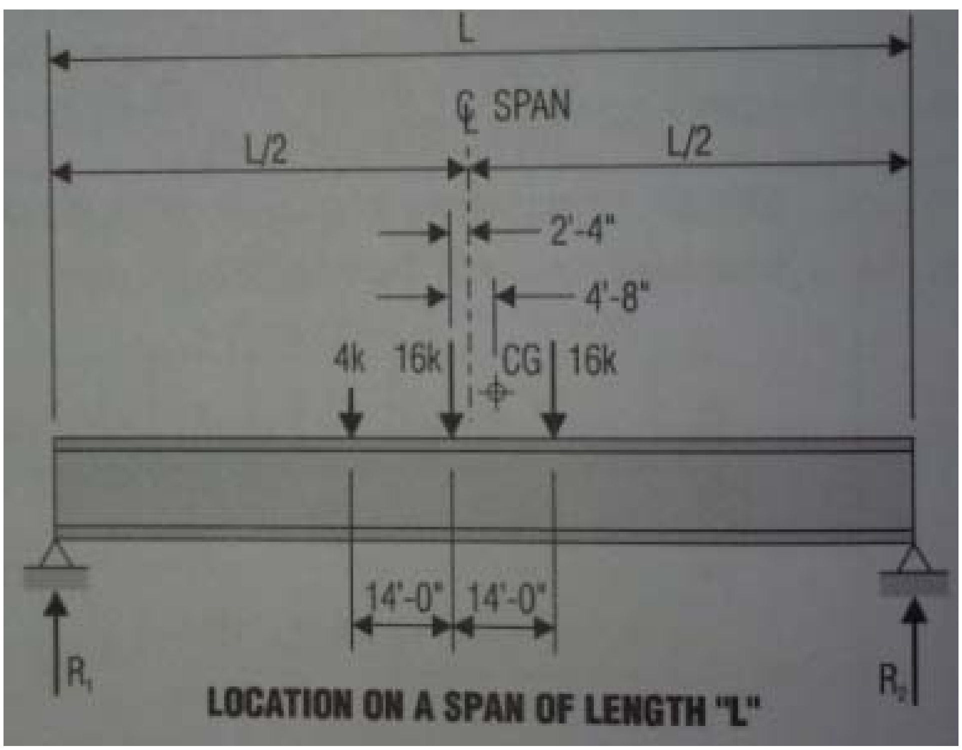
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 720\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720\cdot\text{kip}\cdot\text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 287.28\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

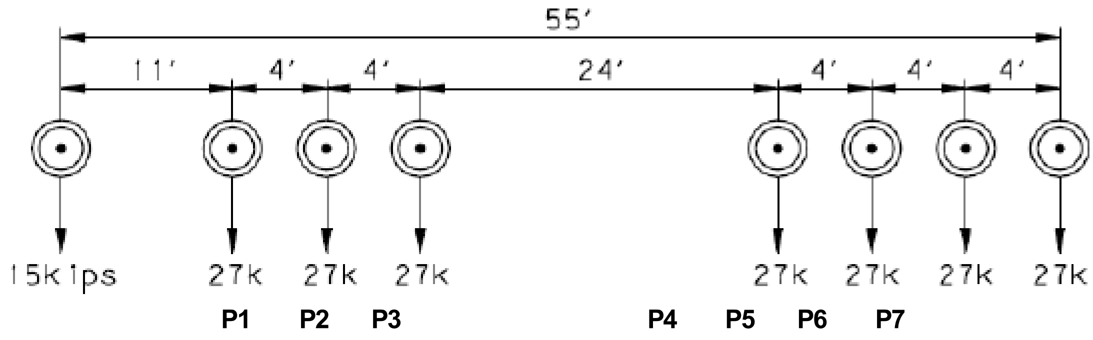
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 60\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.514 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2 \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 18.03 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6699 \cdot in^4$$

Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 18.03 \cdot in$$

N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 13.086 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 4386 \cdot in^4$$

Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 13.086 \cdot in$$

N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 7.527 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.26}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0$ $\qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 21831.18 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 19393 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 7.6 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.2621}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1}$ $\qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 21831 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 19354.3 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1612.9 \cdot kip \cdot ft$ $\qquad Mn = 19354.3 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1\right) = 3.75\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 3.75\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 4.599\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 11.217\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.211\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.796\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 34.073\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 32.278\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 24.98\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 24.977\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.316 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.853 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.005 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.398 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.484 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1011.2 \cdot kip \cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.347 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 956 \cdot kip \cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1822.16 \cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.233 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.853 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.294 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1635.71 \cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 335.185 \cdot in^3$ $\quad$ $S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 371.527 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 403.438 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.101 \times 10^3 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.504 \times 10^3 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 361.029 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 872.266 \cdot kip \cdot ft$ $\quad$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 932.063 \cdot kip \cdot ft$ $\quad$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 1.613 \times 10^3 \cdot kip \cdot ft$ $\quad$ $M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

<u>Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)</u>

$$DF_d := \frac{N_l}{N} = 0.25$$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

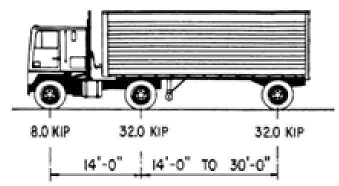
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.668 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot \text{ft}$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

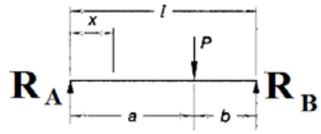
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot \text{ft}$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot \text{ft}$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.069 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.705 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

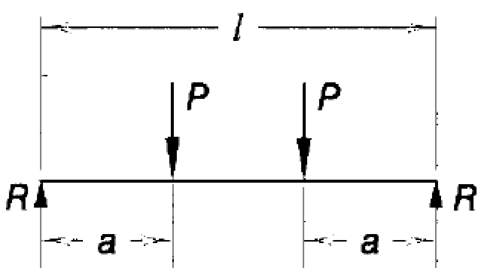
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.606 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.459 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.477 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 160 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.116 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\, \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.717 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 270.732 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.411 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.518 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.423 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.368 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot \left(DF_s\right) = 7.893 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 10.277 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.952 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 114.982 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 65.63 \cdot kip$$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 120.876 \cdot kip$$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 120.876 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$$\frac{D}{t_w} = 50 \qquad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$$

Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 19.9 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\ 3} + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.881 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.944 \times 10^4 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.891 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.996 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

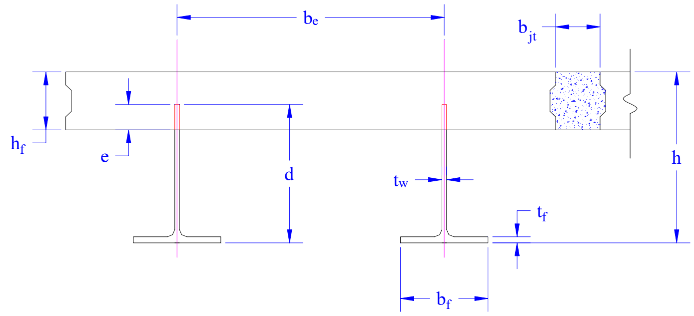
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $modules := \frac{N}{2} = 7$

$L := 50ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.500in$ $t_f := 1.00in$ $b_f := 12in$

$h := 29in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 25 \cdot in$ depth of built up steel section

$h_w := d - t_f = 24 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 409.0176kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 110.862kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.75 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 1.514 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 24 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \dfrac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 554 \cdot in$ $W = 46.167\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 42.792\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 4.328 \cdot \dfrac{kip}{ft}$ DL due to slab

$D_b := .700 \dfrac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \dfrac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.284 \cdot \dfrac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \dfrac{lbf}{ft^3} = 0.082 \cdot \dfrac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.391 \cdot \dfrac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \dfrac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 37.206 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \mathrm{floor}\left(\dfrac{W_{lane}}{12ft}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.33 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.082 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.412 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.092 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.519 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

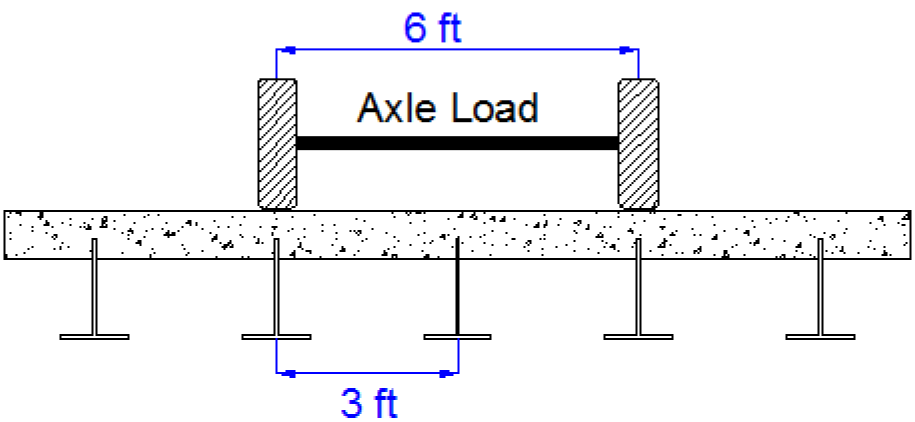
$$M_{DL} := \frac{DL \cdot L^2}{8} = 128.641 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 162.336 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 18.5 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 77828 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 21.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 11.925 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 144 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 5.963 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

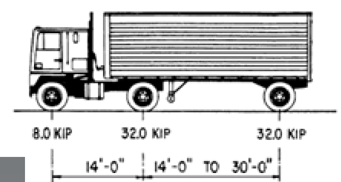
Design truck load in AASHTO (HS20 Truck) is used for this example.

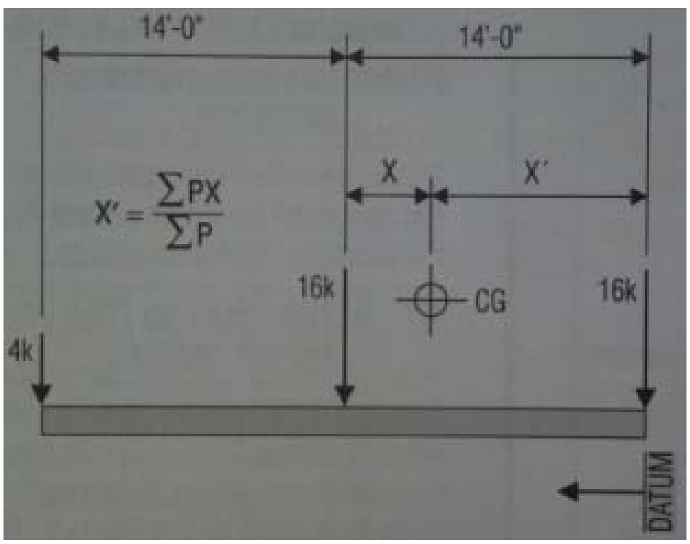
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

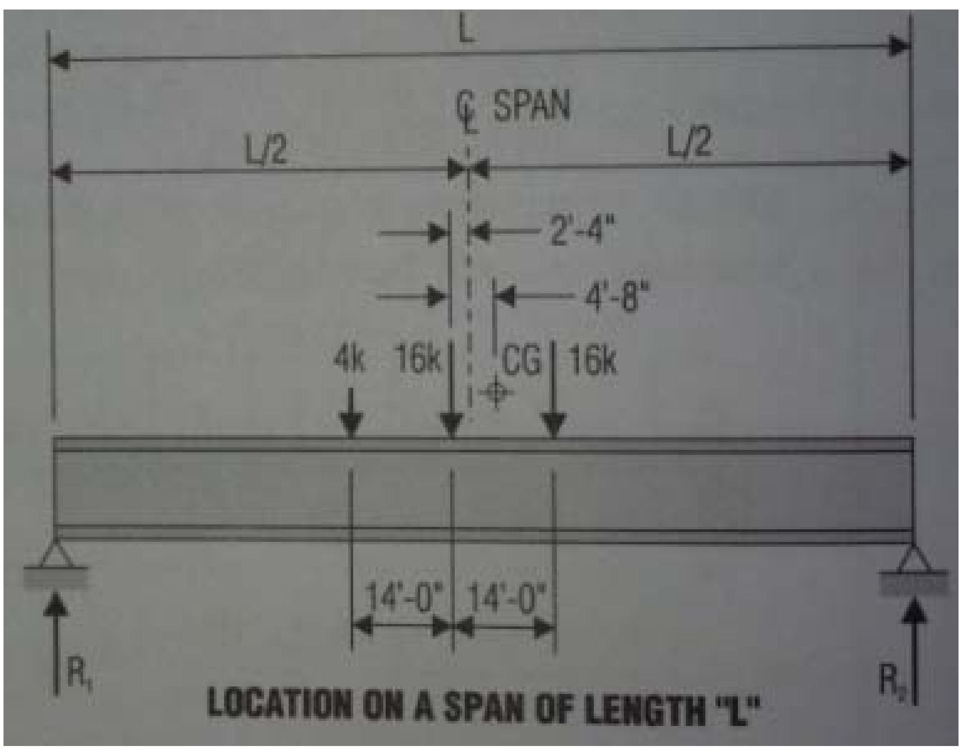
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 32.64 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 627.84 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 720\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 720\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 287.28\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

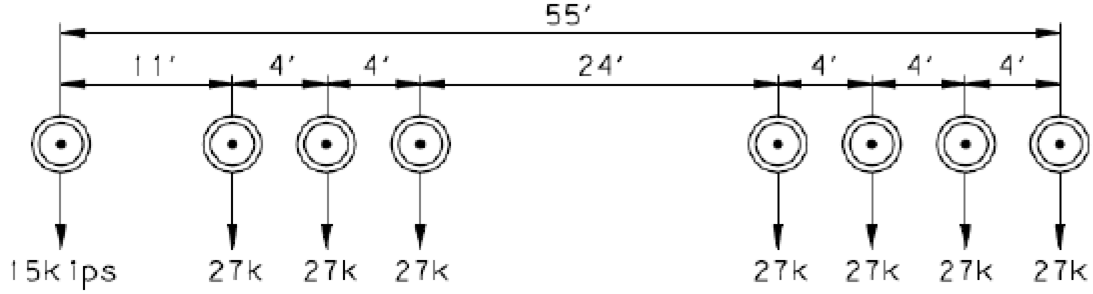
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 60\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 347.28\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



P1 P2 P3 P4 P5 P6 P7

NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 1.514 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $\quad I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 18.03 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6699 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 18.03 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $\quad I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 13.086 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 4386 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 13.086 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 12.18 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 7.527 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.26}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 21831.18 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 19393 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 7.6 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.2621}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 21831 \cdot kip \cdot in}$

$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 19354.3 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1612.9 \cdot kip \cdot ft \qquad Mn = 19354.3 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 3.75 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 3.75 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 4.605 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 11.217 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.211 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.812 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 34.104 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 32.309 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 25 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 24.999 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.316 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.853 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.005 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.399 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.486 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1012 \cdot kip \cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.349 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 956 \cdot kip \cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1823.65 \cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.233 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.853 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.295 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1636.81 \cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 335.185 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 371.527 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 404.305 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.1 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.504 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 361.007 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 873.133 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 932.926 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 1.613 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

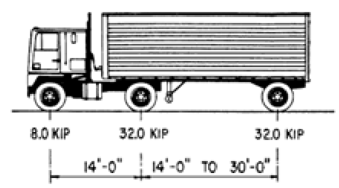
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.668 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 6.333 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

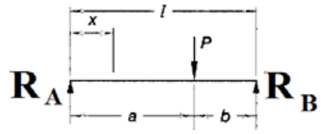
$b2 := \frac{L}{2} - (X_{ASr} - X') = 20.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 15.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.069 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.705 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

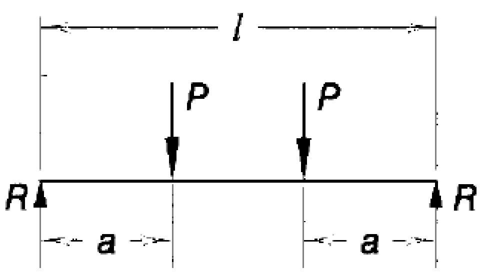
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.606 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.393 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.409 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.099 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.614 \cdot in$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.75 \cdot in$ $\frac{L}{1000} = 0.6 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 270.732 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.412 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.519 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 22 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 36 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 50 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 58.56 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.423 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.368 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 16 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 7.893 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 10.291 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 12.987 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 115.052 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 65.63 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 120.945 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 120.945 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 25 \cdot in$ Thickness of web $t_w = 0.5 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 362.5 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 50$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 362.5 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 362.5 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.25 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 19.9 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.375 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 446.25 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 38.477 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 10.875 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 1.881 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 318.75 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 4.944 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 317.891 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 301.996 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

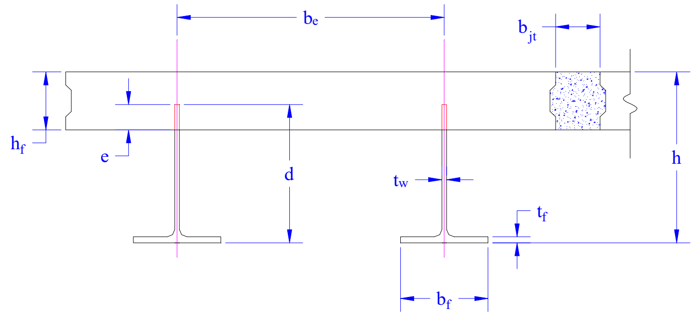
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 60ft$ Span length

$\boxed{b_e := 34in}$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 0.750in$ $\quad b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31 \cdot in$ depth of built up steel section

$h_w := d - t_f = 30.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$\boxed{M_{LL\_IM\_PL82} := 523.6917kip \cdot ft}$ Moment due to PL82 with distribution factor and impact factor!!

$\boxed{V_{PL82span} := 125.11kip}$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.876 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 2.907 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 27.906 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 334 \cdot in$ $W = 27.833\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 24.458\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.609 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.734 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.095 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.356 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 40.591 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \mathrm{floor}\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.279 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.095 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.374 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.073 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.066 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.489 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.25 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.417 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

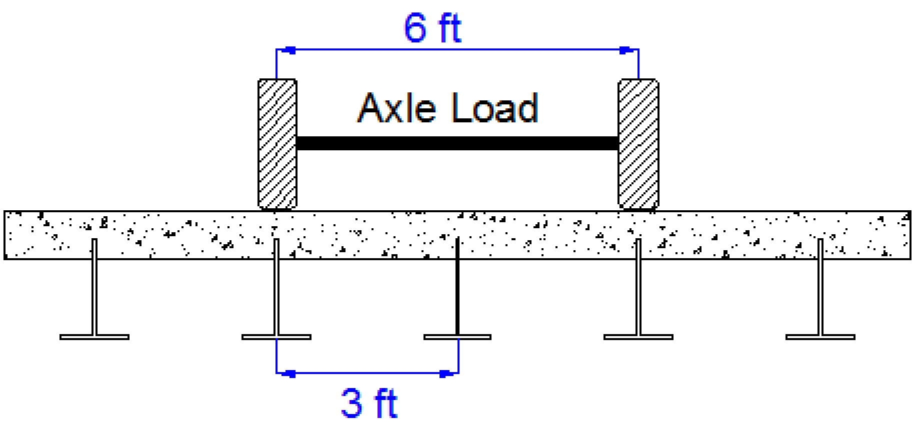
$$M_{DL} := \frac{DL \cdot L^2}{8} = 168.122 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 220.269 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 20.374 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.16 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 27.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^3 \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.643 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

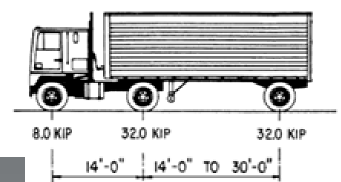
Design truck load in AASHTO (HS20 Truck) is used for this example.

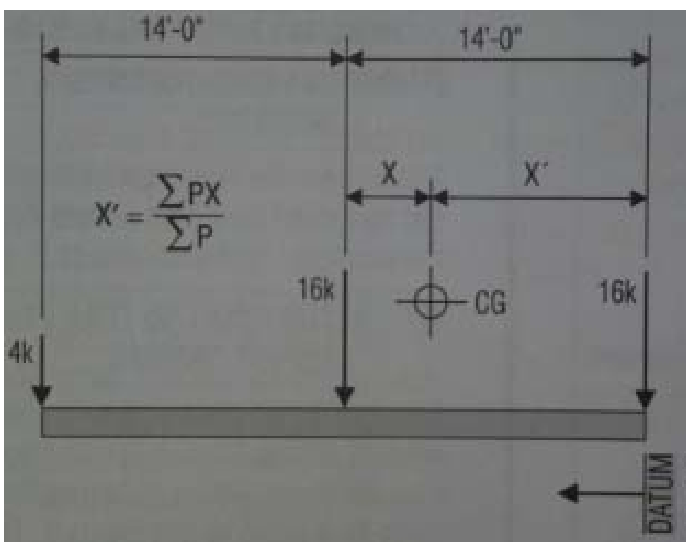
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

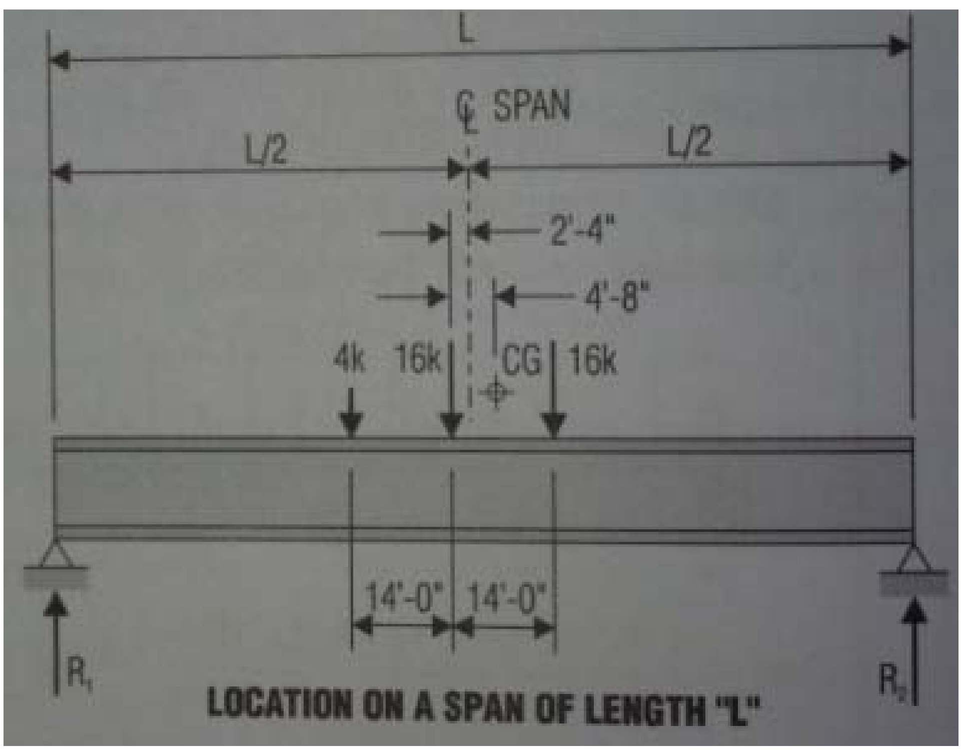
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30.208 \cdot \text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 876.042 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 349.541 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

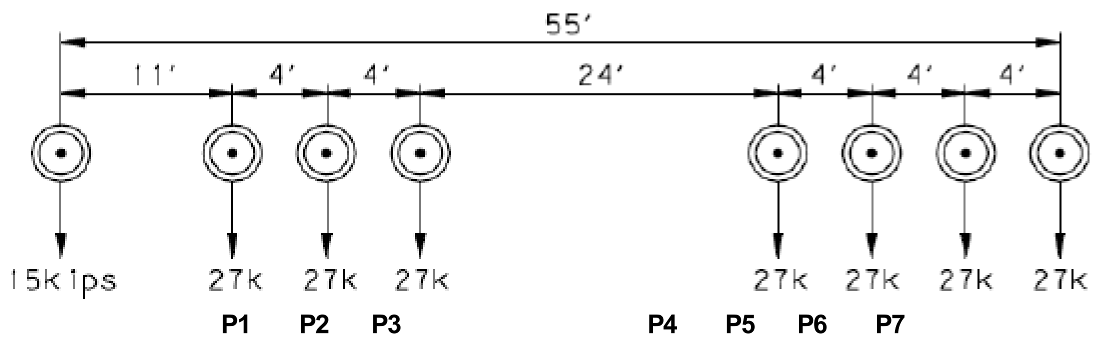
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \dfrac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\dfrac{F_{DLL} \cdot L^2}{8}\right) = 86.4 \cdot \text{kip} \cdot \text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 2.907 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 31.875 \cdot$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 149.414 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 21.739 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9233 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 21.739 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 10.625 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 49.805 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.494 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6151 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.494 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 14.7 \cdot in$

$Dp1 := root(Fp(D_p), D_p) = 9.238 \cdot in$ Must be less than $h_f = 7.5 \cdot in$ $\quad \frac{Dp1}{h} = 0.264$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$Mp1 = 26560.54 \cdot kip \cdot in \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 23512.4 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$Dp2 := root(Fp2(D_p), D_p) = 12.453 \cdot in$ Must be more than $h_f = 7.5 \cdot in$ $\quad \frac{Dp2}{h} = 0.3558$ must be < 0.42

$M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1 \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$Mp2 = 26062.94 \cdot kip \cdot in \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 21396.1 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1783 \cdot kip \cdot ft \qquad Mn = 21396.1 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 8.663 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 8.663 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.41 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 12.317 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 14.797 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 7.088 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 38.95 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 37.37 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 28.51 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 28.51 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.362\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.939\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.128\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.475\cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.808\cdot ksi$$

$$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 1303.5\cdot kip\cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.688\cdot ksi$$

$$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 1248\cdot kip\cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6\cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 2057.67\cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.253\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.939\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.331\cdot ksi$$

$$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1805.18\cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 372.922 \cdot in^3 \qquad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 424.706 \cdot in^3$$

short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 540.556 \cdot kip \cdot ft$$

From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.154 \times 10^3 \cdot kip \cdot ft$$

Additional moment to cause nominal yielding of steel flange

$$My := M_{D2} + M_{AD} = 1.695 \times 10^3 \cdot kip \cdot ft$$

D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$$S_{xt} := \frac{My}{F_y} = 406.691 \cdot in^3$$

Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.129 \times 10^3 \cdot kip \cdot ft$$

Moment due to Strength V limit state

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.294 \times 10^3 \cdot kip \cdot f$$

Equation 6.10.7.1.7-1

$$Mn = 1.783 \times 10^3 \cdot kip \cdot ft \qquad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

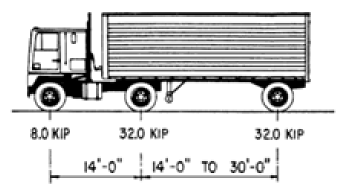
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.091 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

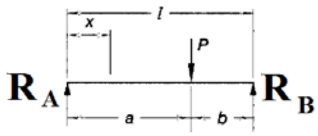
$b2 := \frac{L}{2} - (X_{ASr} - X') = 25.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 20.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.125 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.897 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

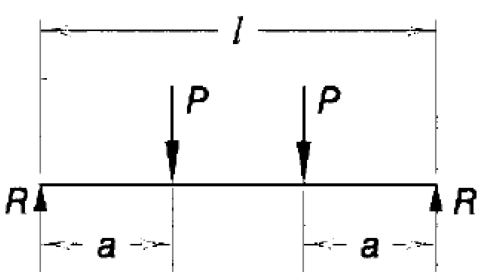
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.808 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.487 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.48 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.139 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.761 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.473}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 303.152 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.374 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.489 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.276 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.034 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 9.088 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 11.208 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 14.685 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 118.923 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 71.062 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 131.971 \cdot kip$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 131.971 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.15 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.749 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.564 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.936 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

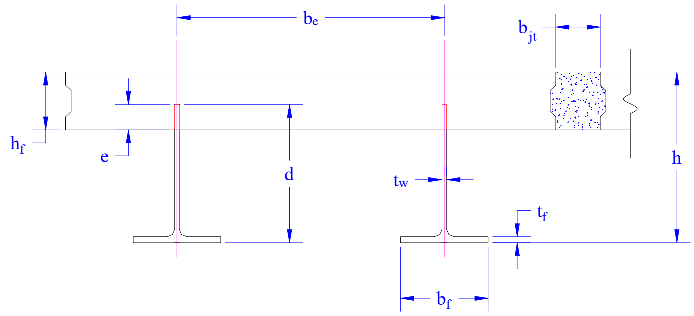
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 60ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 0.750in$ $\quad b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31 \cdot in$ depth of built up steel section

$h_w := d - t_f = 30.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 523.6917kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 125.11kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.876 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 2.907 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 27.906 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 354 \cdot in$ $W = 29.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 26.125\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.766 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.784 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.095 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.372 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 42.466 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \mathrm{floor}\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.295 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.095 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.39 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.078 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.498 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

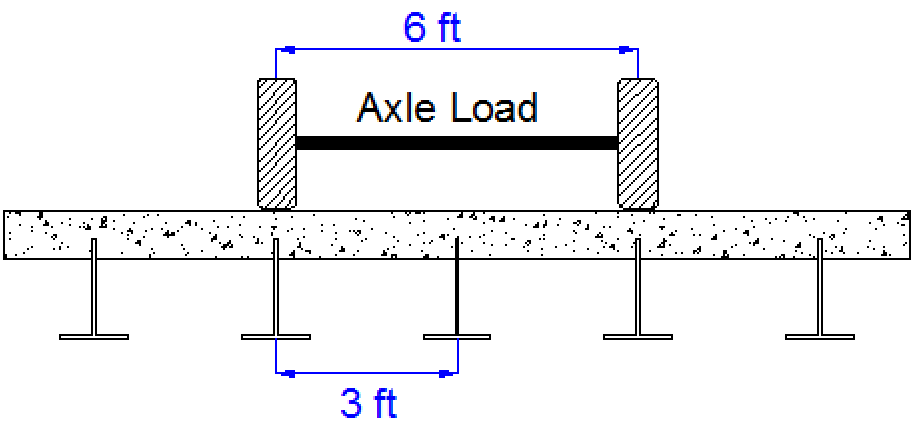
$$M_{DL} := \frac{DL \cdot L^2}{8} = 175.622 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 224.269 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 20.374 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^2\right) = 1.16 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 27.5 \cdot in$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{b_f^{\ 3} \cdot t_f}{12} = 108 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.643 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

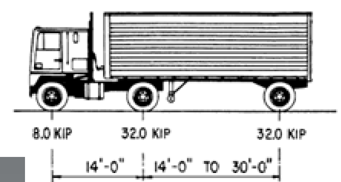
Design truck load in AASHTO (HS20 Truck) is used for this example.

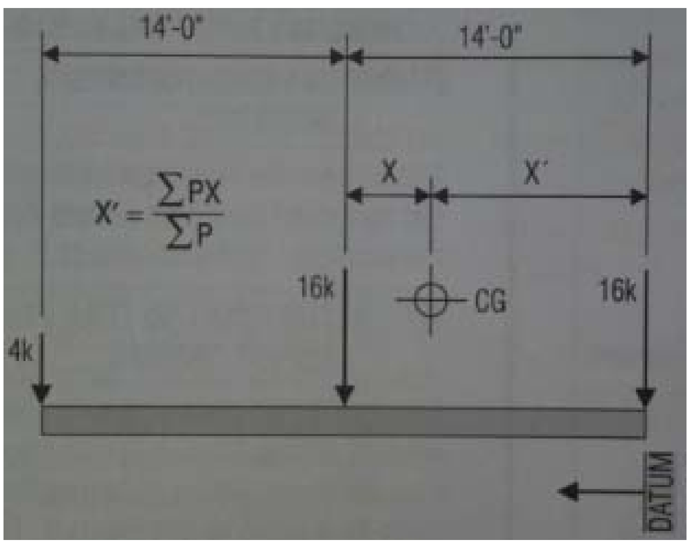
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ $\quad$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

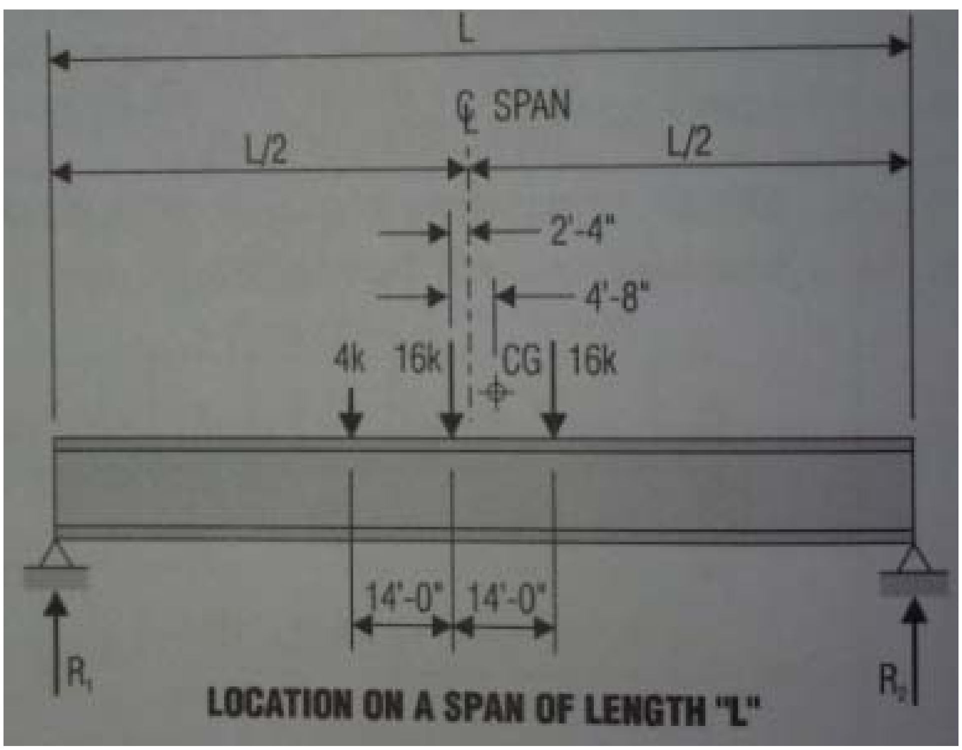
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30.208 \cdot \text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 876.042 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 349.541 \cdot \text{kip} \cdot \text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

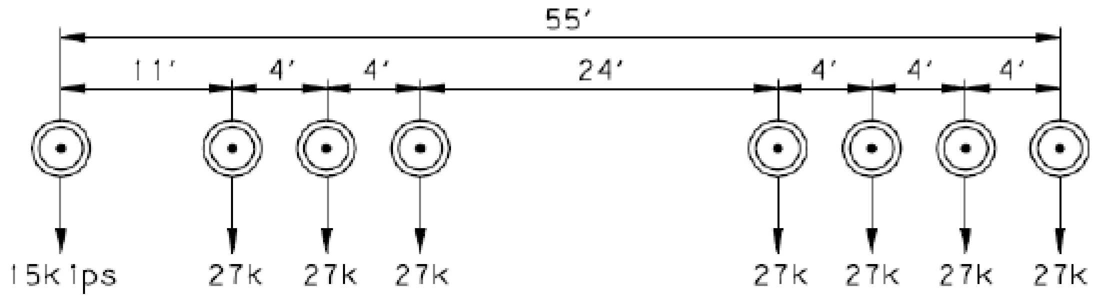
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64 \frac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 86.4 \cdot \text{kip} \cdot \text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 2.907 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot i$ $I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 158.203 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 22.029 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9406 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 22.029 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$

$I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 52.734 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.73 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6288 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.73 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 14.7 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 8.898 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.254}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 26840.03 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 23942.2 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 11.637 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.3325}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 26485.98 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 22175.7 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1848 \cdot kip \cdot ft \qquad Mn = 22175.7 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 7.826\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 7.826\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.607\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 12.251\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 14.718\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 7.16\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 39.19\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 37.618\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 28.69\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 28.694\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.363 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.902 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.083 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.464 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.728 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1318.8 \cdot kip \cdot ft$ $\quad$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.612 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1263 \cdot kip \cdot ft$ $\quad$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1999.51 \cdot psi$ $\quad$ This assumes short term composite section for all calculations $\quad$ $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.255 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.902 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.326 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1753.28 \cdot psi$ $\quad$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 375.851 \cdot in^3$ $\quad S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 426.993 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 555.931 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.148 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.703 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 408.838 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.144 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 1.311 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad Mn = 1.848 \times 10^3 \cdot kip \cdot ft \quad M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

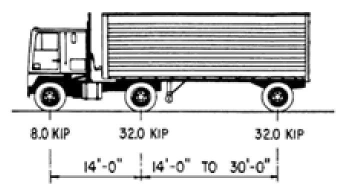
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.053 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

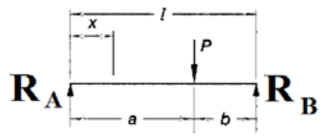
$b2 := \frac{L}{2} - (X_{ASr} - X') = 25.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 20.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.123 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.881 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

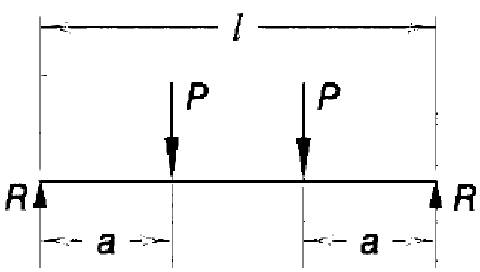
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.794 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.478 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}\left(3 l^2 - 4 a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.471 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.137 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.747 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.48}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 311.223 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.39 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.498 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.815 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.57 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 9.216 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 11.708 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 14.951 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 121.116 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

<u>Strength II Max Positive Shear:</u>

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 72.063 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 134.348 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 134.348 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.749 \times 10^4 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.564 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.936 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

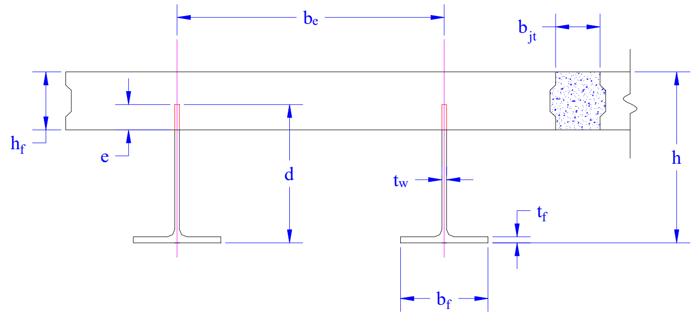
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $\quad$ $modules := \frac{N}{2} = 6$

$L := 60ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad$ $t_f := 0.750in$ $\quad$ $b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31\cdot in$ depth of built up steel section

$h_w := d - t_f = 30.25\cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 523.6917kip\cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 125.11kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2\cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1\cdot t_w = 1$

$$y\_bar := \frac{t_f\cdot b_f\cdot \frac{t_f}{2} + h_w\cdot t_w\cdot\left(t_f + \frac{h_w}{2}\right)}{t_f\cdot b_f + h_w\cdot t_w} = 10.876\cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f\cdot t_f^3}{12} + b_f\cdot t_f\cdot\left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w\cdot h_w^3}{12} + t_w\cdot h_w\cdot\left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 2.907\times 10^3\cdot in^4$$ MOI of steel

$A_{stl} := t_f\cdot b_f + h_w\cdot t_w = 27.906\cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \dfrac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 426 \cdot in$ $W = 35.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 32.125\,ft$

$D_c := (w_c \cdot h_f \cdot W) = 3.328 \cdot \dfrac{kip}{ft}$ DL due to slab

$D_b := .700 \dfrac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \dfrac{lbf}{ft^2} \cdot (W - 2 \cdot b_{par}) = 0.964 \cdot \dfrac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \dfrac{lbf}{ft^3} = 0.095 \cdot \dfrac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \dfrac{(D_c)}{N} + D_s = 0.372 \cdot \dfrac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \dfrac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[(2b_e - b_{jt})(h_f \cdot w_c)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 42.466 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\dfrac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.296 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.095 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.391 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.08 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.5 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

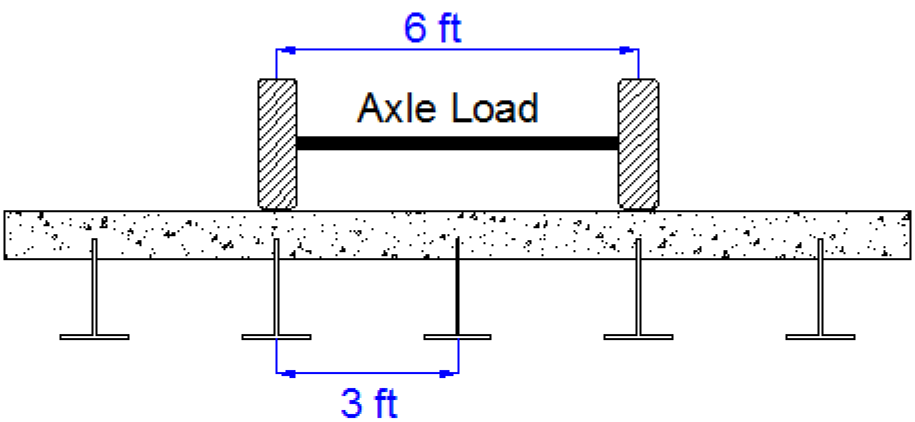
$$M_{DL} := \frac{DL \cdot L^2}{8} = 175.974 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 225.141 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 20.374 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.16 \times 10^5 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 27.5 \cdot in$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.643 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

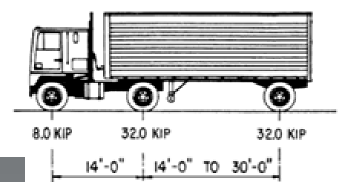
Design truck load in AASHTO (HS20 Truck) is used for this example.

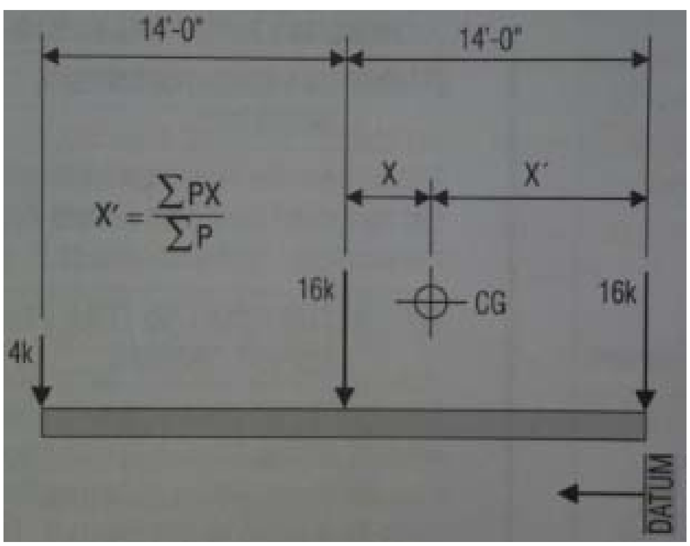
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

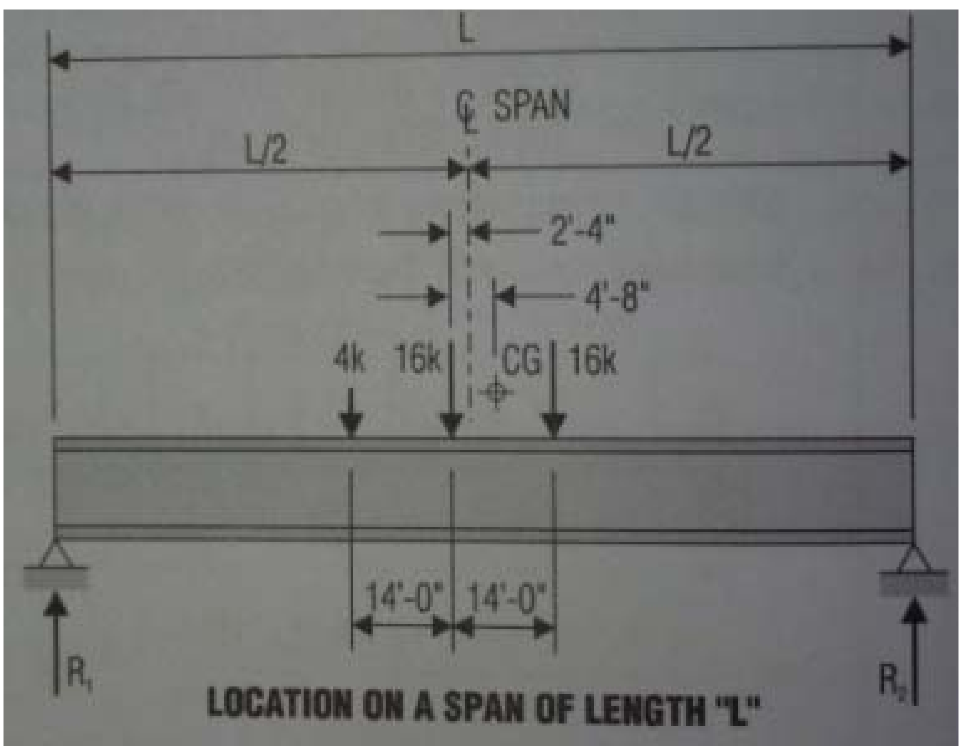
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \frac{F_T\cdot\left(\frac{L}{2} + \frac{X}{2}\right) + F_T\cdot\left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30.208\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2} - \frac{X}{2}\right) = 876.042\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1 + I) = 349.541\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

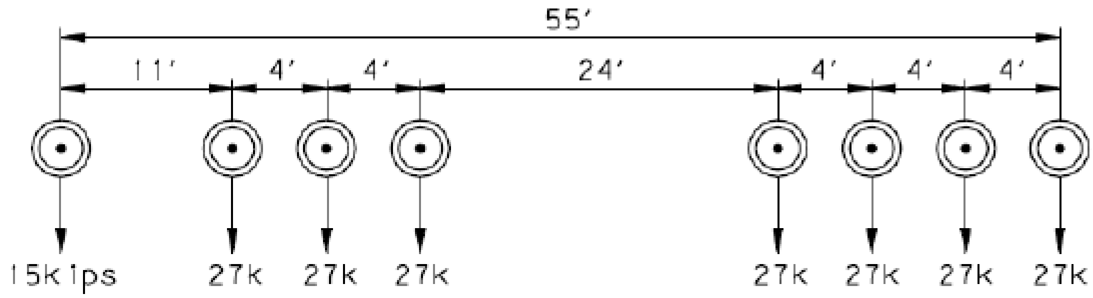
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 86.4\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 2.907 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot i \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 158.203 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 22.029 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9406 \cdot in^4$$

Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 22.029 \cdot in$$

N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 52.734 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.73 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6288 \cdot in^4$$

Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 16.73 \cdot in$$

N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp\left(D_p\right) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 14.7 \cdot in$

$Dp1 := root\left(Fp\left(D_p\right), D_p\right) = 8.898 \cdot in$ Must be less than $h_f = 7.5 \cdot in$ $\quad \frac{Dp1}{h} = 0.254$ must be < 0.42

$M_{n1check} := \left(Dp1 < h_f\right) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot \left(b_f \cdot t_f\right) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot \left(h - Dp1 - t_f\right)\right] \cdot \frac{\left(h - Dp1 - t_f\right)}{2}\right]$$

$Mp1 = 26840.03 \cdot kip \cdot in \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 23942.2 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2\left(D_p\right) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 2.5in$

$Dp2 := root\left(Fp2\left(D_p\right), D_p\right) = 11.637 \cdot in$ Must be more than $h_f = 7.5 \cdot in$ $\quad \frac{Dp2}{h} = 0.3325$ must be < 0.42

$M_{n2check} := \left(Dp2 > h_f\right) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1 \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot \left(b_f \cdot t_f\right) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot \left(h - Dp2 - t_f\right)\right] \cdot \frac{\left(h - Dp2 - t_f\right)}{2}$$

$Mp2 = 26485.98 \cdot kip \cdot in \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 22175.7 \cdot kip \cdot in$

$Mn := max\left(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}\right) = 1848 \cdot kip \cdot ft \qquad Mn = 22175.7 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1\right) = 7.826 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 7.826 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.618 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 12.251 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 14.718 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 7.188 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 39.245 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 37.674 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 28.73 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 28.733 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.364 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.902 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.083 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.466 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.732 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1320.6 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.616 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1265 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2002.04 \cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.256 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.902 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.327 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1755.05 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 375.851 \cdot in^3 \qquad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 426.993 \cdot in^3$$

short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 557.678 \cdot kip \cdot ft$$

From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}} \right) \cdot S_{ST} = 1.146 \times 10^3 \cdot kip \cdot ft$$

Additional moment to cause nominal yielding of steel flange

$$My := M_{D2} + M_{AD} = 1.703 \times 10^3 \cdot kip \cdot ft$$

D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$$S_{xt} := \frac{My}{F_y} = 408.781 \cdot in^3$$

Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 1.146 \times 10^3 \cdot kip \cdot ft$$

Moment due to Strength V limit state

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.312 \times 10^3 \cdot kip \cdot f$$

Equation 6.10.7.1.7-1

$$Mn = 1.848 \times 10^3 \cdot kip \cdot ft$$

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.167$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

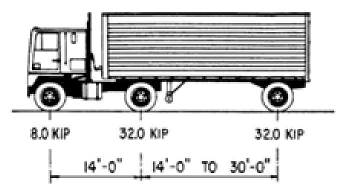
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.053 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

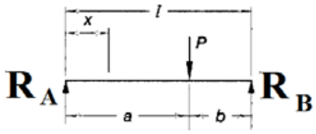
$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

$b2 := \frac{L}{2} - (X_{ASr} - X') = 25.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 20.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support



$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.123 \cdot in$$

Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.881 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

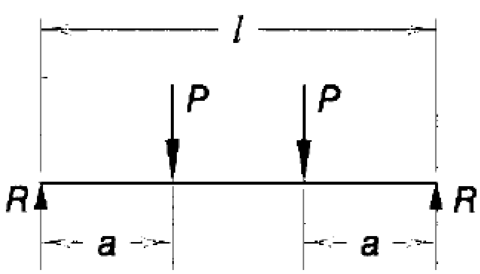
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.794 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.398 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.392 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 106.667 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.114 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.623 \cdot in}$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.48}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 311.223 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.391 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.5 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - \left(X_{ASr} + X_{ASf}\right) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.815 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{\left(L - X_{AS}\right)}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.57 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot \left(DF_s\right) = 9.216 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 11.732 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 15.009 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 121.232 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

<u>Strength II Max Positive Shear:</u>

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 72.063 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 134.464 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 134.464 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements — $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.749 \times 10^4 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) — $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.564 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.936 \cdot kip$ — Compressive capacity of bearing stiffener — $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

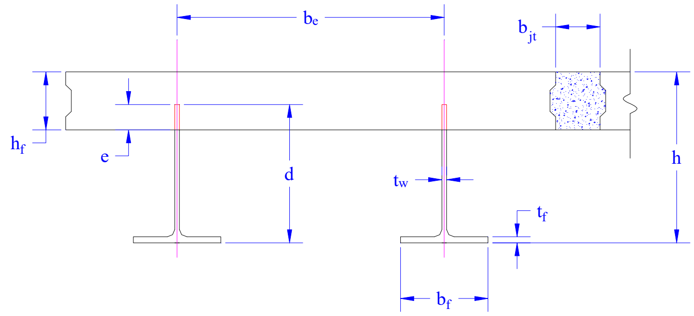
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 60ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 0.875in$ $\quad b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31 \cdot in$ depth of built up steel section

$h_w := d - t_f = 30.125 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 523.6917kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 125.11kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.388 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.044 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.328 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 374 \cdot in$ $W = 31.167\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 27.792\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.922 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.834 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.1 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.392 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 44.921 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.312 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.1 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.412 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.083 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.507 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

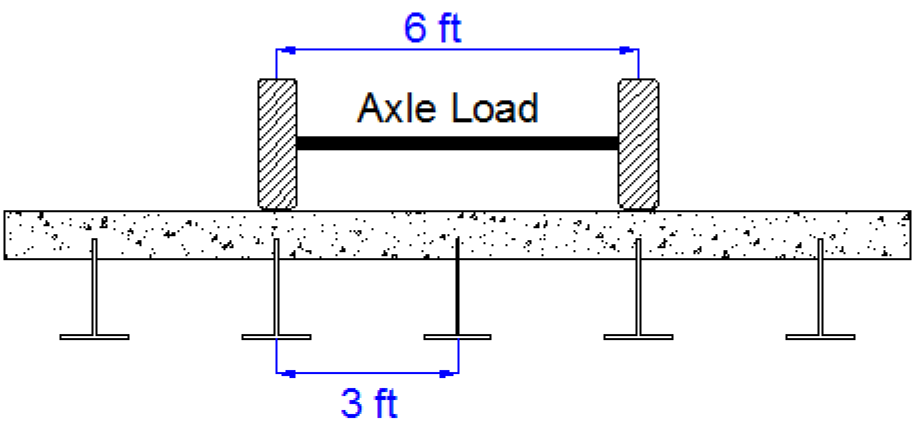
$$M_{DL} := \frac{DL \cdot L^2}{8} = 185.299 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 228.269 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 20.862 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.26 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 27.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 126 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 12.551 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

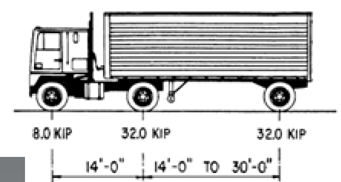
Design truck load in AASHTO (HS20 Truck) is used for this example.

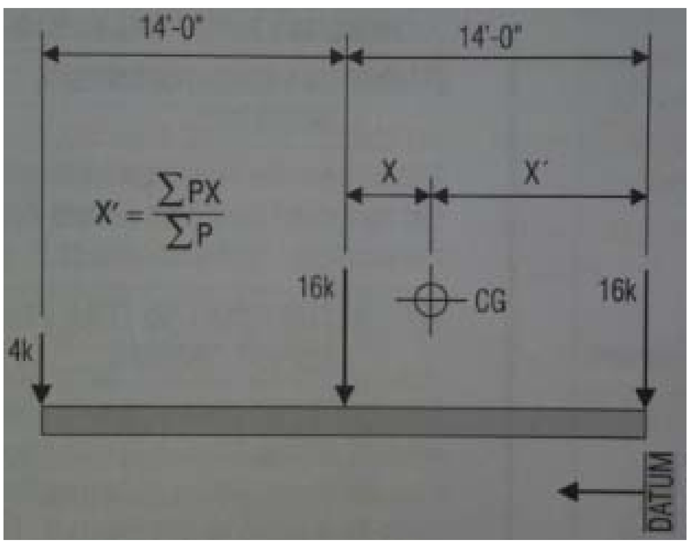
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

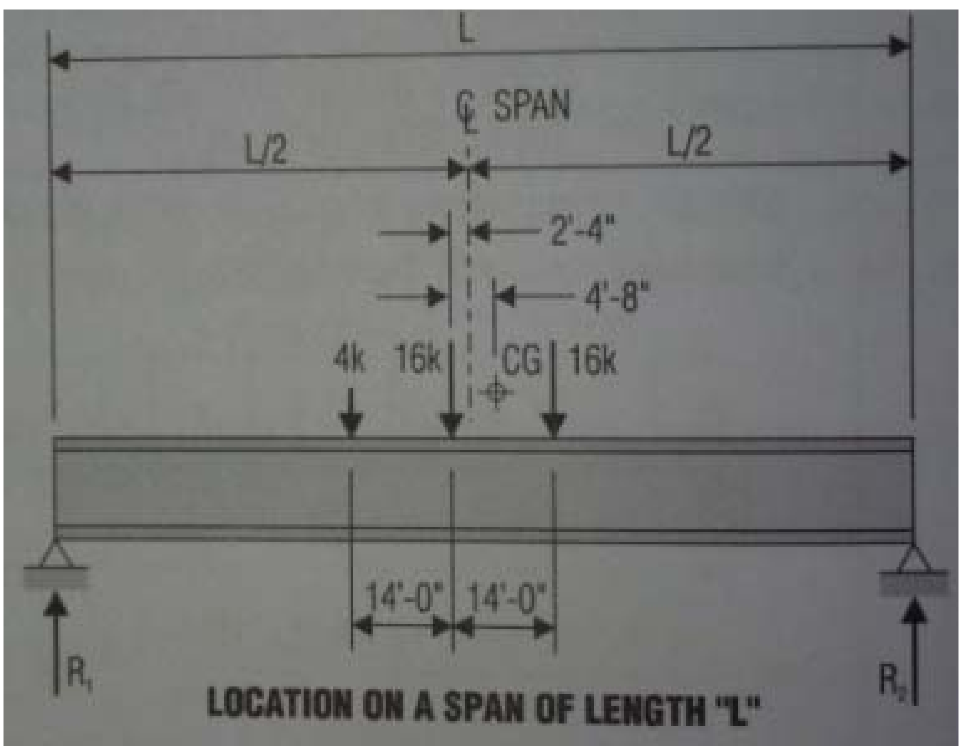
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.208\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 876.042\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 349.541\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

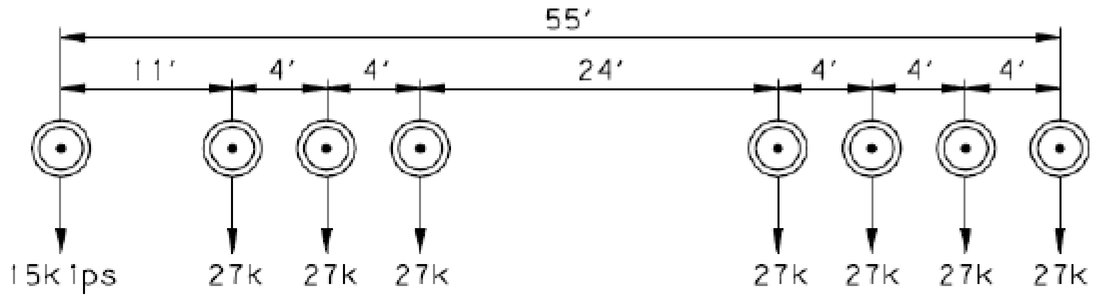
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\dfrac{F_{DLL}\cdot L^2}{8}\right) = 86.4\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.044 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$

$Y' := \dfrac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 21.83 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 10212 \cdot in^4$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 21.83 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$

$Y' := \dfrac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.401 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 6778 \cdot in^4$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.401 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 14.7 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 8.954 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\quad \boxed{\frac{Dp1}{h} = 0.256}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 28906.86 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 25753.9 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 11.959 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\quad \boxed{\frac{Dp2}{h} = 0.3417}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 28488.19 \cdot kip \cdot in}$

$$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 23668.8 \cdot kip \cdot in$$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1972.4 \cdot kip \cdot ft \qquad Mn = 23668.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 8.19 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 8.19 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.38 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 11.183 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.434 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 6.628 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 36.237 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 34.803 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 26.55 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.546 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.358 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.843 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.013 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.442 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.586 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1336.9 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.478 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1281 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1896.36 \cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.254 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.843 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.313 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1663.72 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 413.298 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 467.778 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 574.027 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.299 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.873 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 449.618 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 1.163 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.319 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad Mn = 1.972 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \dfrac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

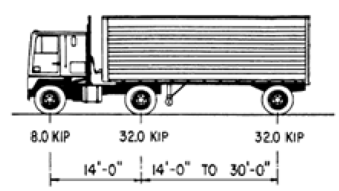
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.891 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \dfrac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

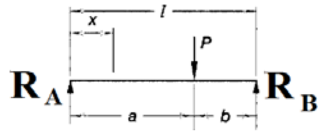
$b2 := \dfrac{L}{2} - (X_{ASr} - X') = 25.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \dfrac{L}{2} - X' = 20.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.113 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.811 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

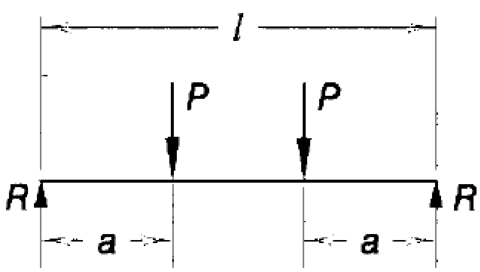
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.731 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.44 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.434 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.126 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.688 \cdot in}$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 335.576 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.412 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.507 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 ***(at left support)*** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.354 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.106 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 9.344 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 12.353 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 15.218 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 123.49 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 73.064 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 136.905 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 136.905 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.025 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.775 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.579 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.95 \cdot kip$ — Compressive capacity of bearing stiffener — $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

Summary of Calculations:

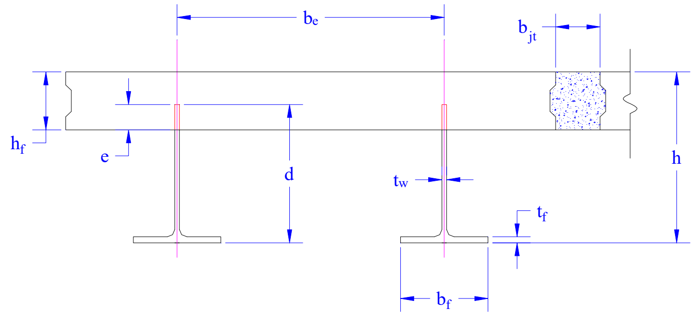
1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:

Number of beams $N := 14$ $\quad modules := \frac{N}{2} = 7$



$L := 60ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 0.875in$ $\quad b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31 \cdot in$ depth of built up steel section

$h_w := d - t_f = 30.125 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 523.6917kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 125.11kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.388 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.044 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.328 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 526 \cdot \text{in}$ $W = 43.833\text{ ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 40.458\text{ ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 4.109 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.214 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.1 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.393 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 44.921 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.313 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.1 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.413 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.087 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.511 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

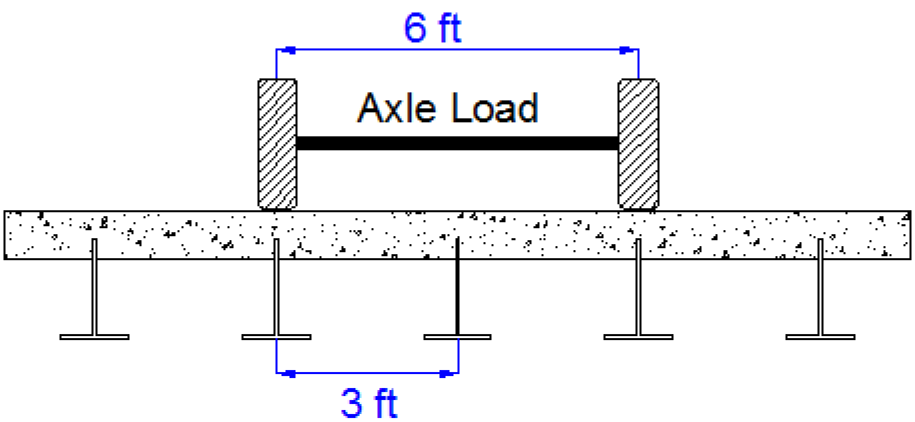
$$M_{DL} := \frac{DL \cdot L^2}{8} = 185.902 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 229.763 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 20.862 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\ 2}\right) = 1.26 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 27.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 126 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 12.551 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

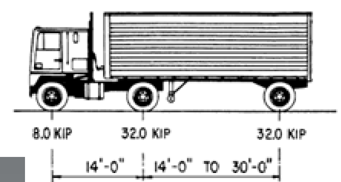
Design truck load in AASHTO (HS20 Truck) is used for this example.

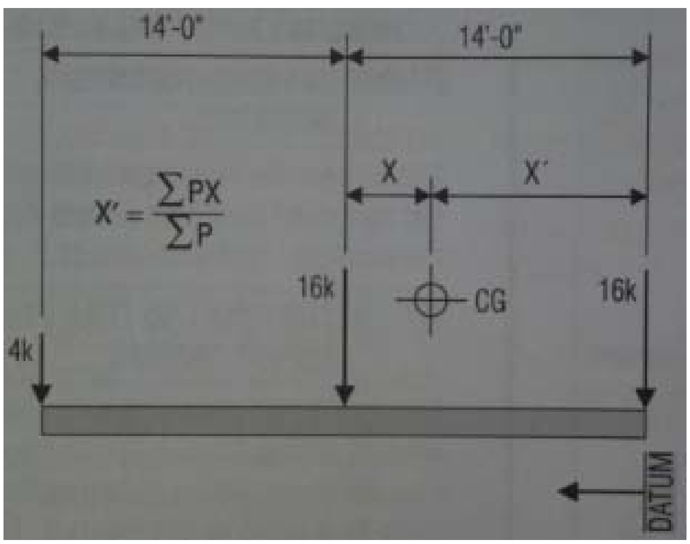
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

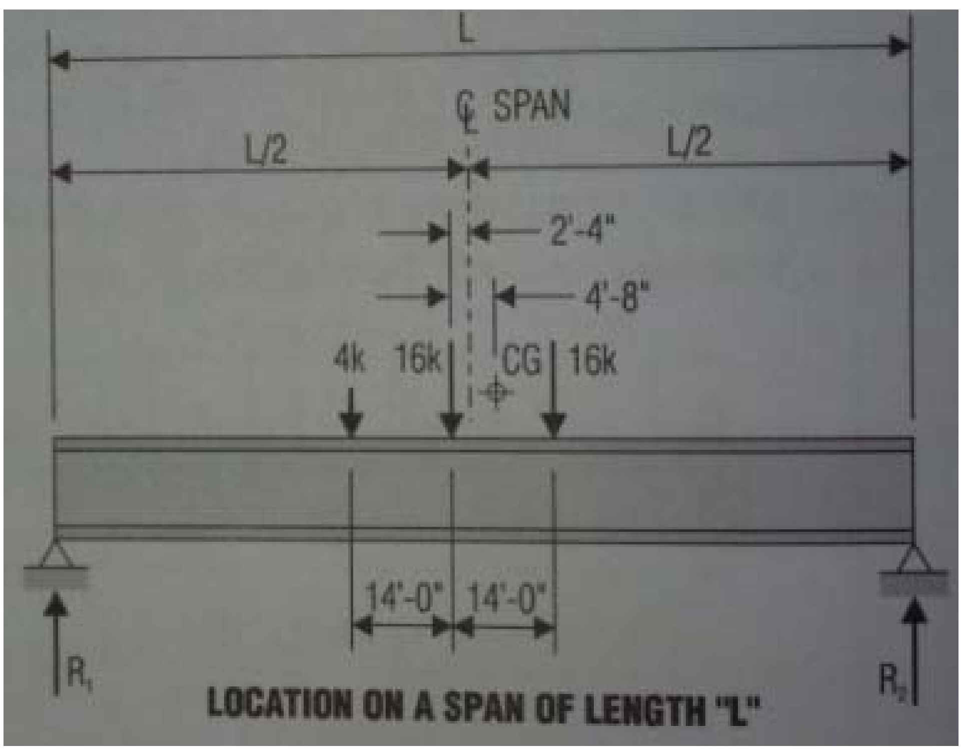
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.208\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 876.042\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 349.541\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

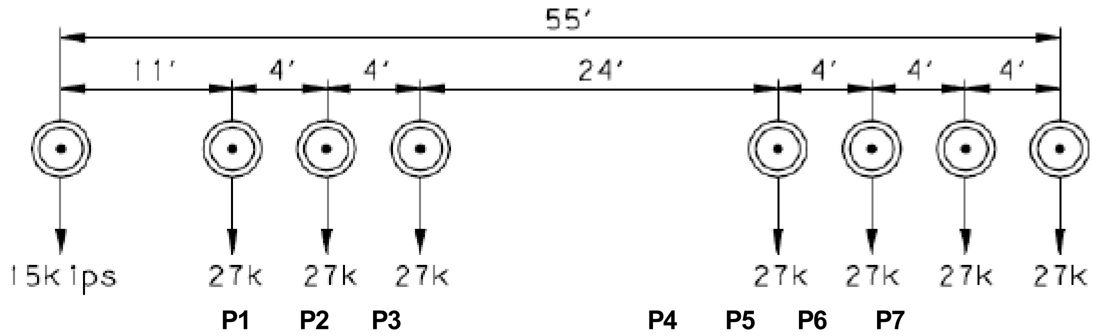
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\dfrac{F_{DLL}\cdot L^2}{8}\right) = 86.4\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.044 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 21.83 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 10212 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 21.83 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.401 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6778 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.401 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 14.7 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 8.954 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.256}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h - d)]] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 28906.86 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 25753.9 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 11.959 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.3417}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h - d)]] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 28488.19 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 23668.8 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 1972.4 \cdot kip \cdot ft \qquad Mn = 23668.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 8.19 \cdot in$ $\quad D_{cp0} := D_{cp} > 0 = 1$

$D_{cp} := D_{cp} \cdot D_{cp0} = 8.19 \cdot in$ $\quad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$ Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.398 \cdot ksi$

$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 11.183 \cdot ksi$ $\quad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.434 \cdot ksi$

$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 6.671 \cdot ksi$

STRENGTH I $\quad f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 36.324 \cdot ksi$

STRENGTH II $\quad f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 34.89 \cdot ksi$

SERVICE II $\quad f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 26.61 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$f_f + \frac{f_{lSII}}{2} = 26.607 \cdot ksi$ $\quad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi$ $\quad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.36 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.843 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.013 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.444 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.592 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1339.9 \cdot kip \cdot ft$ $\quad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.484 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1284 \cdot kip \cdot ft$ $\quad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1900.42 \cdot psi$ This assumes short term composite section for all calculations $\quad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.255 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.843 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.315 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1666.59 \cdot psi$ $\quad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 413.298 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 467.778 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 577.023 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.296 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.873 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 449.523 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.166 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.322 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad Mn = 1.972 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

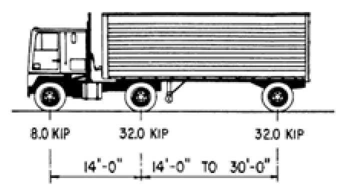
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.891 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

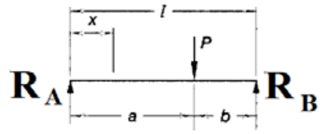
$b2 := \frac{L}{2} - (X_{ASr} - X') = 25.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 20.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.113 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.811 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

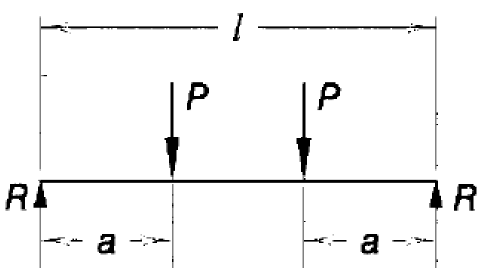
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.731 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.472 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.465 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.135 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.737 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \dfrac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \dfrac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \dfrac{h_f}{2} \right) = 335.576 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.413 \cdot \dfrac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.511 \cdot \dfrac{kip}{ft}$

Design Lane Load: $0.64 \dfrac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.354 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.106 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 9.344 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 12.393 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 15.318 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 123.689 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 73.064 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 137.105 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 137.105 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.025 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \leq 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \geq Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.775 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \geq 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.579 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.95 \cdot kip$ Compressive capacity of bearing stiffener $Pr \geq Vu = 1$

Flexbeam Example Configuration Calculations

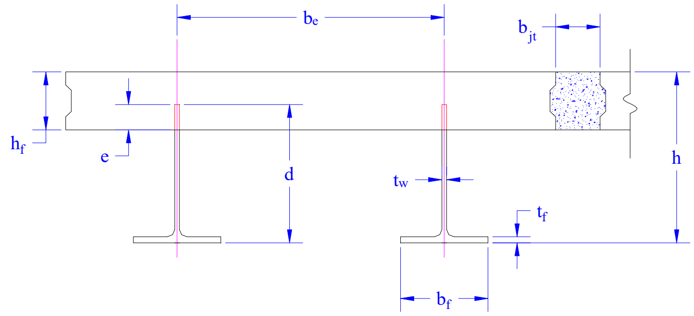
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 60ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 0.875in$ $\quad b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31 \cdot in$ depth of built up steel section

$h_w := d - t_f = 30.125 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 523.6917kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 125.11kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.388 \cdot in$$

Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.044 \times 10^3 \cdot in^4$$

MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.328 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 394 \cdot in$ $W = 32.833\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 29.458\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.078 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.884 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.1 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.408 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 46.796 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$ Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.1 \cdot \frac{kip}{ft}$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.428 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.088 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.516 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

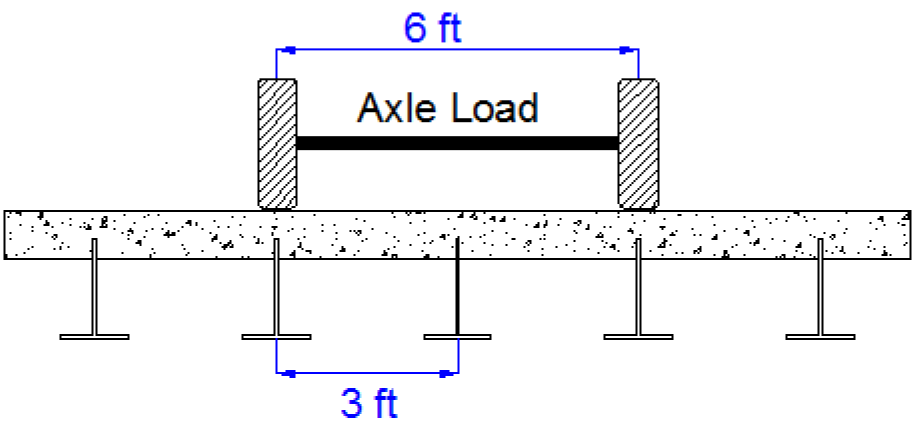
$$M_{DL} := \frac{DL \cdot L^2}{8} = 192.799 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 232.269 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 20.862 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.26 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 27.5 \cdot in$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{{b_f}^3 \cdot t_f}{12} = 126 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 12.551 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

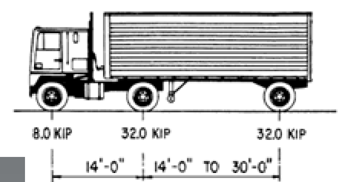
Design truck load in AASHTO (HS20 Truck) is used for this example.

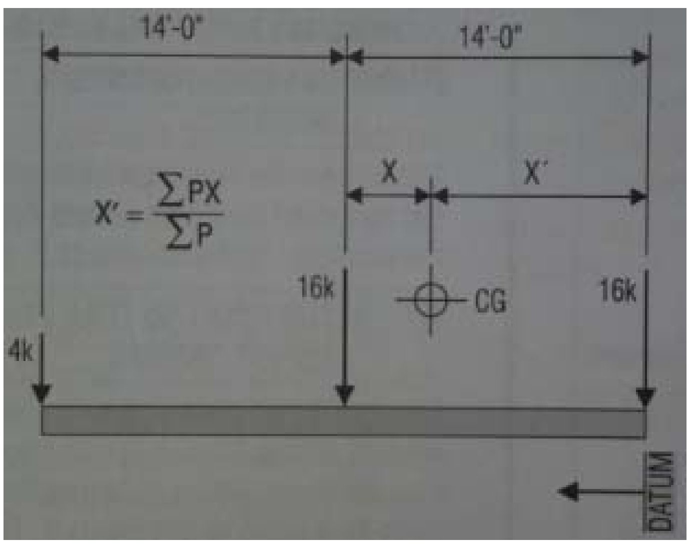
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

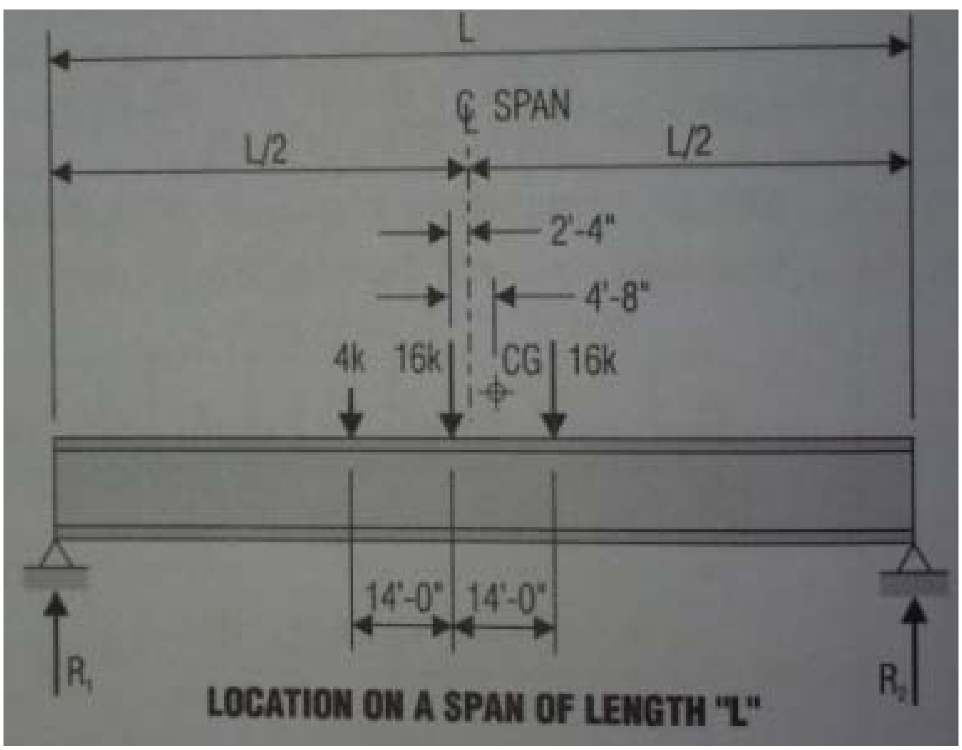
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30.208 \cdot \text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 876.042 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 349.541 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

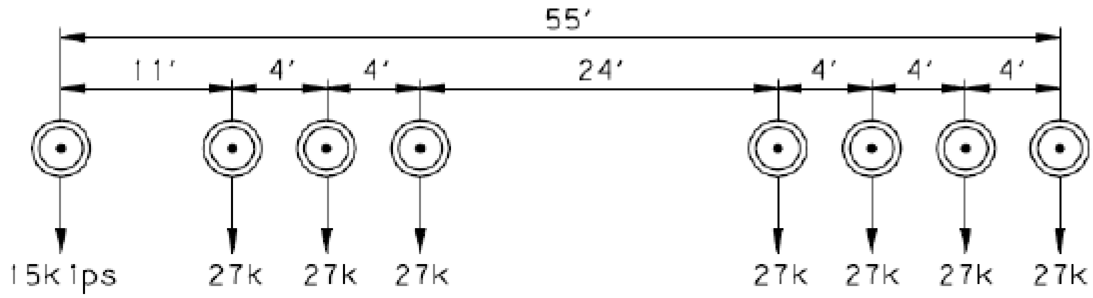
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \dfrac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 86.4 \cdot \text{kip} \cdot \text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.044 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$Y' := \dfrac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 22.095 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 10382 \cdot in^4$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 22.095 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$Y' := \dfrac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.623 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 6917 \cdot in^4$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.623 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 14.7 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 8.647 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.247}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 29170.09 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 26167.4 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming $h_f < D_p$

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 11.143 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.3184}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 28886.02 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 24470.8 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2039.2 \cdot kip \cdot ft \qquad Mn = 24470.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 7.35 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 7.35 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.56 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 11.133 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.374 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 6.698 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 36.479 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.051 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 26.73 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.731 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.359\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.813\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.976\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.433\cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.521\cdot ksi$

$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 1352.3\cdot kip\cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.417\cdot ksi$

$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 1296\cdot kip\cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6\cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1849.23\cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.256\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1\cdot n\cdot I_{c\_short}} = 0.813\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_long})}{k_3\cdot n\cdot I_{c\_long}} = 0.309\cdot ksi$$

$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1621.34\cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 416.124 \cdot in^3$ $\quad$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 469.899 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 589.402 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.292 \times 10^3 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.882 \times 10^3 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 451.619 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.178 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.335 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 2.039 \times 10^3 \cdot kip \cdot ft$ $\quad$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

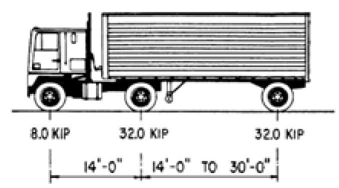
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.86 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

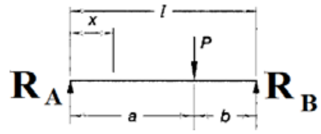
$b2 := \frac{L}{2} - (X_{ASr} - X') = 25.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 20.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.112 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.798 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

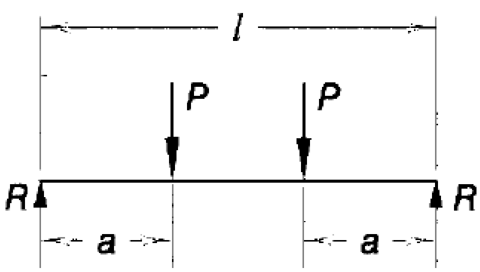
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.719 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.433 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.427 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.124 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.677 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 343.327 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.428 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.516 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.893 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.641 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 9.472 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 12.853 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 15.485 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 125.682 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 74.065 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 139.281 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 139.281 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.025 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.775 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.579 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.95 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

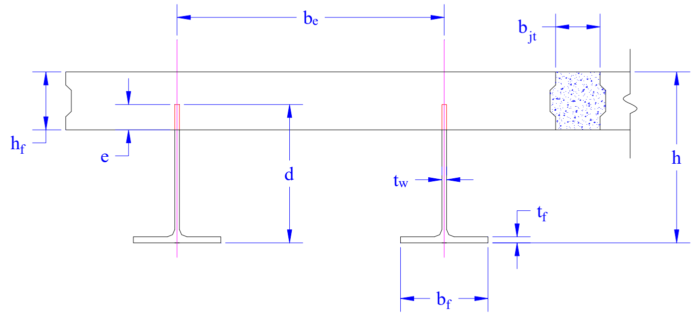
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $modules := \frac{N}{2} = 6$

$L := 60ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $t_f := 0.875in$ $b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31 \cdot in$ depth of built up steel section

$h_w := d - t_f = 30.125 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 523.6917kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 125.11kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.388 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.044 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.328 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 474 \cdot in$ $W = 39.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 36.125\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.703 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.084 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.1 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.408 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 46.796 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12ft}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.1 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.429 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.09 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.518 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

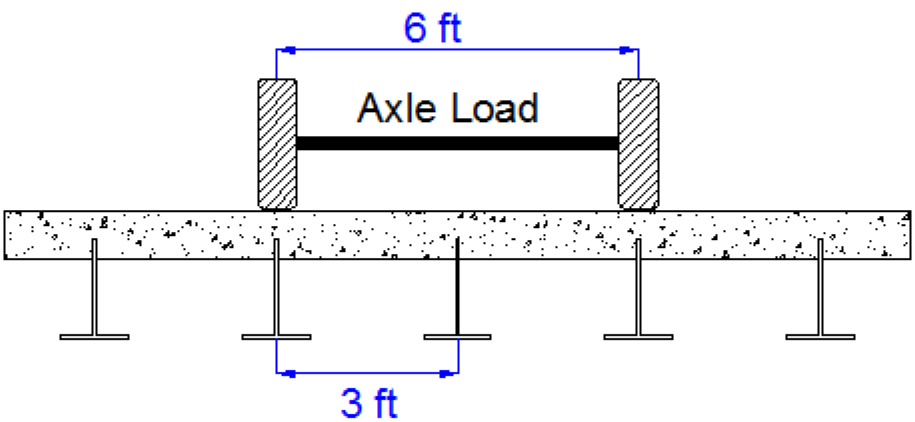
$$M_{DL} := \frac{DL \cdot L^2}{8} = 193.151 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 233.141 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.333$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 20.862 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.26 \times 10^5 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 27.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{{b_f}^3 \cdot t_f}{12} = 126 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 12.551 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

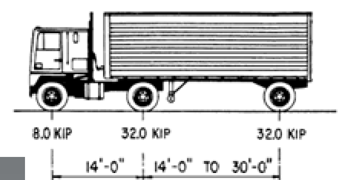
Design truck load in AASHTO (HS20 Truck) is used for this example.

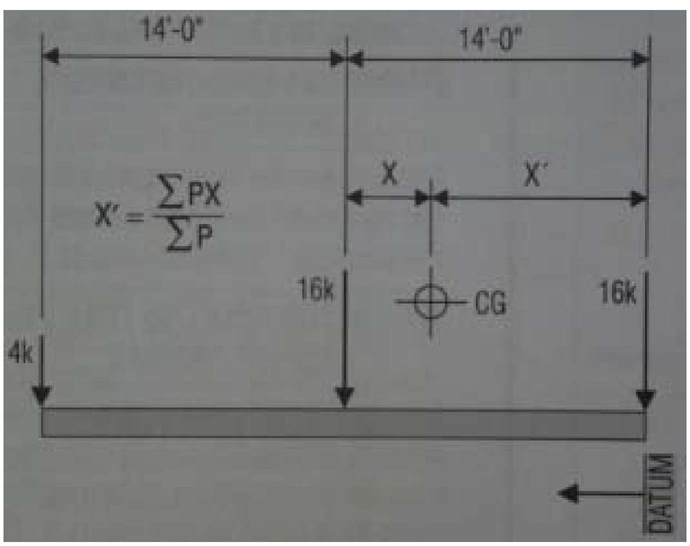
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

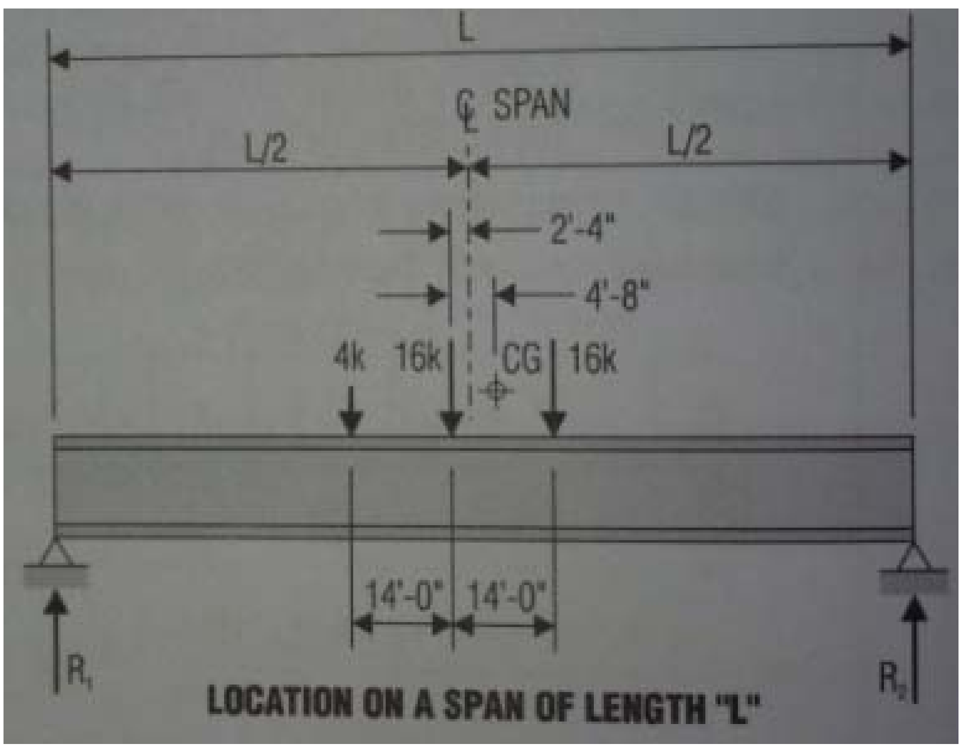
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.208\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 876.042\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 349.541\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

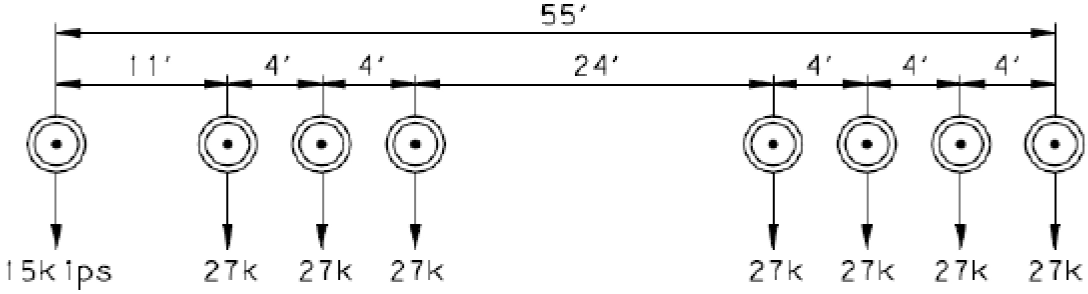
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 86.4\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.044 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2 \quad I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 22.095 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 10382 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 22.095 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2 \qquad I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.623 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6917 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.623 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4\cdot in$

$$Fp(D_p) := 0.85\cdot f_c\cdot b_e\cdot D_p + F_y\cdot\left[t_w\cdot\left[D_p - (h-d)\right]\right] - F_y\cdot\left[A_{stl} - \left[t_w\cdot\left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42\cdot h = 14.7\cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 8.647\cdot in}$ Must be less than $h_f = 7.5\cdot in$ $\boxed{\frac{Dp1}{h} = 0.247}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f)\cdot[(h-d) < Dp1]\cdot(Dp1 < 0.42\cdot h) = 0 \qquad Dp1 \le 0.42\cdot h = 1$

$$Mp1 := 0.85\cdot f_c\cdot b_e\cdot Dp1\cdot\left(\frac{Dp1}{2}\right) + F_y\cdot\left[t_w\cdot[Dp1 - (h-d)]\right]\cdot\frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+\left[F_y\cdot(b_f\cdot t_f)\cdot\left(h - Dp1 - \frac{t_f}{2}\right) + F_y\cdot\left[t_w\cdot(h - Dp1 - t_f)\right]\cdot\frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 29170.09\cdot kip\cdot in} \qquad M_{n1} := Mp1\cdot\left(1.07 - 0.7\cdot\frac{Dp1}{h}\right) = 26167.4\cdot kip\cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming $h_f < D_p$

$$Fp2(D_p) := 0.85\cdot f_c\cdot b_e\cdot h_f + F_y\cdot\left[t_w\cdot\left[D_p - (h-d)\right]\right] - F_y\cdot\left[A_{stl} - \left[t_w\cdot\left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 11.143\cdot in}$ Must be more than $h_f = 7.5\cdot in$ $\boxed{\frac{Dp2}{h} = 0.3184}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f)\cdot(Dp2 < h)\cdot(Dp2 < 0.42\cdot h) = 1} \qquad Dp2 \le 0.42\cdot h = 1$

$$Mp2 := 0.85\cdot f_c\cdot b_e\cdot h_f\cdot\left(Dp2 - \frac{h_f}{2}\right) + F_y\cdot\left[t_w\cdot[Dp2 - (h-d)]\right]\cdot\frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+F_y\cdot(b_f\cdot t_f)\cdot\left(h - Dp2 - \frac{t_f}{2}\right) + F_y\cdot\left[t_w\cdot(h - Dp2 - t_f)\right]\cdot\frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 28886.02\cdot kip\cdot in} \qquad M_{n2} := Mp2\cdot\left(1.07 - 0.7\cdot\frac{Dp2}{h}\right) = 24470.8\cdot kip\cdot in$

$Mn := \max(M_{n1}\cdot M_{n1check}, M_{n2}\cdot M_{n2check}) = 2039.2\cdot kip\cdot ft \qquad Mn = 24470.8\cdot kip\cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 7.35\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 7.35\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.57\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 11.133\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.374\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 6.723\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 36.53\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 35.102\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 26.77\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.766\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.36 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.813 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.976 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.435 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.525 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1354 \cdot kip \cdot ft$ $\quad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.42 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1298 \cdot kip \cdot ft$ $\quad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1851.51 \cdot psi$ This assumes short term composite section for all calculations $\quad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.257 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.813 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.31 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1622.97 \cdot psi$ $\quad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 416.124 \cdot in^3$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 469.899 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 591.15 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.29 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.882 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 451.565 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.18 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.337 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $Mn = 2.039 \times 10^3 \cdot kip \cdot ft$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.25$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

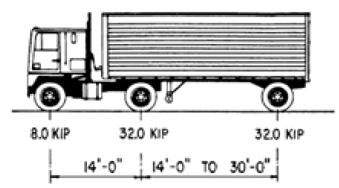
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.86 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot \text{ft}$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

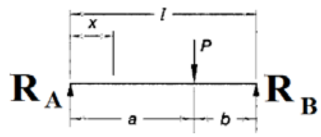
$b2 := \frac{L}{2} - (X_{ASr} - X') = 25.333 \cdot \text{ft}$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 20.667 \cdot \text{ft}$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.112 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.798 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

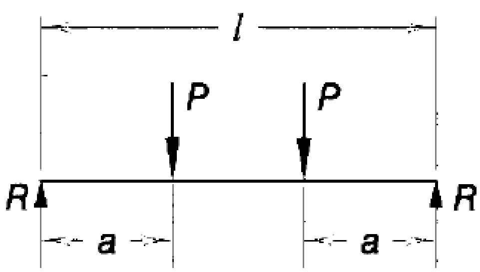
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.719 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.541 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.533 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 160 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.155 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.846 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$DF_s := 0.36 + \frac{S}{25} = 0.493$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 343.327 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.429 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.518 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$X_{ASr} := 14ft$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.893 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.641 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 9.472 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 12.877 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 15.543 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 125.799 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 74.065 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 139.398 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 139.398 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.025 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.775 \times 10^4 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.579 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.95 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

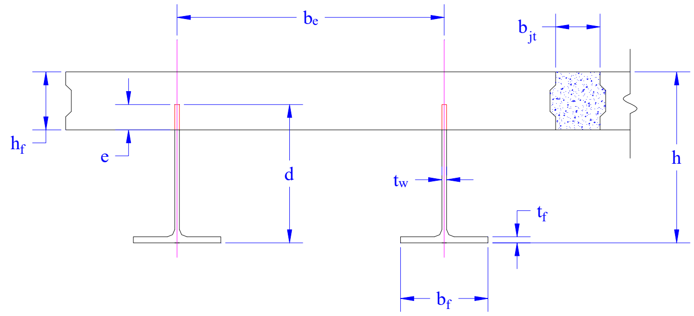
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  - AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  - PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $\text{modules} := \frac{N}{2} = 7$

$L := 60ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $t_f := 0.875in$ $b_f := 12in$

$h := 35in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 31 \cdot in$ depth of built up steel section

$h_w := d - t_f = 30.125 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 523.6917kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 125.11kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.388 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.044 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.328 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 554 \cdot in$ $W = 46.167\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 42.792\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 4.328 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700\,\frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.284 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{lbf}{ft^3} = 0.1 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.409 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 46.796 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.33 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.1 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.43 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.092 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.519 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

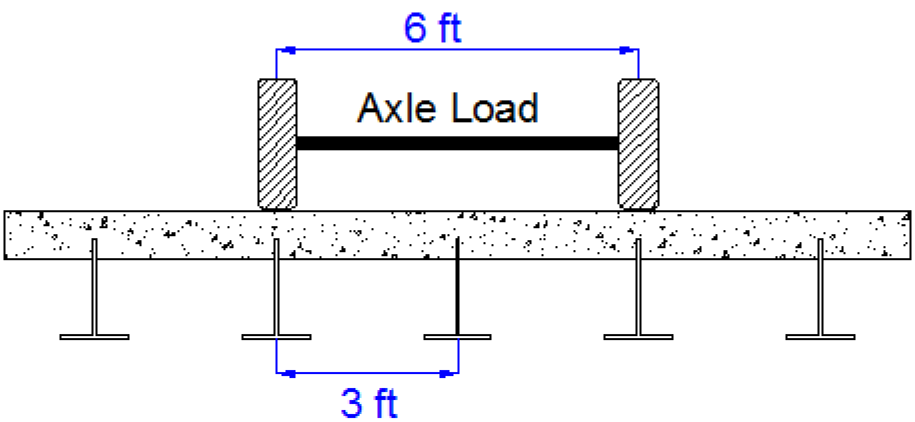
$$M_{DL} := \frac{DL \cdot L^2}{8} = 193.402 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 233.763 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 20.862 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.26 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 27.5 \cdot in$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 21.965 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 126 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 12.551 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

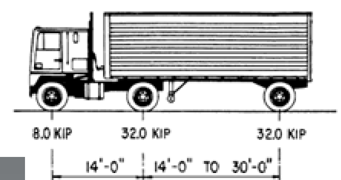
Design truck load in AASHTO (HS20 Truck) is used for this example.

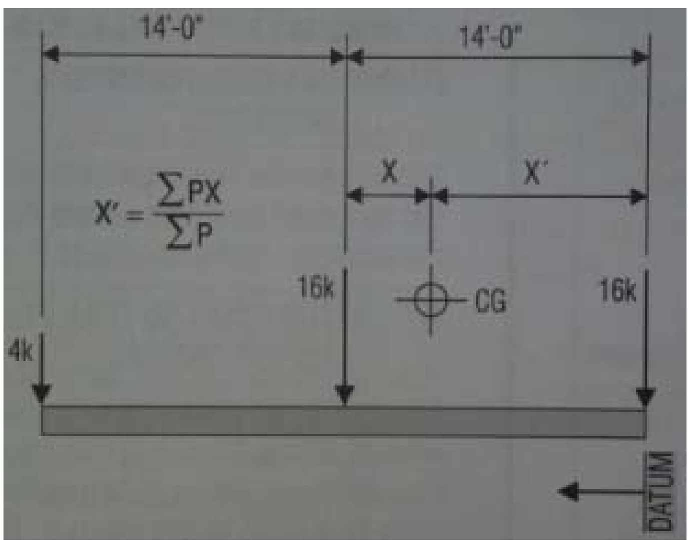
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

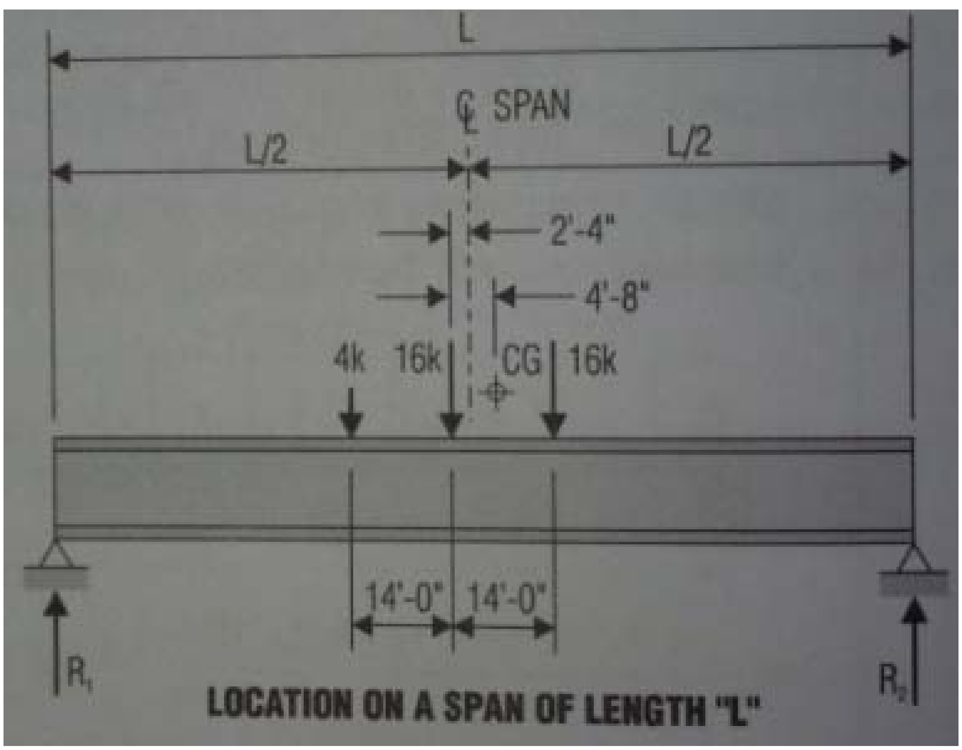
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.2 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 806.533 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.208\cdot\text{kip}$$

Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 876.042\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 876.042\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 349.541\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

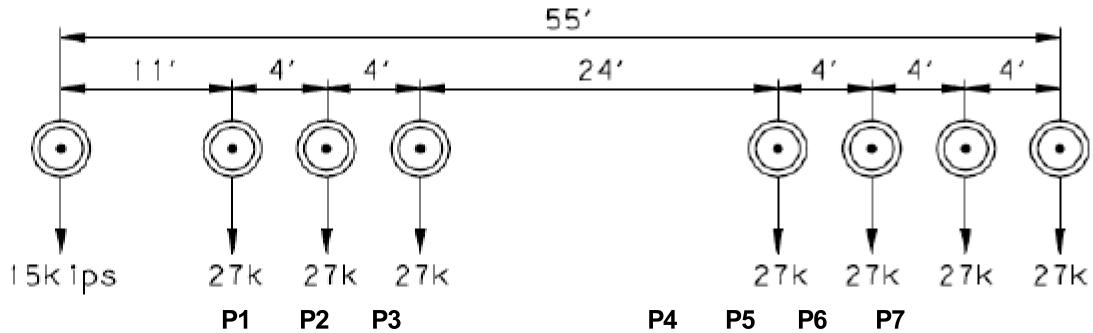
$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 86.4\cdot\text{kip}\cdot\text{ft}$$

Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 435.941\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.044 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2 \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^3}{12} = 175.781 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 22.095 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 10382 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 22.095 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^3}{12} = 58.594 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.623 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 6917 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.623 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 14.7 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 8.647 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.247}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0$ $\qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 29170.09 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 26167.4 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 11.143 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.3184}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1}$ $\qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 28886.02 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 24470.8 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2039.2 \cdot kip \cdot ft$ $\qquad Mn = 24470.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 7.35\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 7.35\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 5.577\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 11.133\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.374\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 6.741\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 36.566\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 35.138\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 26.79\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.791\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.361 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.813 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.976 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.436 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.527 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1355.3 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.423 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1299 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1853.14 \cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.257 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.813 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.311 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1624.13 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 416.124 \cdot in^3 \qquad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 469.899 \cdot in^3$$

short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 592.398 \cdot kip \cdot ft$$

From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.289 \times 10^3 \cdot kip \cdot ft$$

Additional moment to cause nominal yielding of steel flange

$$My := M_{D2} + M_{AD} = 1.881 \times 10^3 \cdot kip \cdot ft$$

D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$$S_{xt} := \frac{My}{F_y} = 451.526 \cdot in^3$$

Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.181 \times 10^3 \cdot kip \cdot ft$$

Moment due to Strength V limit state

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.338 \times 10^3 \cdot kip \cdot f$$

Equation 6.10.7.1.7-1

$$Mn = 2.039 \times 10^3 \cdot kip \cdot ft$$

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$$

STEP 9: *Compute Deflection*

<u>Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)</u>

$DF_d := \frac{N_l}{N} = 0.214$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

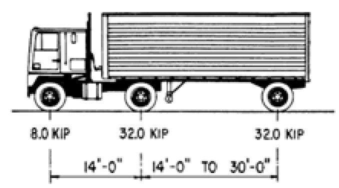
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.86 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 11.333 \cdot \text{ft}$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

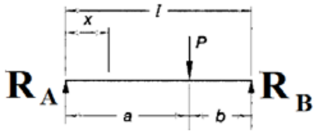
$b2 := \frac{L}{2} - (X_{ASr} - X') = 25.333 \cdot \text{ft}$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 20.667 \cdot \text{ft}$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.112 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.798 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

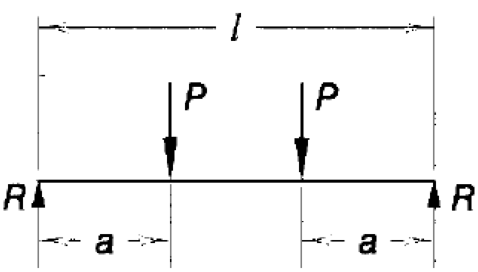
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.719 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.464 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.457 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.133 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.725 \cdot in$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.9 \cdot in$ $\frac{L}{1000} = 0.72 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 343.327 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.43 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.519 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 32 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 46 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 60 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 60.8 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 39.893 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.417 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 39.641 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 19.2 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 9.472 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 12.893 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 15.584 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1} \right) \right) = 125.882 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 74.065 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$\boxed{V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 139.481 \cdot kip}$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 139.481 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 31 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 561.875 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 49.6 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 561.875 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 561.875 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 26.025 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.775 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.579 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.95 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

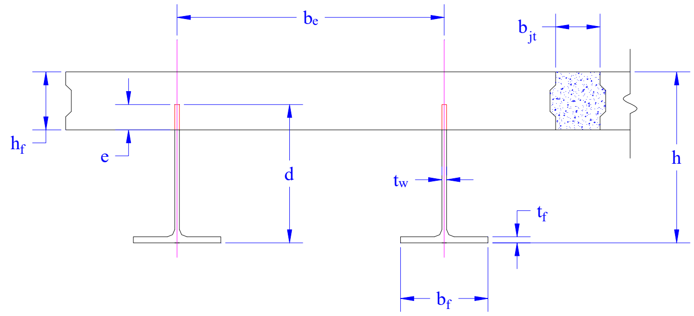
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad$ $modules := \frac{N}{2} = 5$

$L := 70ft$ Span length

$\boxed{b_e := 34in}$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad$ $t_f := 0.75in$ $\quad$ $b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 33.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$\boxed{M_{LL\_IM\_PL82} := 684.78kip \cdot ft}$ Moment due to PL82 with distribution factor and impact factor!!

$\boxed{V_{PL82span} := 136.38kip}$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 12.238 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.73 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.781 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 334 \cdot in$ $W = 27.833\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 24.458\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.609 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.734 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.101 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.362 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 48.227 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.279 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.101 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.38 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.073 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.066 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.489 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.25 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.417 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

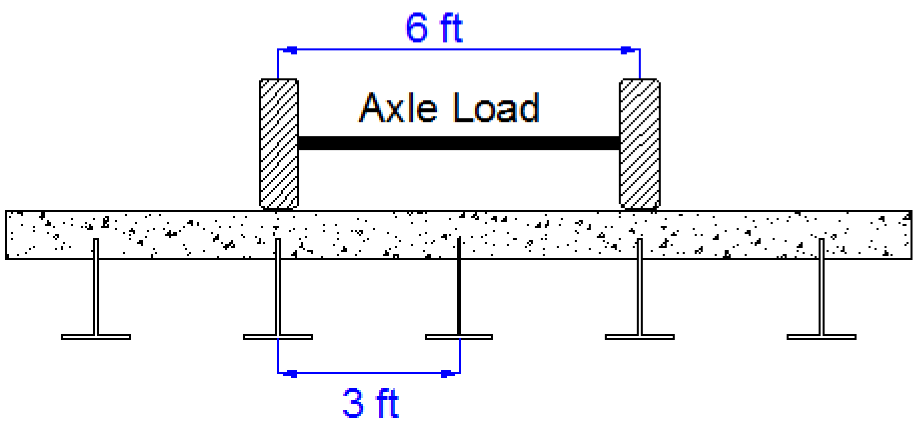
$$M_{DL} := \frac{DL \cdot L^2}{8} = 232.741 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 299.81 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 2.833$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 22.012 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.45 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 30.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 22.105 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

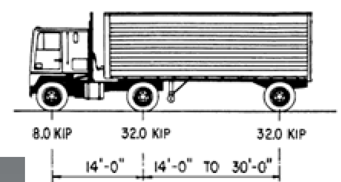
Design truck load in AASHTO (HS20 Truck) is used for this example.

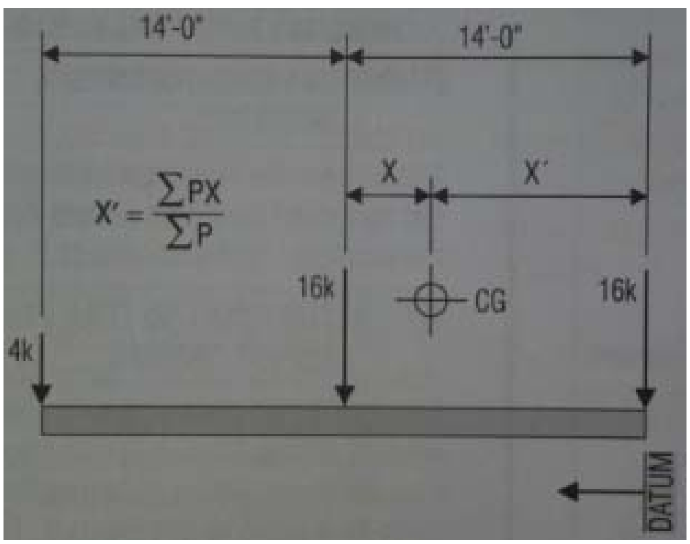
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

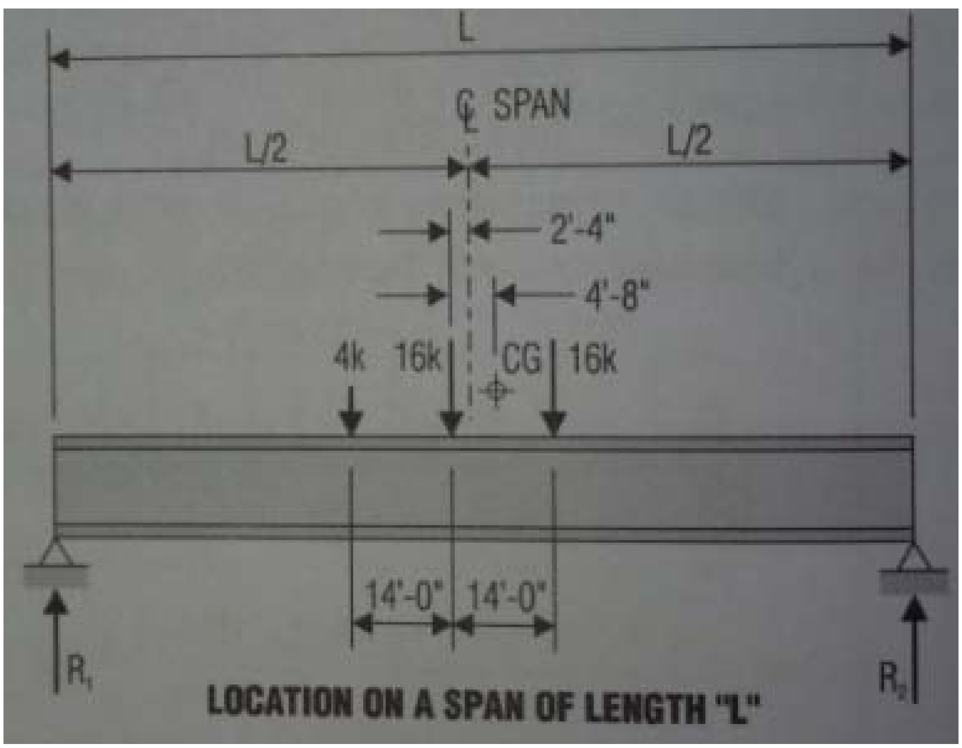
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.357\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 411.825\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

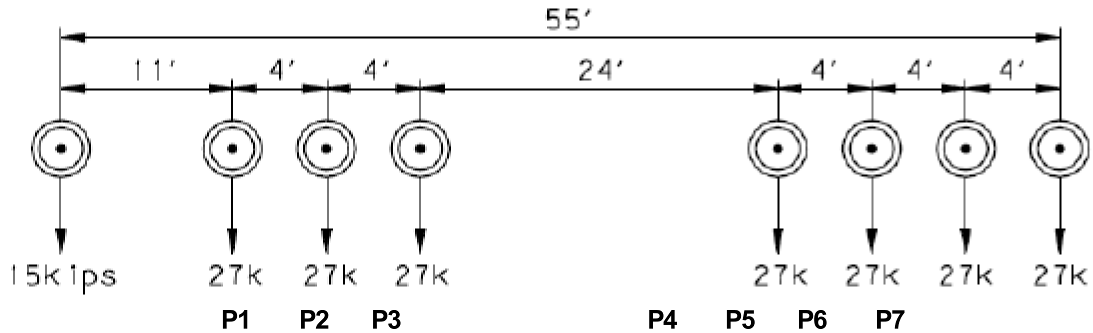
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 117.6\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.73 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 31.875 \cdot \; I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 149.414 \cdot in^4$

$Y' := \dfrac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 23.618 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 11340 \cdot in^4$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 23.618 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 10.625 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 49.805 \cdot in^4$

$Y' := \dfrac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 18.026 \cdot in$ S.G. of the total flex beam (Datum is bottom)

$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \dfrac{h_f}{2}\right) - Y'\right]^2 = 7574 \cdot in^4$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 18.026 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 15.96 \cdot in$

$Dp1 := root(Fp(D_p), D_p) = 9.765 \cdot in$ Must be less than $h_f = 7.5 \cdot in$ $\quad \frac{Dp1}{h} = 0.257$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$Mp1 = 30371.35 \cdot kip \cdot in \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 27034.4 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$Dp2 := root(Fp2(D_p), D_p) = 13.953 \cdot in$ Must be more than $h_f = 7.5 \cdot in$ $\quad \frac{Dp2}{h} = 0.3672$ must be < 0.42

$M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1 \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$Mp2 = 29526.72 \cdot kip \cdot in \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 24004.4 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2000.4 \cdot kip \cdot ft \qquad Mn = 24004.4 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 10.178\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 10.178\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 6.647\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.232\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.115\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 8.562\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 44.307\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 44.256\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 32.41\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 32.41\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.443 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.007 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.303 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.57 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 3.172 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1667.1 \cdot kip \cdot ft$ $\quad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 3.168 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1665 \cdot kip \cdot ft$ $\quad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2322.59 \cdot psi$ This assumes short term composite section for all calculations $\quad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.307 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.007 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.395 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2011.6 \cdot psi$ $\quad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 420.193 \cdot in^3$ $\quad$ $S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 480.136 \cdot in^3$ $\quad$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 740.641 \cdot kip \cdot ft$ $\quad$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.154 \times 10^3 \cdot kip \cdot ft$ $\quad$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.895 \times 10^3 \cdot kip \cdot ft$ $\quad$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 454.779 \cdot in^3$ $\quad$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.455 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 1.735 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad$ $Mn = 2 \times 10^3 \cdot kip \cdot ft$ $\quad$ $M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$$DF_d := \frac{N_l}{N} = 0.2$$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

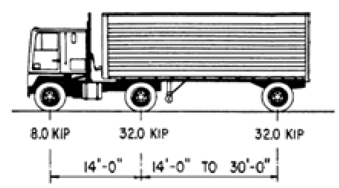
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.704 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot \text{ft}$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

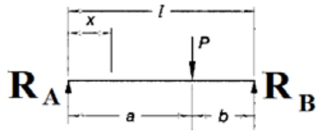
$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot \text{ft}$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot \text{ft}$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.195 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 1.171 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

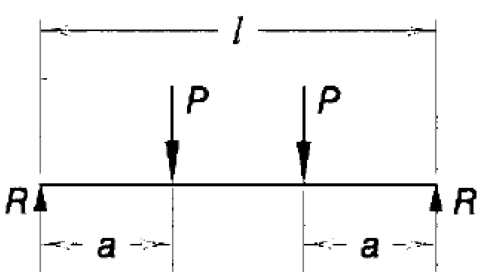
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 1.085 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.652 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.621 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.21 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 1.019 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \dfrac{b_e}{12in} = 2.833$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \dfrac{S}{25} = 0.473}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \dfrac{h_f}{2}\right) = 338.91 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.38 \cdot \dfrac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.489 \cdot \dfrac{kip}{ft}$

Design Lane Load: $0.64 \dfrac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0* *(at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.283 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.222 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 10.603 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 13.299 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 17.132 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 129.622 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 77.464 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 146.899 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 146.899 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 54.4 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 29.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.212 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.202 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.592 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

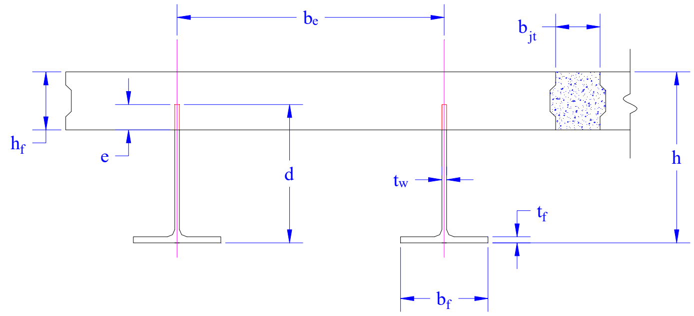
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  - AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  - PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to to be conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $modules := \frac{N}{2} = 5$

$L := 70ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $t_f := 0.75in$ $b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 33.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 684.78kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 136.38kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 12.238 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.73 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.781 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 354 \cdot in$ $W = 29.5\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 26.125\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.766 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.784 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.101 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.378 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 50.415 \cdot kip}$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.295 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.101 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.397 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.078 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.498 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

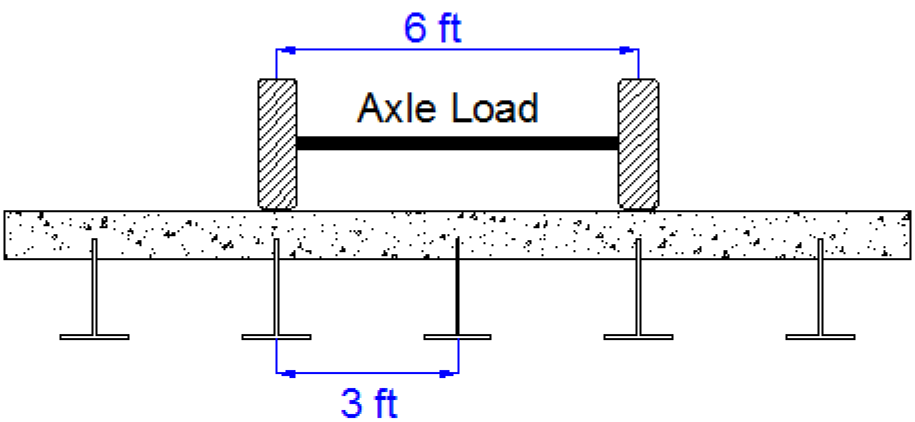
$$M_{DL} := \frac{DL \cdot L^2}{8} = 242.949 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 305.255 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 22.012 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.45 \times 10^5 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 30.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 22.105 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

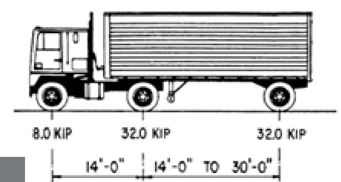
Design truck load in AASHTO (HS20 Truck) is used for this example.

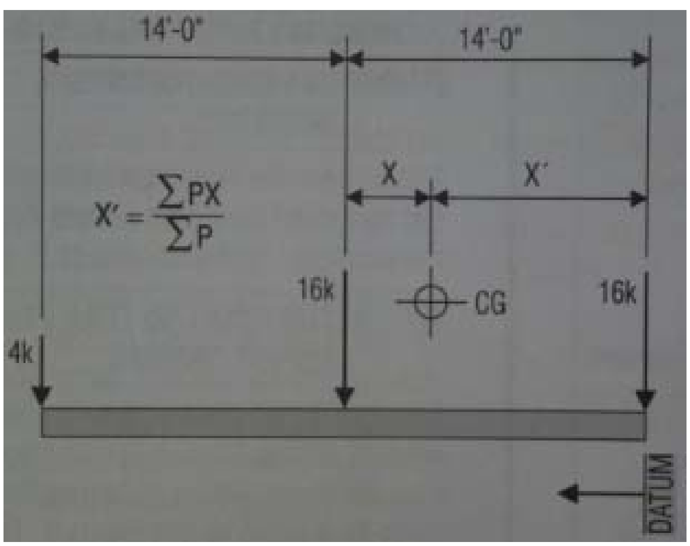
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

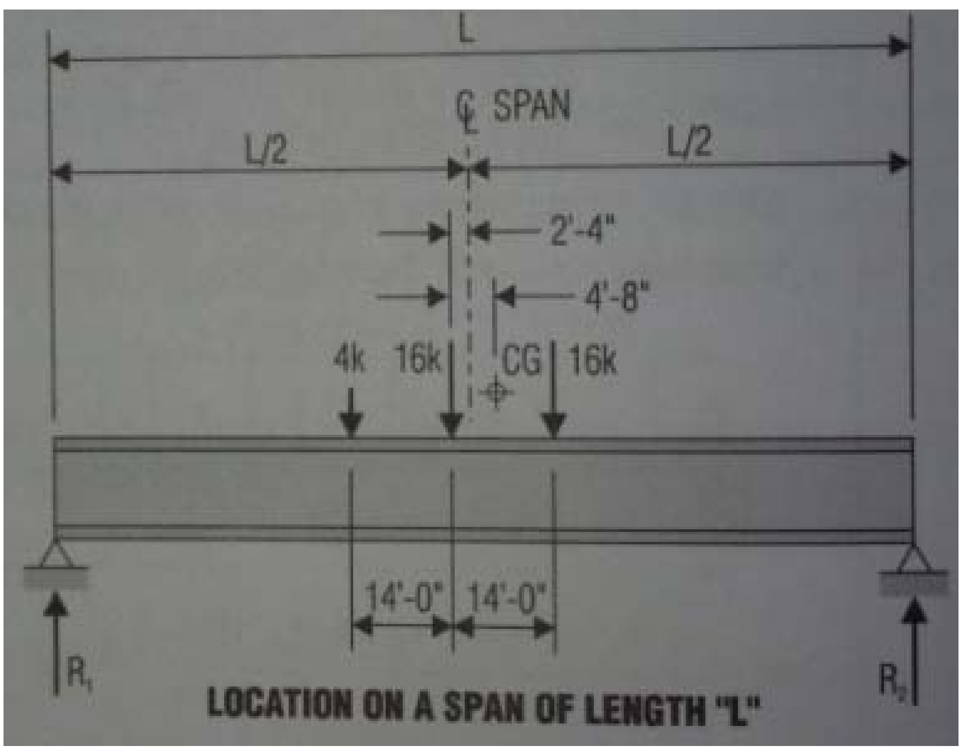
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.357\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 1.032\times 10^3\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032\times 10^3\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 411.825\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

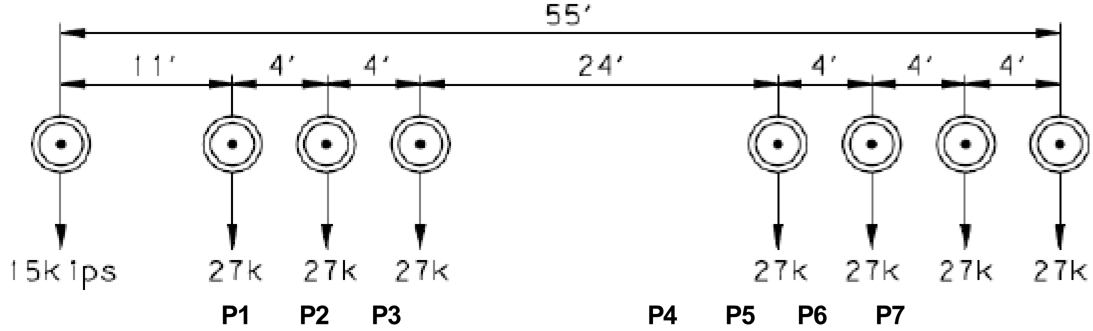
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 117.6\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.73 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot i$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\ 3}}{12} = 158.203 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 23.931 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 11554 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 23.931 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\ 3}}{12} = 52.734 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 18.273 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 7739 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 18.273 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 15.96 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 9.405 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.248}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 30683.61 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 27515.3 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 13.137 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.3457}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 30026.27 \cdot kip \cdot in}$

$$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 24861.8 \cdot kip \cdot in$$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2071.8 \cdot kip \cdot ft \qquad Mn = 24861.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 9.343\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 9.343\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 6.883\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.159\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.02\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 8.649\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 44.605\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 44.554\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 32.64\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 32.639\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.444 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.967 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.251 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.558 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 3.083 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1688.1 \cdot kip \cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 3.079 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1686 \cdot kip \cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2258.32 \cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.31 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.967 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.389 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1955.73 \cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 423.537 \cdot in^3$ $\quad S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 482.804 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 761.568 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.144 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 1.905 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 457.227 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.476 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 1.757 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad Mn = 2.072 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

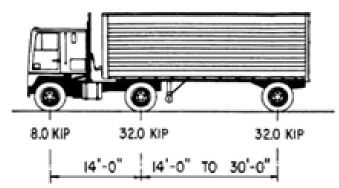
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.653 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

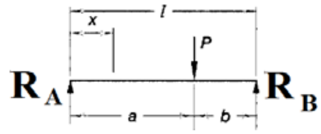
$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.191 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 1.149 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

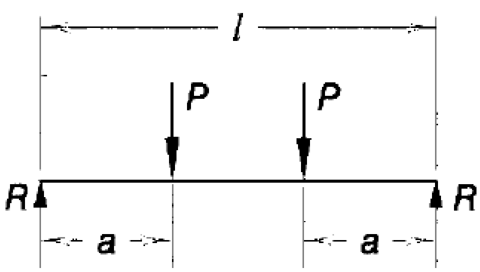
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 1.065 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.64 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.61 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.206 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\ max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 1.000 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$DF_s := 0.36 + \frac{S}{25} = 0.48$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 348.255 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.397 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.498 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$X_{ASr} := 14ft$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 ***(at left support)*** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.836 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.76 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 10.752 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 13.883 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 17.443 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 132.047 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 78.555 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 149.567 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 149.567 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 54.4 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 29.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.212 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.202 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.592 \cdot kip$ — Compressive capacity of bearing stiffener

$$Pr \ge Vu = 1$$

Flexbeam Example Configuration Calculations

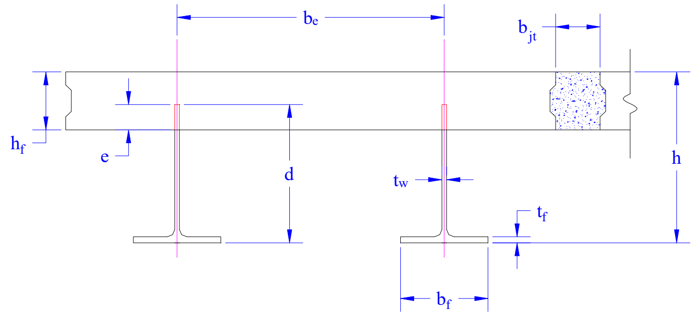
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- TThe steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to to be conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $\quad$ $modules := \frac{N}{2} = 6$

$L := 70ft$ Span length

$b_e := 36in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad$ $t_f := 0.75in$ $\quad$ $b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 33.25 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 684.78kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 136.38kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 12.238 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 3.73 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 29.781 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 426 \cdot \text{in}$ $W = 35.5\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 32.125\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.328 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700 \frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.964 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{\text{lbf}}{\text{ft}^3} = 0.101 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.379 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 50.415 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.296 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.101 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.397 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.08 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$

distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.07 \cdot \frac{kip}{ft}$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.5 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.5 \cdot in$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.5 \cdot in$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

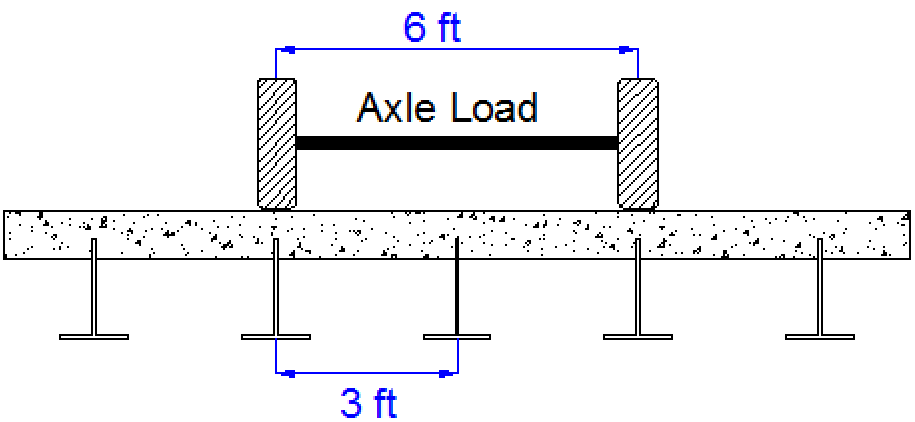
$$M_{DL} := \frac{DL \cdot L^2}{8} = 243.428 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 306.441 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 22.012 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.45 \times 10^5 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 30.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 108 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 22.105 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

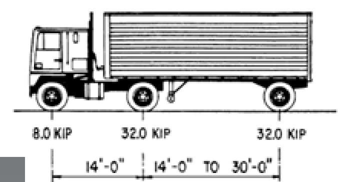
Design truck load in AASHTO (HS20 Truck) is used for this example.

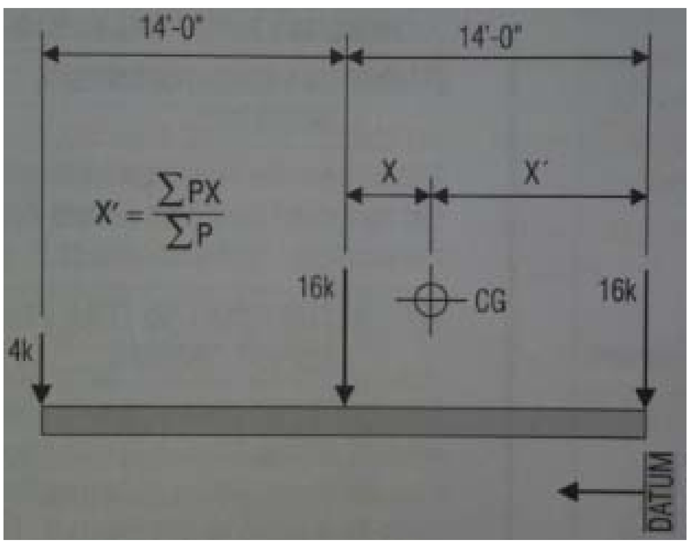
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

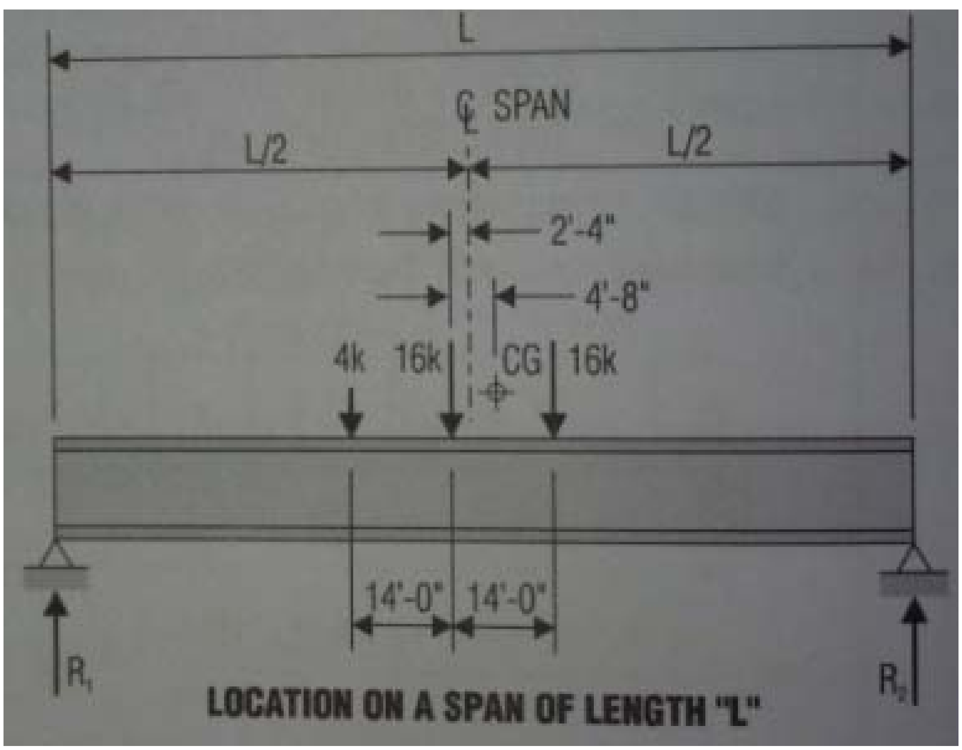
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.357\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 1.032\times10^{3}\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032\times10^{3}\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 411.825\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

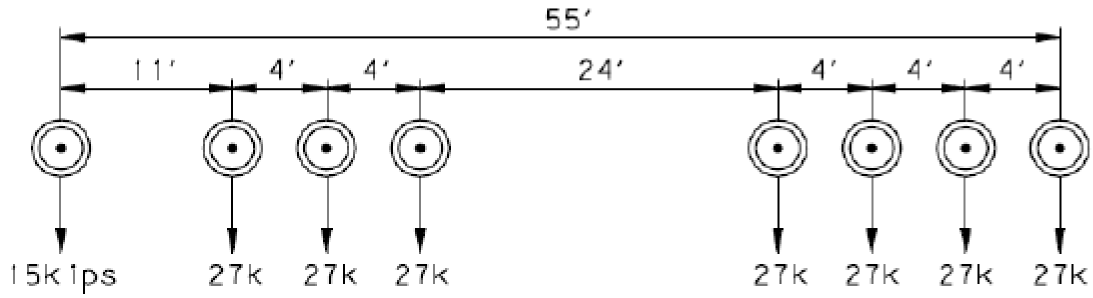
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\dfrac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 117.6\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 3.73 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 33.75 \cdot in$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 158.203 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 23.931 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 11554 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 23.931 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.25 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 52.734 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 18.273 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 7739 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 18.273 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad$ $h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in$ $\qquad$ $0.42 \cdot h = 15.96 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 9.405 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.248}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0$ $\qquad$ $Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 30683.61 \cdot kip \cdot in}$ $\qquad$ $M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 27515.3 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 13.137 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.3457}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1}$ $\qquad$ $Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 30026.27 \cdot kip \cdot in}$

$M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 24861.8 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2071.8 \cdot kip \cdot ft$ $\qquad$ $Mn = 24861.8 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 9.343\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 9.343\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 6.897\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.159\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 17.02\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 8.682\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 44.673\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 44.622\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 32.69\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 32.686\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.445 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.967 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.251 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.56 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 3.088 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1690.4 \cdot kip \cdot ft$ $M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 3.084 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1688 \cdot kip \cdot ft$ $M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2261.36 \cdot psi$ This assumes short term composite section for all calculations $f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.31 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.967 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.391 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1957.85 \cdot psi$ $f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 423.537 \cdot in^3 \qquad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 482.804 \cdot in^3$$

short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 763.947 \cdot kip \cdot ft$$

From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.141 \times 10^3 \cdot kip \cdot ft$$

Additional moment to cause nominal yielding of steel flange

$$My := M_{D2} + M_{AD} = 1.905 \times 10^3 \cdot kip \cdot ft$$

D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$$S_{xt} := \frac{My}{F_y} = 457.147 \cdot in^3$$

Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 1.479 \times 10^3 \cdot kip \cdot ft$$

Moment due to Strength V limit state

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.759 \times 10^3 \cdot kip \cdot f$$

Equation 6.10.7.1.7-1

$$Mn = 2.072 \times 10^3 \cdot kip \cdot ft$$

$$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.167$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

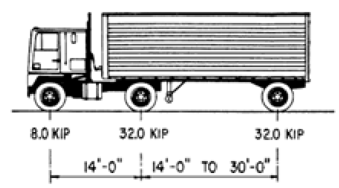
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.653 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

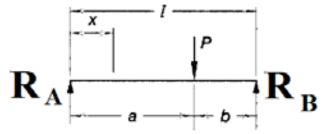
$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.191 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 1.149 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

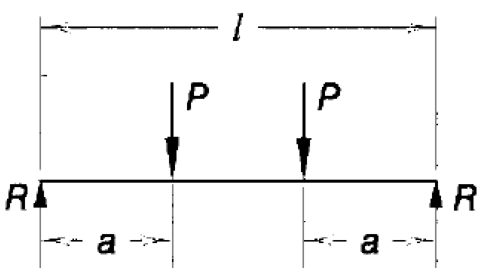
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 1.065 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.533 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24 EI} (3 l^2 - 4 a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.508 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 106.667 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.172 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\, max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.833 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.48}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 348.255 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.397 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.5 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 *(at left support)*

(Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - \left(X_{ASr} + X_{ASf}\right) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.836 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{\left(L - X_{AS}\right)}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 38.76 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot \left(DF_s\right) = 10.752 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 13.91 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 17.511 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 132.183 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 78.555 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 149.703 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 149.703 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 54.4 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 29.15 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.212 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.202 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.592 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

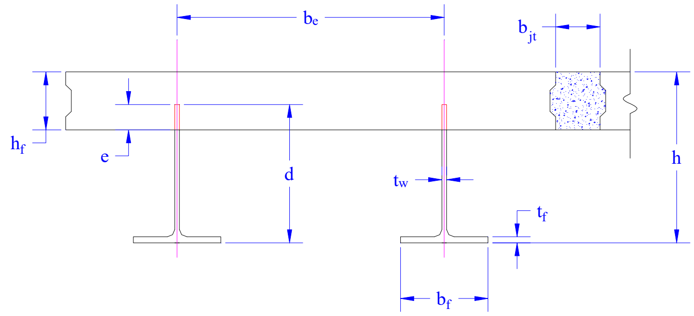
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to to be conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 70ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 1.125in$ $\quad b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 32.875 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 684.78kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 136.38kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.822 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^{\,3}}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 4.206 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 34.047 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 374 \cdot in$ $W = 31.167\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 27.792\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.922 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.834 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.116 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.408 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 54.634 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \mathrm{floor}\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.312 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.116 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.428 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.083 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.507 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

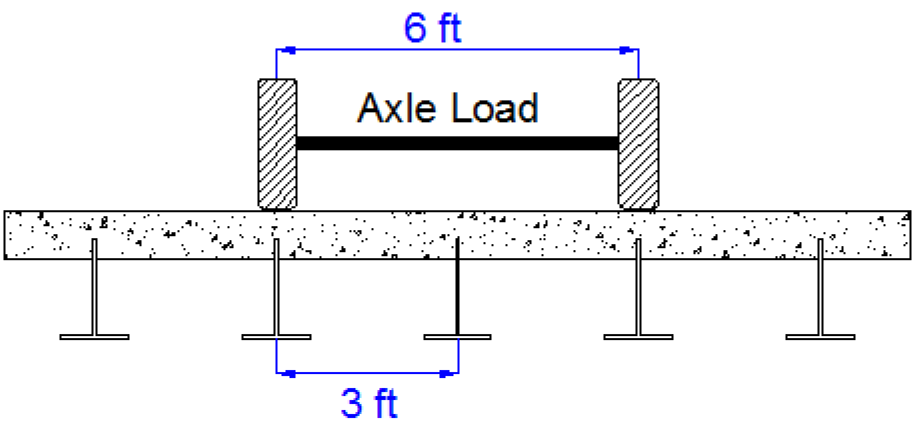
$$M_{DL} := \frac{DL \cdot L^2}{8} = 262.048 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 310.699 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.167$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 23.428 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.83 \times 10^5 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 30.5 \cdot in$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{{b_f}^3 \cdot t_f}{12} = 162 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.737 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

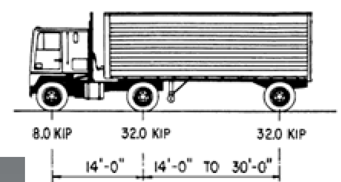
Design truck load in AASHTO (HS20 Truck) is used for this example.

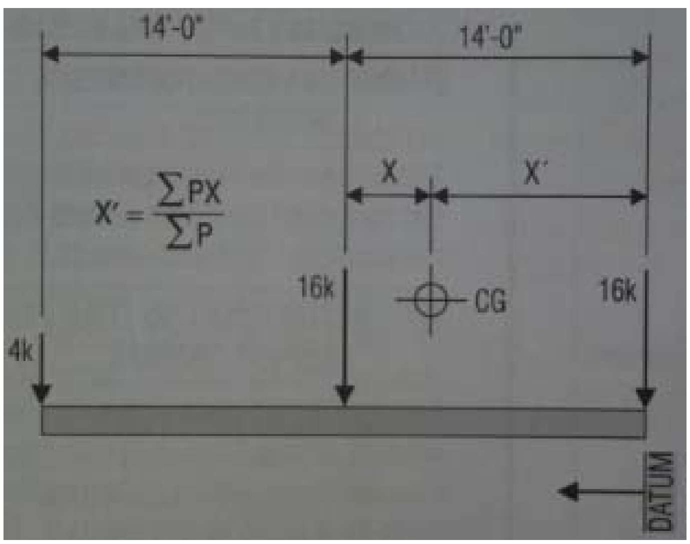
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

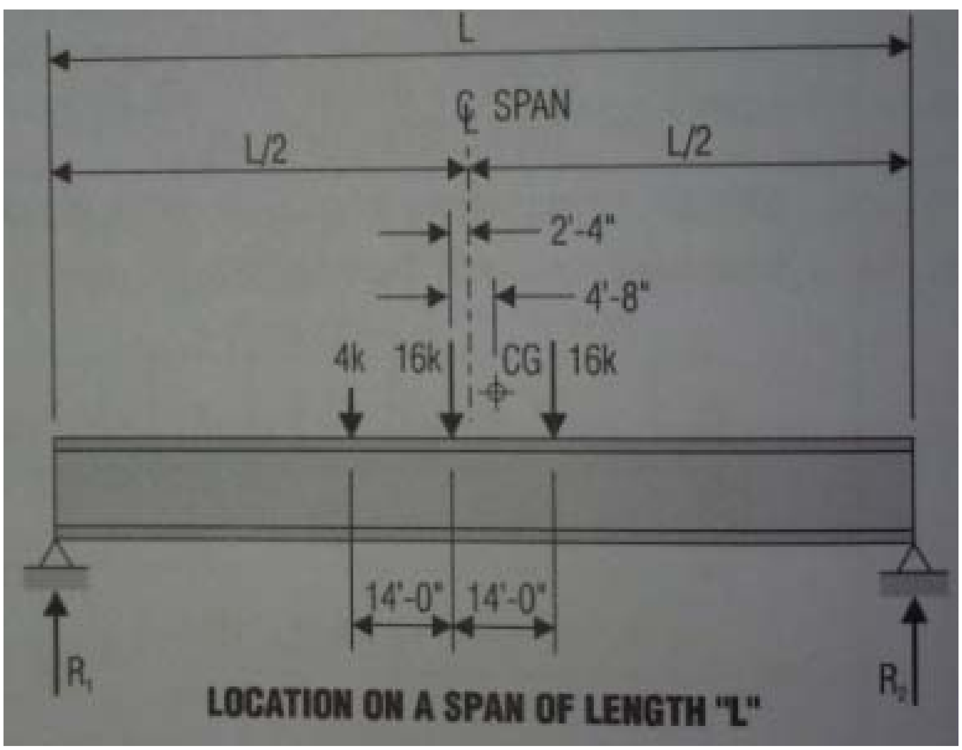
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30.357 \cdot \text{kip}$$

Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 1.032 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032 \times 10^3 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 411.825 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

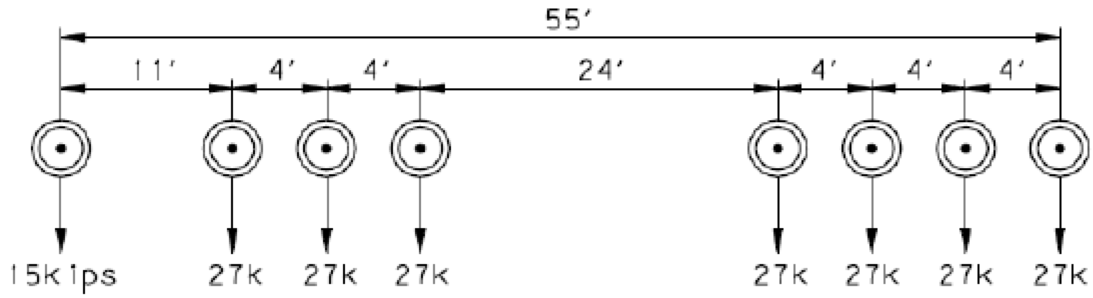
$F_{DLL} := 0.64 \dfrac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 117.6 \cdot \text{kip} \cdot \text{ft}$$

Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 4.206 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 22.801 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 13929 \cdot in^4$$

Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 22.801 \cdot in$$

N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.88 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9095 \cdot in^4$$

Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 16.88 \cdot in$$

N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 15.96 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 10.184 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.268}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h-d)]\right] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp1 - t_f)\right] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 36824.95 \cdot kip \cdot in} \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 32494.1 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h-d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 15.734 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.414}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h-d)]\right] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot (h - Dp2 - t_f)\right] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 35397.18 \cdot kip \cdot in} \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 27615.9 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2301.3 \cdot kip \cdot ft \qquad Mn = 27615.9 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2} \cdot \left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1 \right) = 12.135 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 12.135 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.837 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 10.4 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.452 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 6.92 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 35.876 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.835 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 26.28 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.276 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.429 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.121 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.509 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.815 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1720.1 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.812 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1718 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2063.94 \cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.304 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.361 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1791.52 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 538.775 \cdot in^3$ $S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 610.885 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 793.608 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.646 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 2.439 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 585.393 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.508 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.748 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $Mn = 2.301 \times 10^3 \cdot kip \cdot ft$ $M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

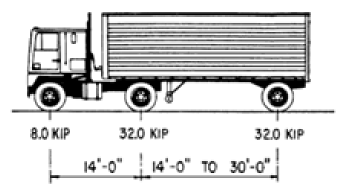
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.201 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

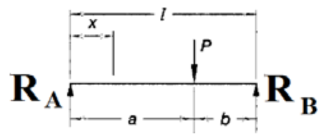
$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.159 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.953 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

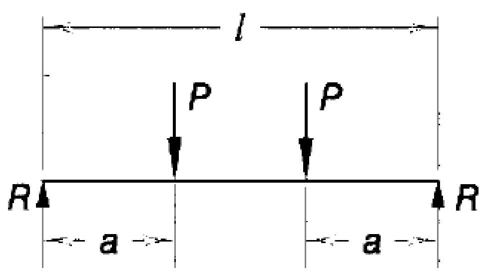
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.883 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.531 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.506 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.171 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.829 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 407.862 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.428 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.507 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

***At x=0* (at left support)** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 40.389 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.298 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 10.901 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 14.974 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 17.754 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 135.108 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 79.646 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 152.871 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 152.871 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 54.4 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 28.775 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.27 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.249 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.637 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

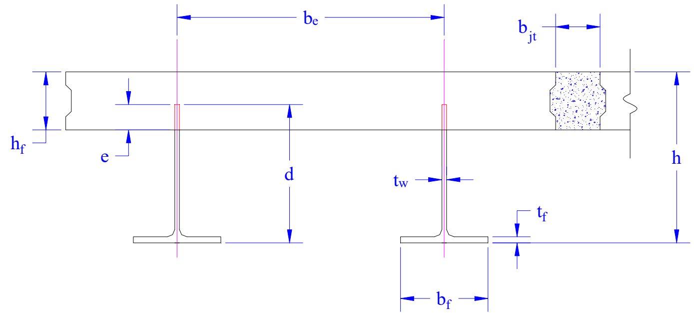
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to to be conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 14$ $\quad modules := \frac{N}{2} = 7$

$L := 70ft$ Span length

$b_e := 38in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 1.125in$ $\quad b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 32.875 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 684.78kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 136.38kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.822 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 4.206 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 34.047 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 526 \cdot in$ $W = 43.833\,ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 40.458\,ft$

$D_c := (w_c \cdot h_f \cdot W) = 4.109 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot (W - 2 \cdot b_{par}) = 1.214 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.116 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{(D_c)}{N} + D_s = 0.409 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[(2b_e - b_{jt})(h_f \cdot w_c)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 54.634 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.313 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.116 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.429 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.087 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.074 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.511 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 4.75 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.583 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

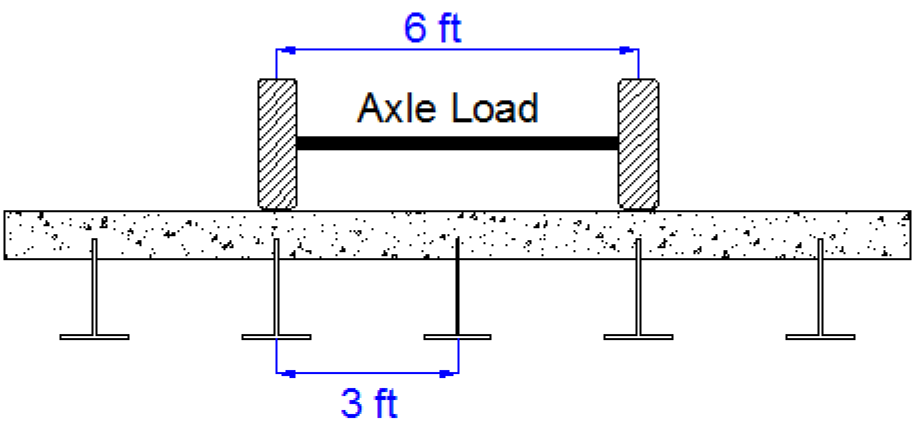
$$M_{DL} := \frac{DL \cdot L^2}{8} = 262.868 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 312.734 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.167$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 23.428 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.83 \times 10^5 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***<u>Per Lane</u>*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 30.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 162 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.737 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

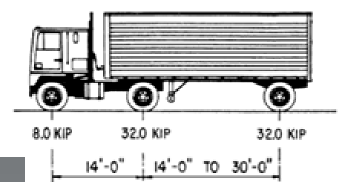
Design truck load in AASHTO (HS20 Truck) is used for this example.

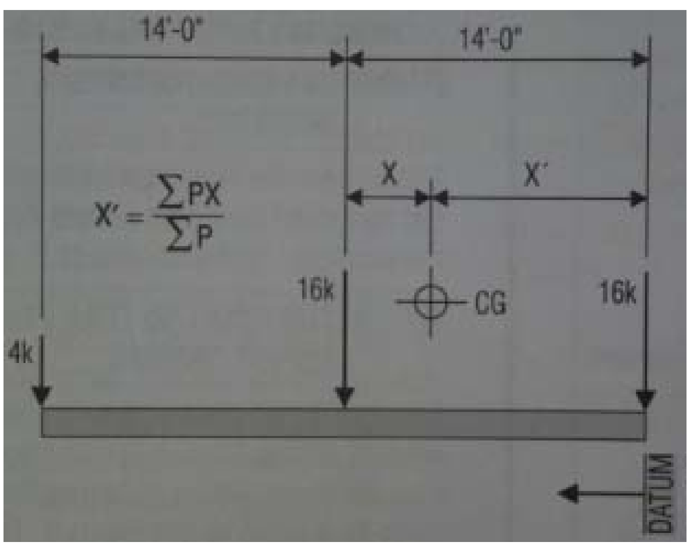
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

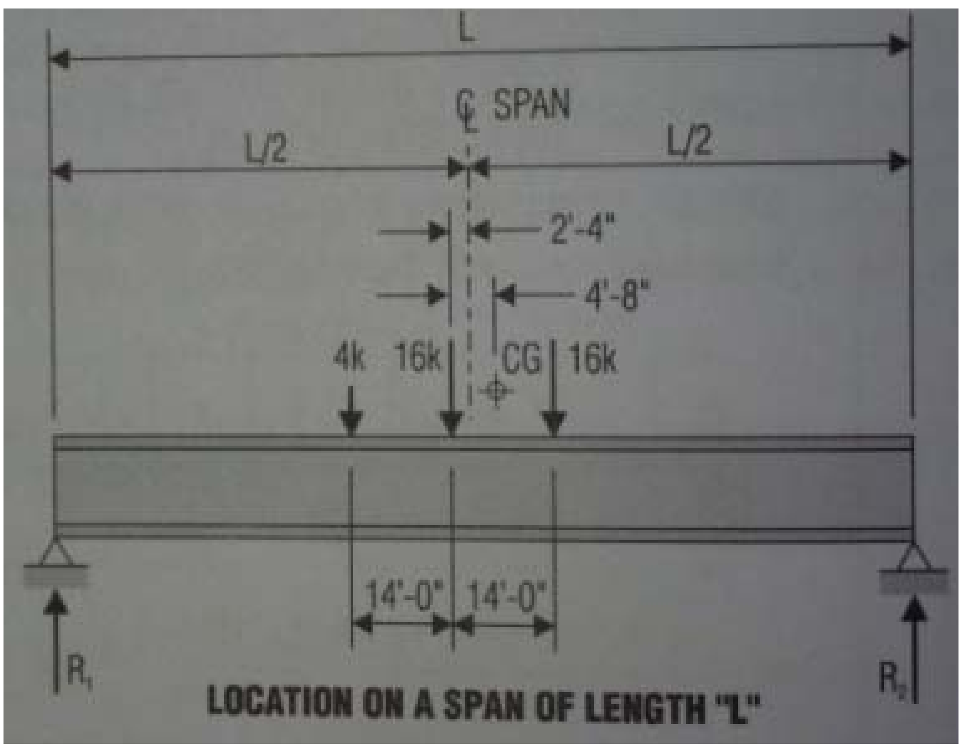
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.357\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 411.825\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

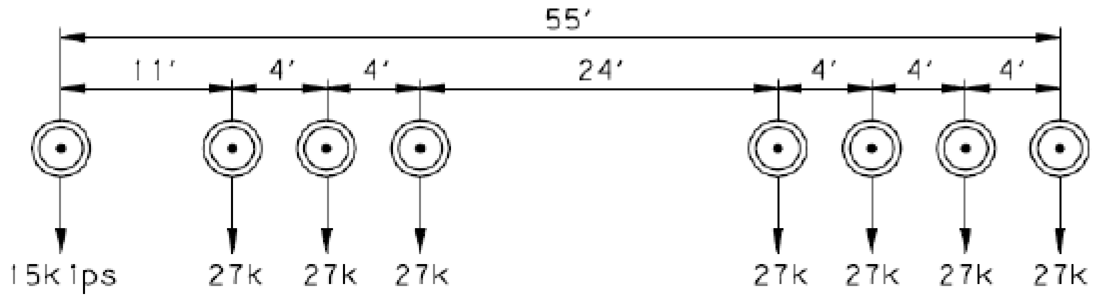
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 117.6\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 4.206 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 35.625 \cdot$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 166.992 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 22.801 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 13929 \cdot in^4$$ Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 22.801 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 11.875 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 55.664 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 16.88 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9095 \cdot in^4$$ Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 16.88 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad$ $h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in$ $\qquad$ $0.42 \cdot h = 15.96 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 10.184 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.268}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0$ $\qquad$ $Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 36824.95 \cdot kip \cdot in}$ $\qquad$ $M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 32494.1 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 15.734 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.414}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1}$ $\qquad$ $Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 35397.18 \cdot kip \cdot in}$ $\qquad$ $M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 27615.9 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2301.3 \cdot kip \cdot ft$ $\qquad$ $Mn = 27615.9 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1\right) = 12.135 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 12.135 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 5.855 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 10.4 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.452 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 6.965 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 35.966 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 35.926 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 26.34 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.34 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.43\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot(h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.121\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.512\cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.822\cdot ksi$

$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 1724.2\cdot kip\cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.819\cdot ksi$

$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 1722\cdot kip\cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6\cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2068.61\cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot(h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.305\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot(h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.867\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot(h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.363\cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1794.84\cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 538.775 \cdot in^3$ $\quad S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 610.885 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 797.685 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.641 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 2.439 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 585.262 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.512 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 1.752 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad Mn = 2.301 \times 10^3 \cdot kip \cdot ft \quad M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

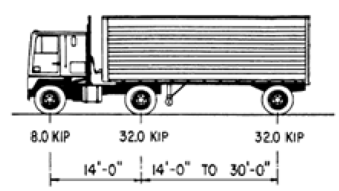
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.201 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

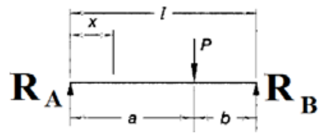
$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.159 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.953 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

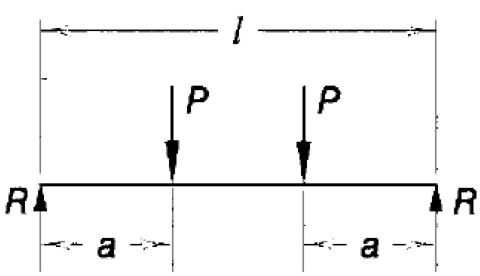
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.883 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.569 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.542 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.183 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.888 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.167$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.487}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 407.862 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.429 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.511 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 40.389 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.298 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 10.901 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 15.021 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 17.87 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 135.341 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 79.646 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 153.104 \cdot kip$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 153.104 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 54.4$ $1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314$ $C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 28.775 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\;3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$$

Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.27 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.249 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.637 \cdot kip$ — Compressive capacity of bearing stiffener — $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

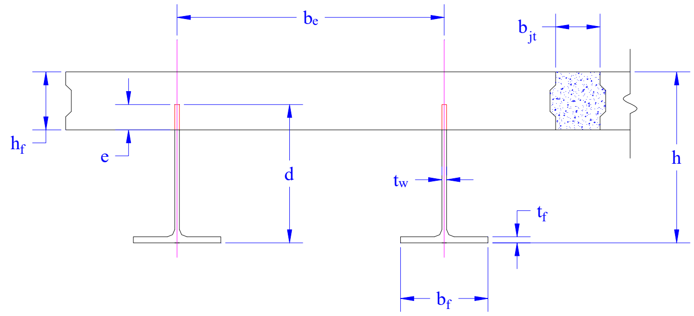
- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to to be conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:

Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 70ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)



Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 1.125in$ $\quad b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 32.875 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 684.78kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 136.38kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.822 \cdot in$$

Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 4.206 \times 10^3 \cdot in^4$$

MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 34.047 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6in$

Parapet width: $b_{par} := 20.25in$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 394 \cdot in$ $W = 32.833\ ft$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 29.458\ ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.078 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .700 \frac{kip}{ft}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.884 \cdot \frac{kip}{ft}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.116 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.424 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 56.822 \cdot kip}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := floor\left(\frac{W_{lane}}{12ft}\right) = 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.116 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.444 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.088 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.516 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

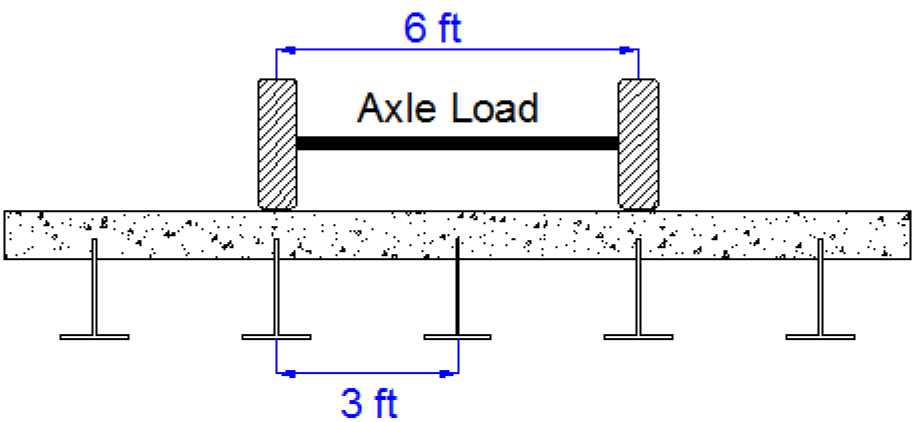
$$M_{DL} := \frac{DL \cdot L^2}{8} = 272.256 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 316.144 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$$S := \frac{b_e}{12in} = 3.333$$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$$e_g := h - y\_bar - \frac{h_f}{2} = 23.428 \cdot in$$ Distance between the centers of gravity of the basic beam and deck

$$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.83 \times 10^5 \cdot in^4$$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 30.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 162 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.737 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

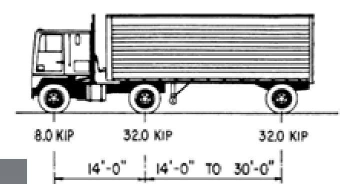
Design truck load in AASHTO (HS20 Truck) is used for this example.

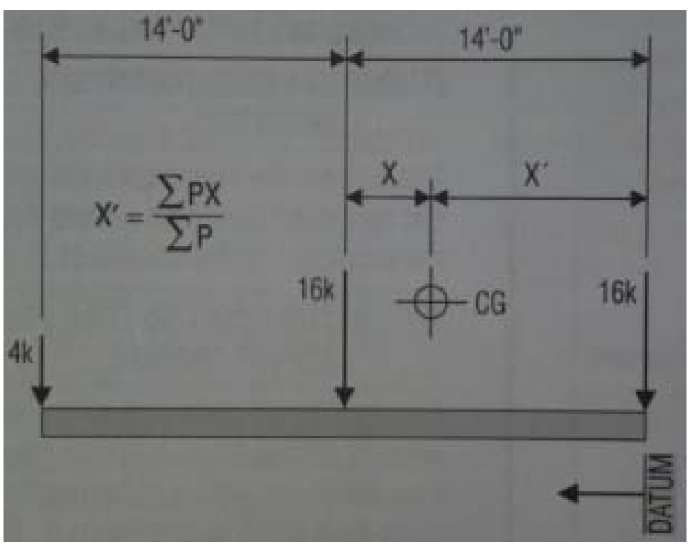
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

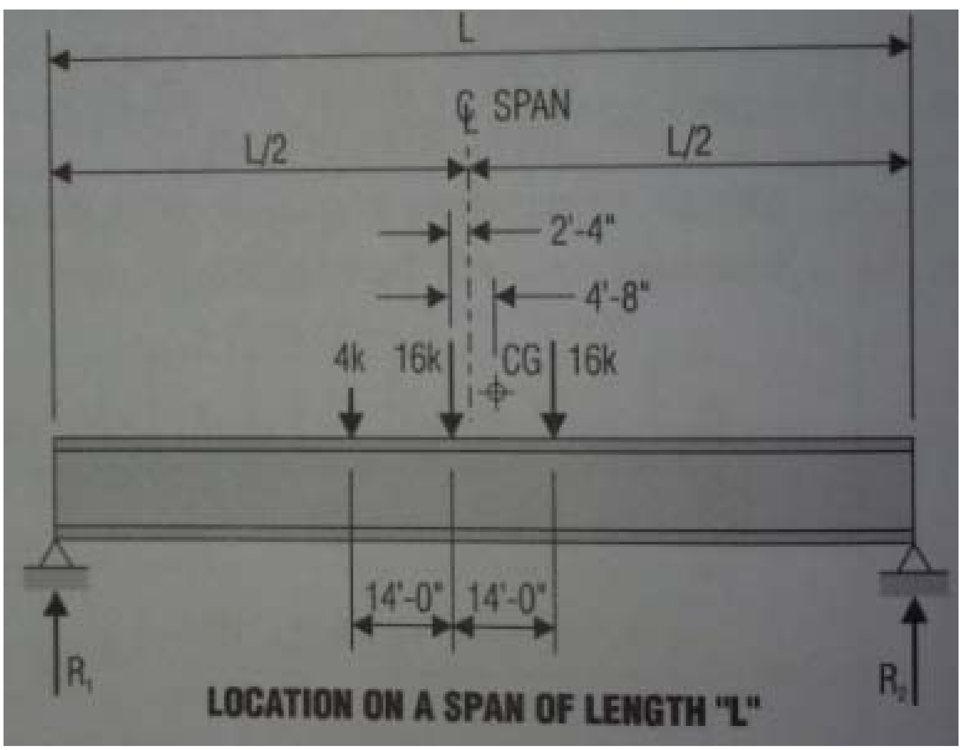
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2} + \frac{X}{2}\right) + F_T\cdot\left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30.357\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2} - \frac{X}{2}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1 + I) = 411.825\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

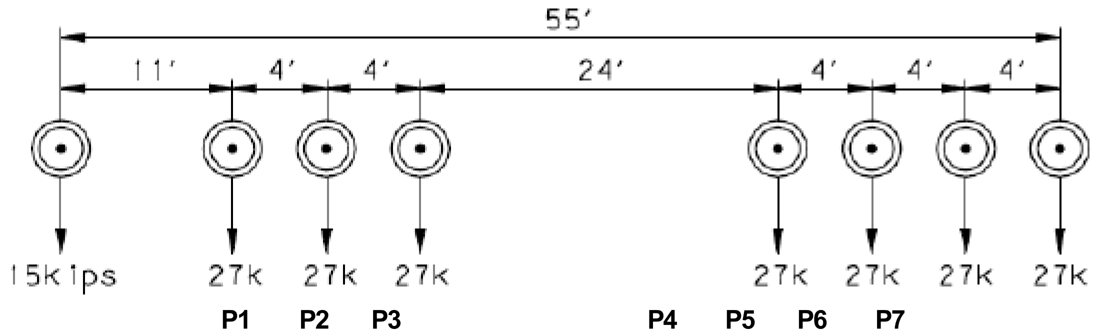
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 117.6\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $$\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425\cdot\text{kip}\cdot\text{ft}}$$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 4.206 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 23.101 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 14177 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 23.101 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 17.113 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9284 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 17.113 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\quad h - d = 4\cdot in$

$$Fp(D_p) := 0.85\cdot f_c\cdot b_e\cdot D_p + F_y\cdot\left[t_w\cdot\left[D_p - (h-d)\right]\right] - F_y\cdot\left[A_{stl} - \left[t_w\cdot\left[D_p - (h-d)\right]\right]\right]$$

$D_p := 7.5in \qquad 0.42\cdot h = 15.96\cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 9.835\cdot in}$ Must be less than $h_f = 7.5\cdot in$ $\boxed{\frac{Dp1}{h} = 0.259}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f)\cdot[(h-d) < Dp1]\cdot(Dp1 < 0.42\cdot h) = 0 \qquad Dp1 \le 0.42\cdot h = 1$

$$Mp1 := 0.85\cdot f_c\cdot b_e\cdot Dp1\cdot\left(\frac{Dp1}{2}\right) + F_y\cdot\left[t_w\cdot[Dp1 - (h-d)]\right]\cdot\frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+\left[F_y\cdot(b_f\cdot t_f)\cdot\left(h - Dp1 - \frac{t_f}{2}\right) + F_y\cdot\left[t_w\cdot(h - Dp1 - t_f)\right]\cdot\frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 37165.53\cdot kip\cdot in}$ $\qquad M_{n1} := Mp1\cdot\left(1.07 - 0.7\cdot\frac{Dp1}{h}\right) = 33033.5\cdot kip\cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85\cdot f_c\cdot b_e\cdot h_f + F_y\cdot\left[t_w\cdot\left[D_p - (h-d)\right]\right] - F_y\cdot\left[A_{stl} - \left[t_w\cdot\left[D_p - (h-d)\right]\right]\right]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 14.918\cdot in}$ Must be more than $h_f = 7.5\cdot in$ $\boxed{\frac{Dp2}{h} = 0.3926}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f)\cdot(Dp2 < h)\cdot(Dp2 < 0.42\cdot h) = 1} \qquad Dp2 \le 0.42\cdot h = 1$

$$Mp2 := 0.85\cdot f_c\cdot b_e\cdot h_f\cdot\left(Dp2 - \frac{h_f}{2}\right) + F_y\cdot\left[t_w\cdot[Dp2 - (h-d)]\right]\cdot\frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+F_y\cdot(b_f\cdot t_f)\cdot\left(h - Dp2 - \frac{t_f}{2}\right) + F_y\cdot\left[t_w\cdot(h - Dp2 - t_f)\right]\cdot\frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 35987.53\cdot kip\cdot in}$

$$M_{n2} := Mp2\cdot\left(1.07 - 0.7\cdot\frac{Dp2}{h}\right) = 28617.4\cdot kip\cdot in$$

$Mn := \max(M_{n1}\cdot M_{n1check}, M_{n2}\cdot M_{n2check}) = 2384.8\cdot kip\cdot ft \qquad Mn = 28617.4\cdot kip\cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y \cdot t_f \cdot b_f - 0.85 \cdot f_c \cdot b_e \cdot h_f}{F_y \cdot h_w \cdot t_w} + 1\right) = 11.291 \cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp} \cdot D_{cp0} = 11.291 \cdot in \qquad \frac{2 \cdot D_{cp}}{t_w} < 3.76 \cdot \sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL} \cdot Y'_{c\_long}}{I_{c\_long}} = 6.023 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot Y'_{c\_short}}{I_{c\_short}} = 10.352 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot Y'_{c\_short}}{I_{c\_short}} = 13.39 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot Y'_{c\_long}}{I_{c\_long}} = 6.993 \cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 36.135 \cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 36.095 \cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3 f_{LL\_I} = 26.47 \cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0 ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.474 \cdot ksi \qquad 0.95 \cdot R_h \cdot F_y = 47.5 \cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95 R_h \cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.429 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.835 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 1.079 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.498 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.744 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1741 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.741 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1739 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\,psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2012.46 \cdot psi$ $\qquad$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot \left(h - Y'_{c\_long}\right)}{k_3 \cdot n \cdot I_{c\_long}} = 0.306 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot \left(h - Y'_{c\_short}\right)}{k_1 \cdot n \cdot I_{c\_short}} = 0.835 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot \left(h - Y'_{c\_long}\right)}{k_3 \cdot n \cdot I_{c\_long}} = 0.356 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1746.84 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 542.475 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 613.692 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 814.536 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.636 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 2.45 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 588.028 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 1.529 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.77 \times 10^3 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 2.385 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

<u>Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)</u>

$$DF_d := \frac{N_l}{N} = 0.2$$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

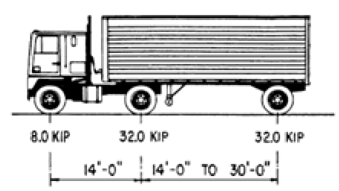
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ $\quad$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ $\quad$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ $\quad$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.162 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot \text{ft}$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 1$

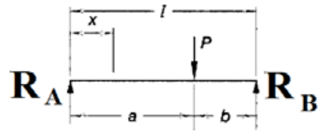
$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot \text{ft}$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot \text{ft}$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.156 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.937 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

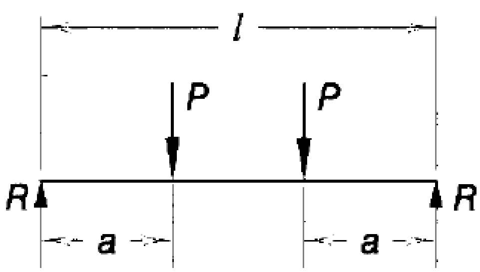
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.868 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.521 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}(3l^2 - 4a^2)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.497 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.168 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\, \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.815 \cdot in}$$ $$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \dfrac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \dfrac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \dfrac{h_f}{2} \right) = 418.078 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.444 \cdot \dfrac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.516 \cdot \dfrac{kip}{ft}$

Design Lane Load: $0.64 \dfrac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 40.943 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.837 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 11.051 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 15.557 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 18.065 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 137.533 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 80.737 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 155.54 \cdot kip$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 155.54 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 54.4 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 28.775 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.27 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.249 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.637 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

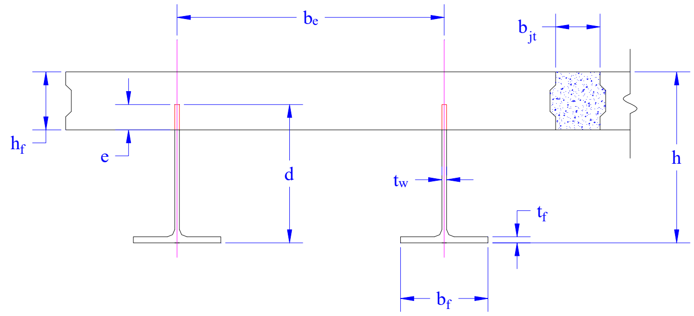
Summary of Calculations:

1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to to be conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:



Number of beams $N := 12$ $\quad modules := \frac{N}{2} = 6$

$L := 70ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $\quad t_f := 1.125in$ $\quad b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 32.875 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 684.78kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 136.38kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.822 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 4.206 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 34.047 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 474 \cdot \text{in}$ $W = 39.5\,\text{ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 36.125\,\text{ft}$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 3.703 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot \left(W - 2 \cdot b_{par}\right) = 1.084 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.116 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.424 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 56.822 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.329 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.116 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.445 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.09 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.518 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

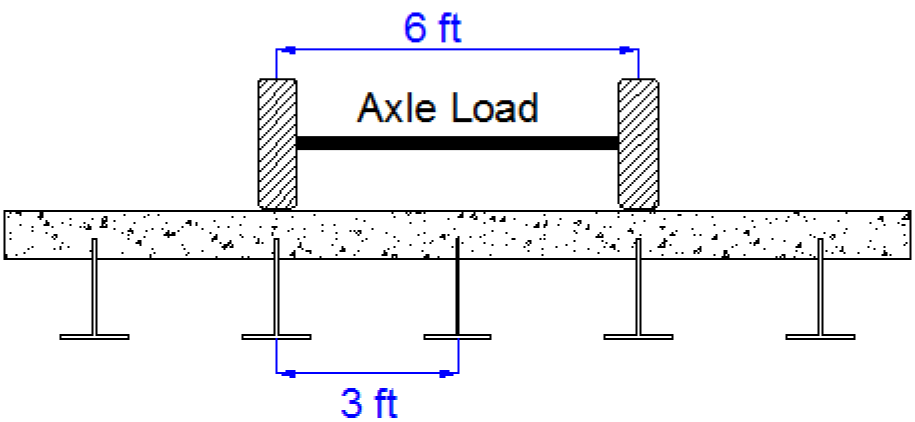
$$M_{DL} := \frac{DL \cdot L^2}{8} = 272.735 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 317.33 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \dfrac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \dfrac{h_f}{2} = 23.428 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 1.83 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***<u>Per Lane</u>*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 30.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 162 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.737 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

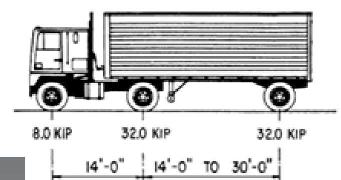
Design truck load in AASHTO (HS20 Truck) is used for this example.

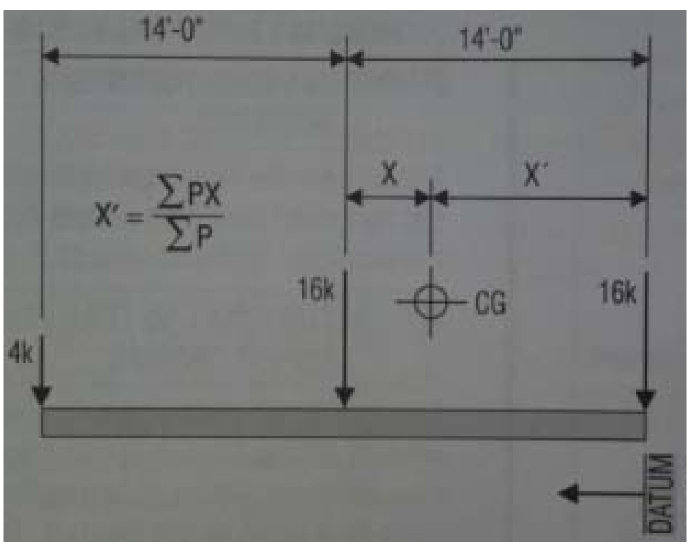
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

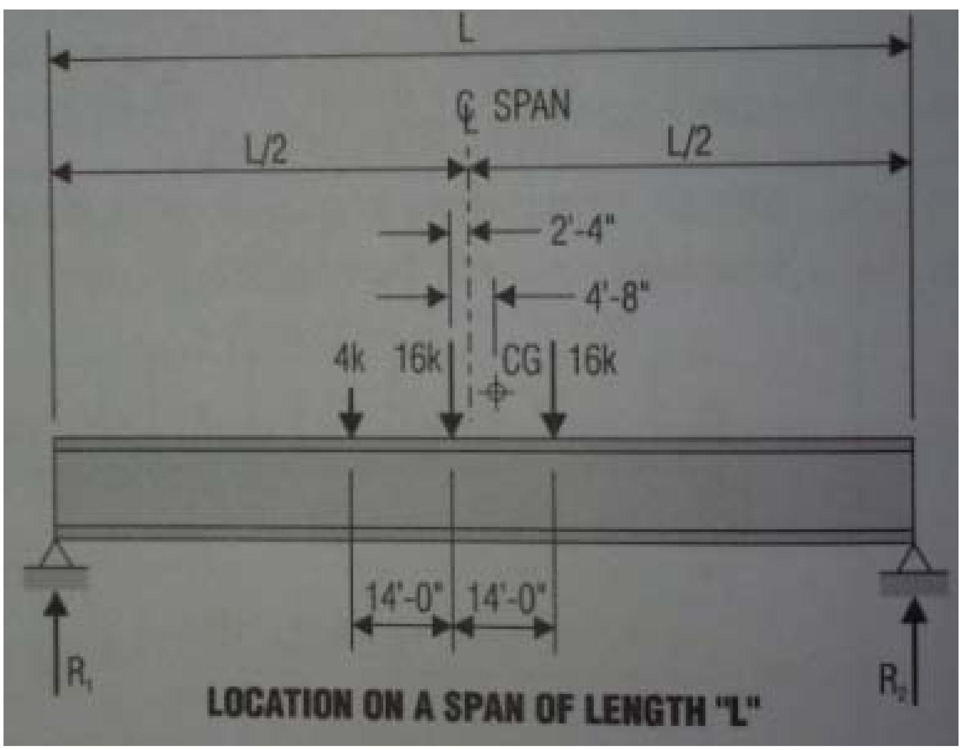
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25kip$$

$X_{AS} := 4ft$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2 \cdot ft$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \dfrac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 30.357 \cdot kip$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 1.032 \times 10^3 \cdot kip \cdot ft$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032 \times 10^3 \cdot kip \cdot ft}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 411.825 \cdot kip \cdot ft$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

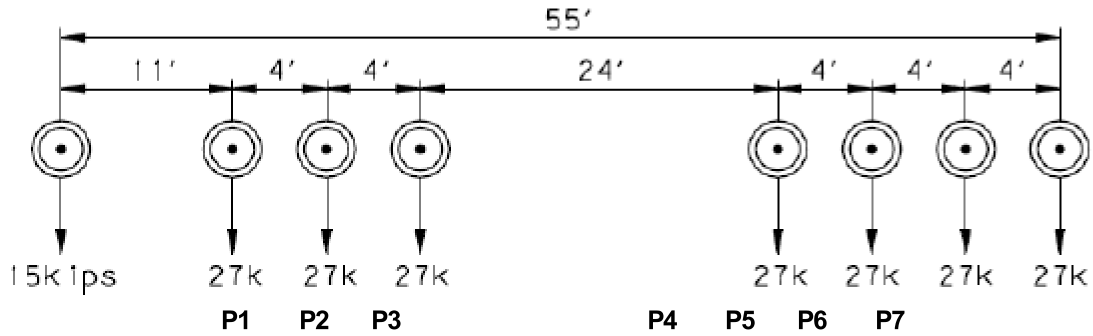
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \dfrac{kip}{ft} \cdot DF = 0.192 \cdot \dfrac{kip}{ft}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\dfrac{F_{DLL} \cdot L^2}{8}\right) = 117.6 \cdot kip \cdot ft$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425 \cdot kip \cdot ft}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 4.206 \times 10^3 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2 \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^3}{12} = 175.781 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 23.101 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 14177 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 23.101 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^3}{12} = 58.594 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 17.113 \cdot in$$

S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot (Y'_{nc} - Y')^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9284 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 17.113 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f

$$h - d = 4 \cdot in$$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$$D_p := 7.5in \qquad 0.42 \cdot h = 15.96 \cdot in$$

$\boxed{Dp1 := root\left(Fp(D_p), D_p\right) = 9.835 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.259}$ must be < 0.42

$$M_{n1check} := \left(Dp1 < h_f\right) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot \left[t_w \cdot [Dp1 - (h - d)]\right] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot \left(b_f \cdot t_f\right) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot \left(h - Dp1 - t_f\right)\right] \cdot \frac{\left(h - Dp1 - t_f\right)}{2}\right]$$

$\boxed{Mp1 = 37165.53 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 33033.5 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot \left[t_w \cdot \left[D_p - (h - d)\right]\right] - F_y \cdot \left[A_{stl} - \left[t_w \cdot \left[D_p - (h - d)\right]\right]\right]$$

$$D_p := 2.5in$$

$\boxed{Dp2 := root\left(Fp2(D_p), D_p\right) = 14.918 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.3926}$ must be < 0.42

$$\boxed{M_{n2check} := \left(Dp2 > h_f\right) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1} \qquad Dp2 \le 0.42 \cdot h = 1$$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot \left[t_w \cdot [Dp2 - (h - d)]\right] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot \left(b_f \cdot t_f\right) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot \left[t_w \cdot \left(h - Dp2 - t_f\right)\right] \cdot \frac{\left(h - Dp2 - t_f\right)}{2}$$

$\boxed{Mp2 = 35987.53 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 28617.4 \cdot kip \cdot in$

$$Mn := \max\left(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}\right) = 2384.8 \cdot kip \cdot ft \qquad Mn = 28617.4 \cdot kip \cdot in$$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 11.291\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 11.291\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 6.033\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 10.352\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.39\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 7.02\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 36.187\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 36.147\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 26.51\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.511\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.43 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.835 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.079 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.5 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.748 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1743.4 \cdot kip \cdot ft$ $\qquad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.745 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1741 \cdot kip \cdot ft$ $\qquad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2015.09 \cdot psi$ This assumes short term composite section for all calculations $\qquad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.307 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.835 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.357 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1748.71 \cdot psi$ $\qquad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 542.475 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 613.692 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 816.914 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.633 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 2.45 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 587.953 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 1.532 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.772 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad Mn = 2.385 \times 10^3 \cdot kip \cdot ft \quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.25$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

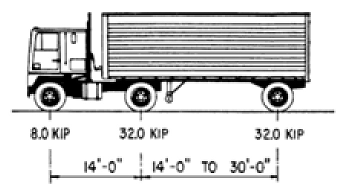
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.162 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

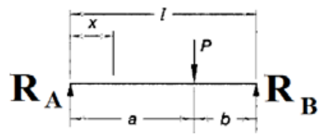
$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support



$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.156 \cdot in$$

Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.937 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

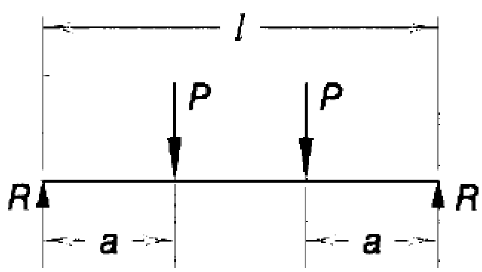
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.868 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.652 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3l^2 - 4a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.621 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 160 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.21 \cdot in}$$

Live Load Deflection:

$$\boxed{\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 1.018 \cdot in}$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2}\right) = 418.078 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.445 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.518 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 ***(at left support)*** (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 40.943 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 39.837 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 11.051 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 15.585 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 18.133 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 137.669 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot (1 + I_{PL}) = 80.737 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot (V_{PL82}) = 155.676 \cdot kip$

Max Shear Demand $Vu := \max(V_{str1}, V_{strII\_PL}) = 155.676 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 54.4 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 28.775 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$$I_s := \frac{18 \cdot t_w \cdot t_w^{\ 3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$$

Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.27 \times 10^4 \cdot kip$$

Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$$PePo := \frac{Pe}{Po} \ge 0.44 = 1$$

$$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.249 \cdot kip$$

Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.637 \cdot kip$ — Compressive capacity of bearing stiffener

$Pr \ge Vu = 1$

Flexbeam Example Configuration Calculations

Summary of Calculations:

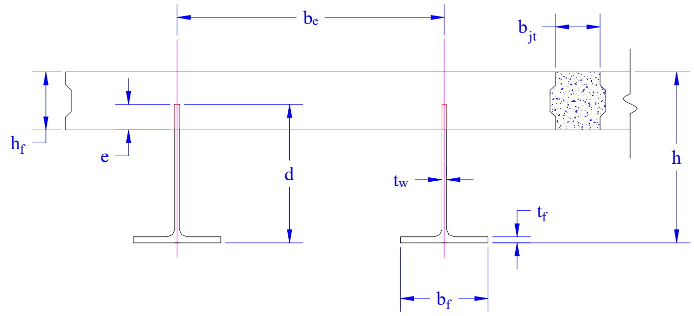
1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- Example configurations are determined for two girder spacings, 36" and 40".
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to to be conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 45 in. tall F-Shape barrier is used, Weight = 700 lb/ft (Distributed to exterior two beams).

Variable Definitions:

Number of beams $N := 14$ $modules := \frac{N}{2} = 7$



$L := 70ft$ Span length

$b_e := 40in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.625in$ $t_f := 1.125in$ $b_f := 12in$

$h := 38in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 34 \cdot in$ depth of built up steel section

$h_w := d - t_f = 32.875 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 684.78kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 136.38kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 10.822 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 4.206 \times 10^3 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 34.047 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\frac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 6\text{in}$

Parapet width: $b_{par} := 20.25\text{in}$ Parapet width per PennDOT F-shape barrier

Width of the bridge $W := b_e \cdot N - b_{jt} = 554 \cdot \text{in}$ $W = 46.167\text{ ft}$

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 42.792\text{ ft}$

$D_c := (w_c \cdot h_f \cdot W) = 4.328 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .700\,\frac{\text{kip}}{\text{ft}}$ DL due to each 45" F-Shape Barrier (per email 11/1/18)

$D_{FWS} := 30\,\frac{\text{lbf}}{\text{ft}^2} \cdot (W - 2 \cdot b_{par}) = 1.284 \cdot \frac{\text{kip}}{\text{ft}}$ DL due to Future Wearing Surface (FWS)

$D_s := A_{stl} \cdot 490\,\frac{\text{lbf}}{\text{ft}^3} = 0.116 \cdot \frac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \frac{(D_c)}{N} + D_s = 0.425 \cdot \frac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\frac{\text{lbf}}{\text{ft}} \cdot 32\text{in} = 0.067 \cdot \text{kip}$

$$\boxed{\left[\left[(2b_e - b_{jt})(h_f \cdot w_c)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 56.822 \cdot \text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1)

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := \text{floor}\left(\frac{W_{lane}}{12\text{ft}}\right) = 3$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.33 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.116 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.446 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.092 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.35 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.078 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.519 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 5 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.667 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments (Based on first interior girder demands)*

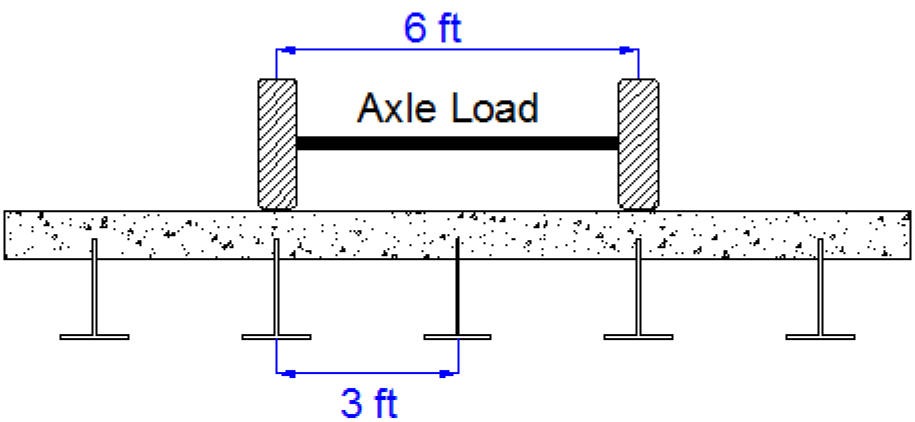
$$M_{DL} := \frac{DL \cdot L^2}{8} = 273.076 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 318.178 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 23.428 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\ 2}\right) = 1.83 \times 10^5 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 30.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 33.158 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 162 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 14.737 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

<u>Design Truck</u>

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

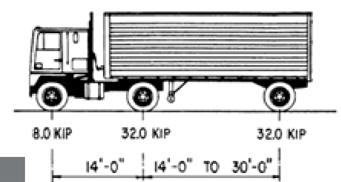
Design truck load in AASHTO (HS20 Truck) is used for this example.

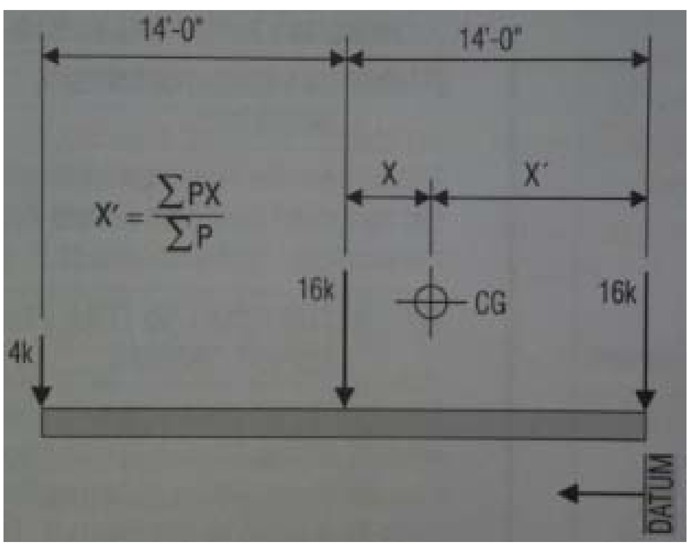
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $\quad P_2 := 32\text{kip}$ $\quad P_3 := 32\text{kip}$ $\quad$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ $\quad$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ $\quad$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

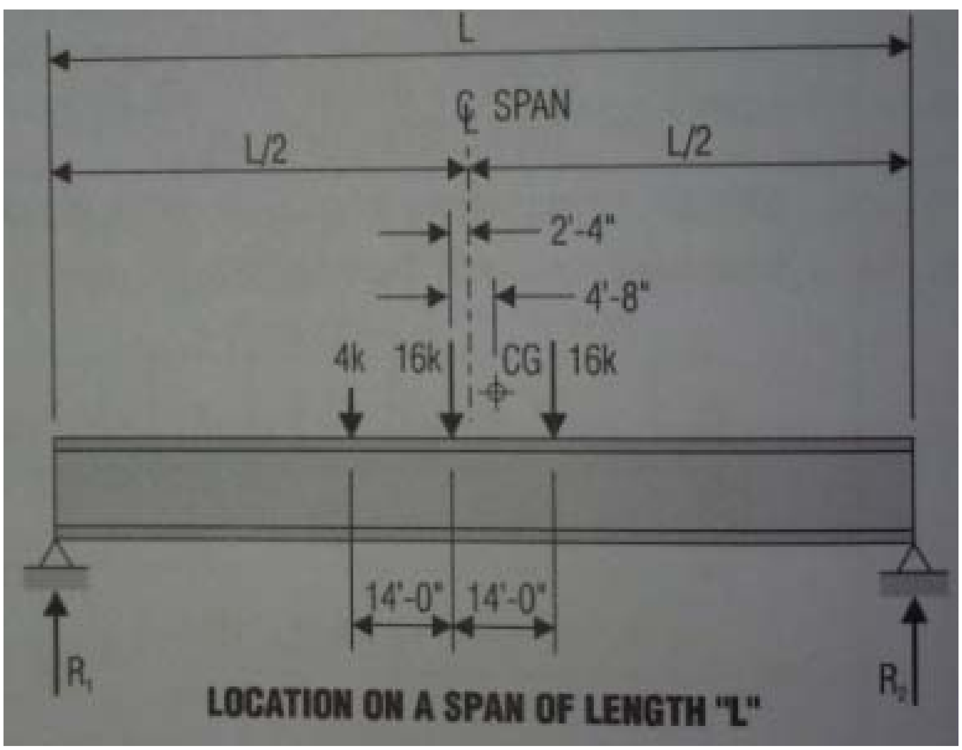
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 33.6 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 985.6 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$$F_T := 31.25\text{kip}$$

$$X_{AS} := 4\text{ft}$$ Axle spacing

Center of gravity (CG) of loads:

$$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 30.357\cdot\text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 1.032\times10^3\cdot\text{kip}\cdot\text{ft}}$$

Apply the impact and wheel load distribution:

$$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 411.825\cdot\text{kip}\cdot\text{ft}$$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

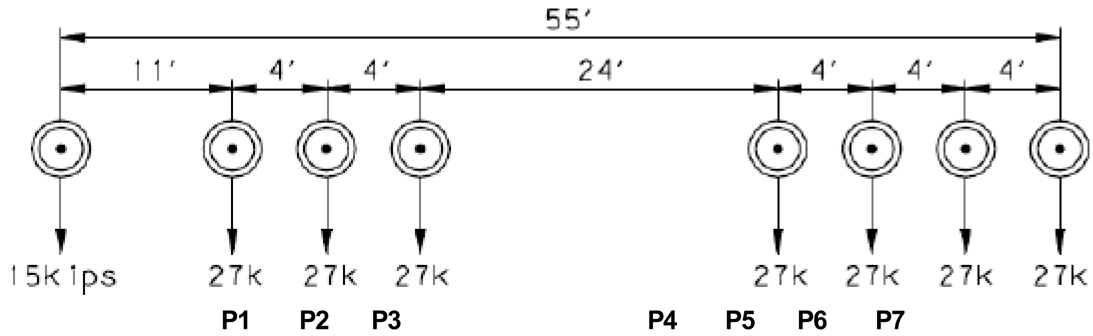
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 117.6\cdot\text{kip}\cdot\text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 529.425\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 4.206 \times 10^{3} \cdot in^{4}$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$A_{ctr1} := h_f \cdot b_{tr1} = 37.5 \cdot in^2$ $I_{tr1} := \dfrac{b_{tr1} \cdot h_f^{\,3}}{12} = 175.781 \cdot in^4$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 23.101 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 14177 \cdot in^4$$

Composite section modulus (short-term)

$Y'_{c\_short} := Y' = 23.101 \cdot in$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$A_{ctr3} := h_f \cdot b_{tr3} = 12.5 \cdot in^2$ $I_{tr3} := \dfrac{b_{tr3} \cdot h_f^{\,3}}{12} = 58.594 \cdot in^4$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \dfrac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 17.113 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 9284 \cdot in^4$$

Composite section modulus (long-term)

$Y'_{c\_long} := Y' = 17.113 \cdot in$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 15.96 \cdot in$

$Dp1 := root(Fp(D_p), D_p) = 9.835 \cdot in$ Must be less than $h_f = 7.5 \cdot in$ $\quad \frac{Dp1}{h} = 0.259$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 0 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h - d)]] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$Mp1 = 37165.53 \cdot kip \cdot in \qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 33033.5 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 2.5in$

$Dp2 := root(Fp2(D_p), D_p) = 14.918 \cdot in$ Must be more than $h_f = 7.5 \cdot in$ $\quad \frac{Dp2}{h} = 0.3926$ must be < 0.42

$M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 1 \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h - d)]] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$Mp2 = 35987.53 \cdot kip \cdot in \qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 28617.4 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 2384.8 \cdot kip \cdot ft \qquad Mn = 28617.4 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = 11.291\cdot in \qquad D_{cp0} := D_{cp} > 0 = 1$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 11.291\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 6.041\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 10.352\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 13.39\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 7.038\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 36.225\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 36.185\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 26.54\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 26.537\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.43 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.835 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.079 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.502 \cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.751 \cdot ksi$$

$$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 1745.1 \cdot kip \cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.748 \cdot ksi$$

$$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 1743 \cdot kip \cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6 \cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 2016.96 \cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.307 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.835 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.358 \cdot ksi$$

$$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1750.05 \cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 542.475 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 613.692 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 818.612 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 1.631 \times 10^3 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 2.45 \times 10^3 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 587.899 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5 M_{SDL} + 1.35 M_{LL\_IM} = 1.533 \times 10^3 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 1.774 \times 10^3 \cdot kip \cdot f$ Equation 6.10.7.1.7-1 $\quad Mn = 2.385 \times 10^3 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.214$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

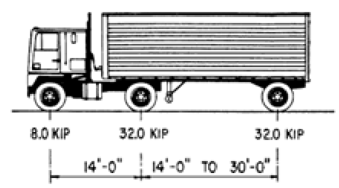
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 2.162 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = 16.333 \cdot ft$ distance btw load P1 and support A $b1check := b1 > 0 = 1$

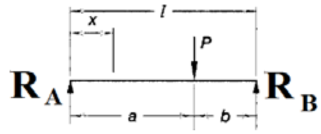
$b2 := \frac{L}{2} - (X_{ASr} - X') = 30.333 \cdot ft$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 25.667 \cdot ft$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.156 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.937 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

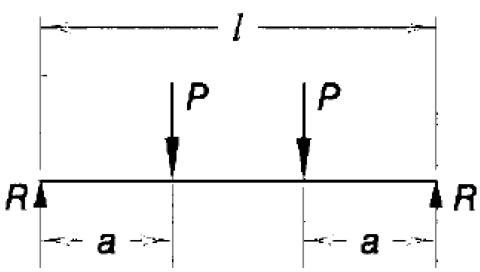
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.868 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta1 + \Delta2 + \Delta3) \cdot DF_d \cdot (1 + I) = 0.559 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.532 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 137.143 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.18 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25 \max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.873 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 1.05 \cdot in$ $\frac{L}{1000} = 0.84 \cdot in$

<u>Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)</u>

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 3.333$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.493}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 418.078 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.446 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.519 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 42 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 56 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 70 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 62.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 40.943 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 60.714 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I) \right] = 39.837 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 22.4 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 11.051 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 15.604 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 18.182 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1} \right) \right) = 137.766 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 80.737 \cdot kip$$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 155.773 \cdot kip$$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 155.773 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 34 \cdot in$ Thickness of web $t_w = 0.625 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 616.25 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$$\frac{D}{t_w} = 54.4 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$$

Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 616.25 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 616.25 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ — Yield strength of stiffener

$t_p := 0.75in$ — Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.188 \cdot in$ — Effective width of stiffener

$D_{st} := h_w - 4.1in = 28.775 \cdot in$ — Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.281 \cdot in^2$ — Projected area of stiffeners

Check that effective width of stiffener meets requirements

$b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 439.688 \cdot kip$ — Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ — Bearing capacity check

$Q_s := 1.0$ — Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 36.943 \cdot in^4$ — Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 13.313 \cdot in^2$ — Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 1.666 \cdot in$ — Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 314.062 \cdot kip$ — Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 2.27 \times 10^4 \cdot kip$ — Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1)

$PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 312.249 \cdot kip$ — Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ — phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 296.637 \cdot kip$ — Compressive capacity of bearing stiffener

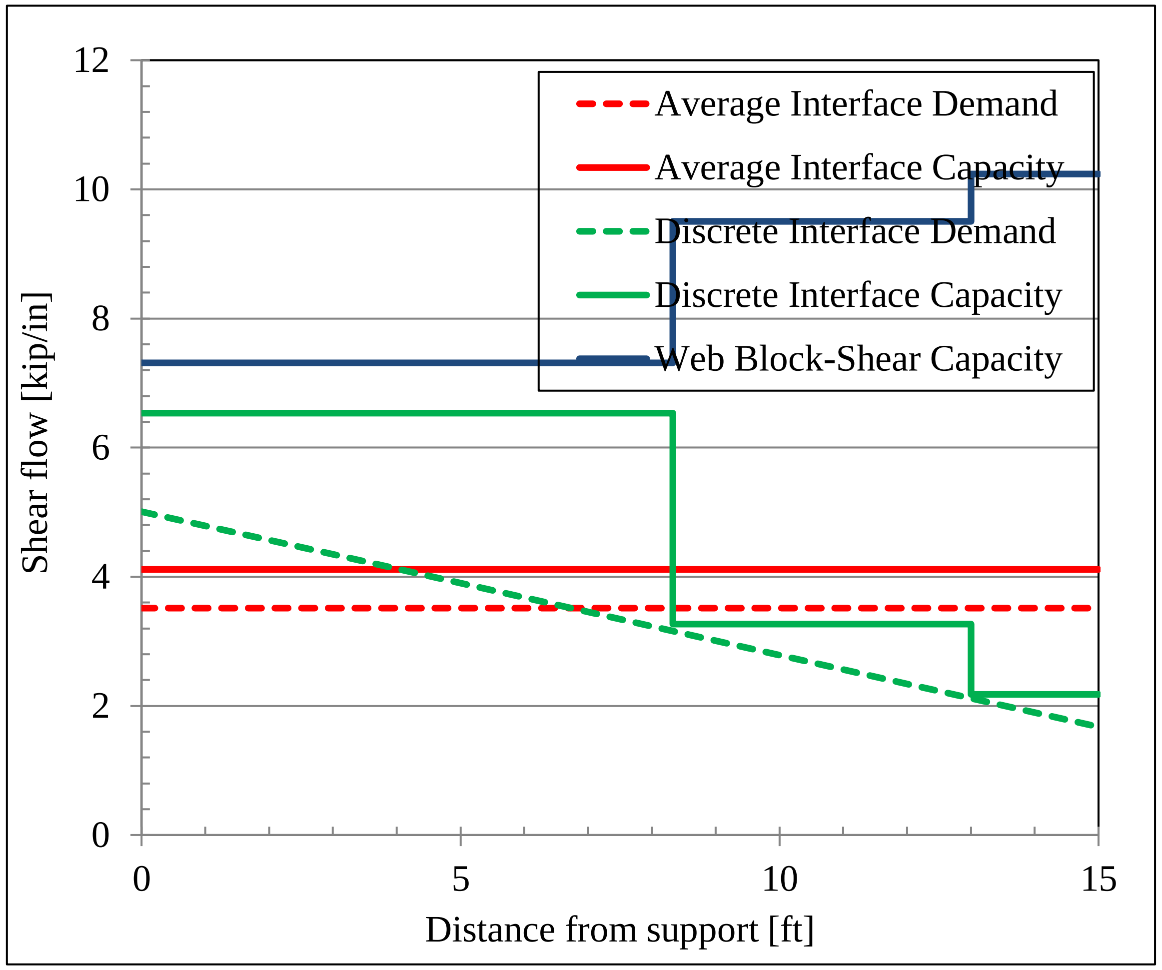
$Pr \ge Vu = 1$

3. Appendix B – Example Configuration – Shear Calculations

The interface shear calculations are included in this section. Shear transfer rebar spacing is limited to 4, 8, and 12 inches. The spacing of the rebar is specified to transfer the entire horizontal shear force in the deck to the steel section. The interface shear meets both the maximum discrete shear demands as well as the average shear demand due to the plastic moment occurring at midspan.

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	30 ft
Width of deck:	36 in
Q_{short}:	121.75 in^3
I_{short}:	2199 in^4
Area of steel:	12.652 in^2
Web thickness:	0.375 in
Web tensile:	65 ksi
Area of deck:	270 in^2
Avg. interface shear:	3.51 kip/in
Block shear ϕ:	0.80



Calculated Shear Demand Envelope from Matlab

Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	90.446	5.01	3.51
3	78.547	4.35	3.51
6	66.445	3.68	3.51
9	54.344	3.01	3.51
12	42.243	2.34	3.51
15	30.258	1.68	3.51

Shear Dowel Spacing Design

Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	25.0	6.534	7.3125	4.113
100	8.3	4		6.534	7.3125	4.113
100	8.3	8	7.0	3.267	9.50625	4.113
156	13.0	8		3.267	9.50625	4.113
156	13.0	12	2.0	2.178	10.2375	4.113
180	15.0	12		2.178	10.2375	4.113

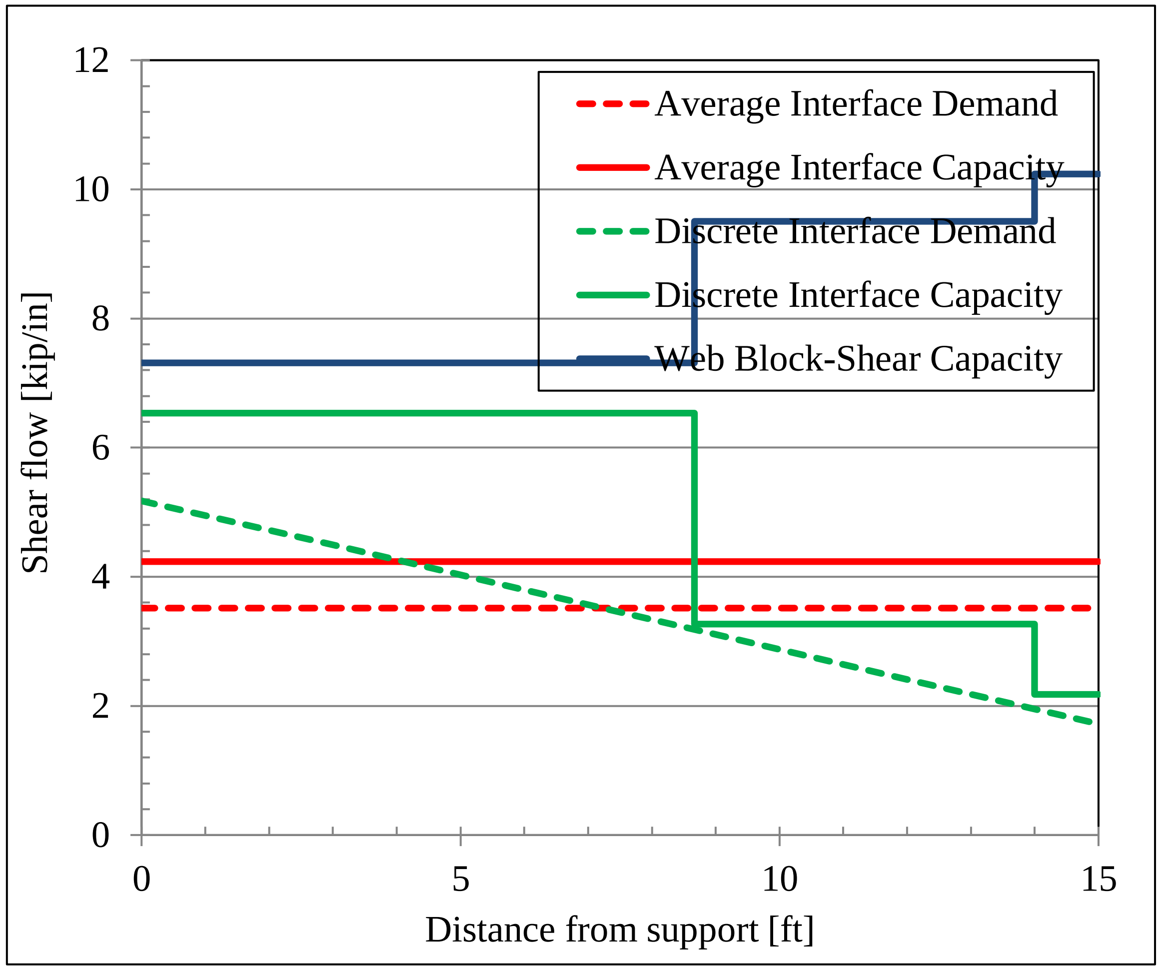
Total Number of Holes	34	holes on half span
Total Force Capacity	740	kip
Average Shear Capacity	4.11	kip/in

Capacity of Dowel

Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	30 ft
Width of deck:	40 in
Q_{short}:	125.14 in^3
I_{short}:	2261 in^4
Area of steel:	12.652 in^2
Web thickness:	0.375 in
Web tensile:	65 ksi
Area of deck:	300 in^2
Avg. interface shear:	3.51 kip/in
Block shear ϕ:	0.80



Calculated Shear Demand Envelope from Matlab			
Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	93.495	5.17	3.51
3	81.159	4.49	3.51
6	68.614	3.80	3.51
9	56.069	3.10	3.51
12	43.524	2.41	3.51
15	31.098	1.72	3.51

Shear Dowel Spacing Design						
Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	26.0	6.534	7.3125	4.235
104	8.7	4		6.534	7.3125	4.235
104	8.7	8	8.0	3.267	9.50625	4.235
168	14.0	8		3.267	9.50625	4.235
168	14.0	12	1.0	2.178	10.2375	4.235
180	15.0	12		2.178	10.2375	4.235
	Total Number of Holes		35	holes on half span		
	Total Force Capacity		762	kip		
	Average Shear Capacity		4.23	kip/in		

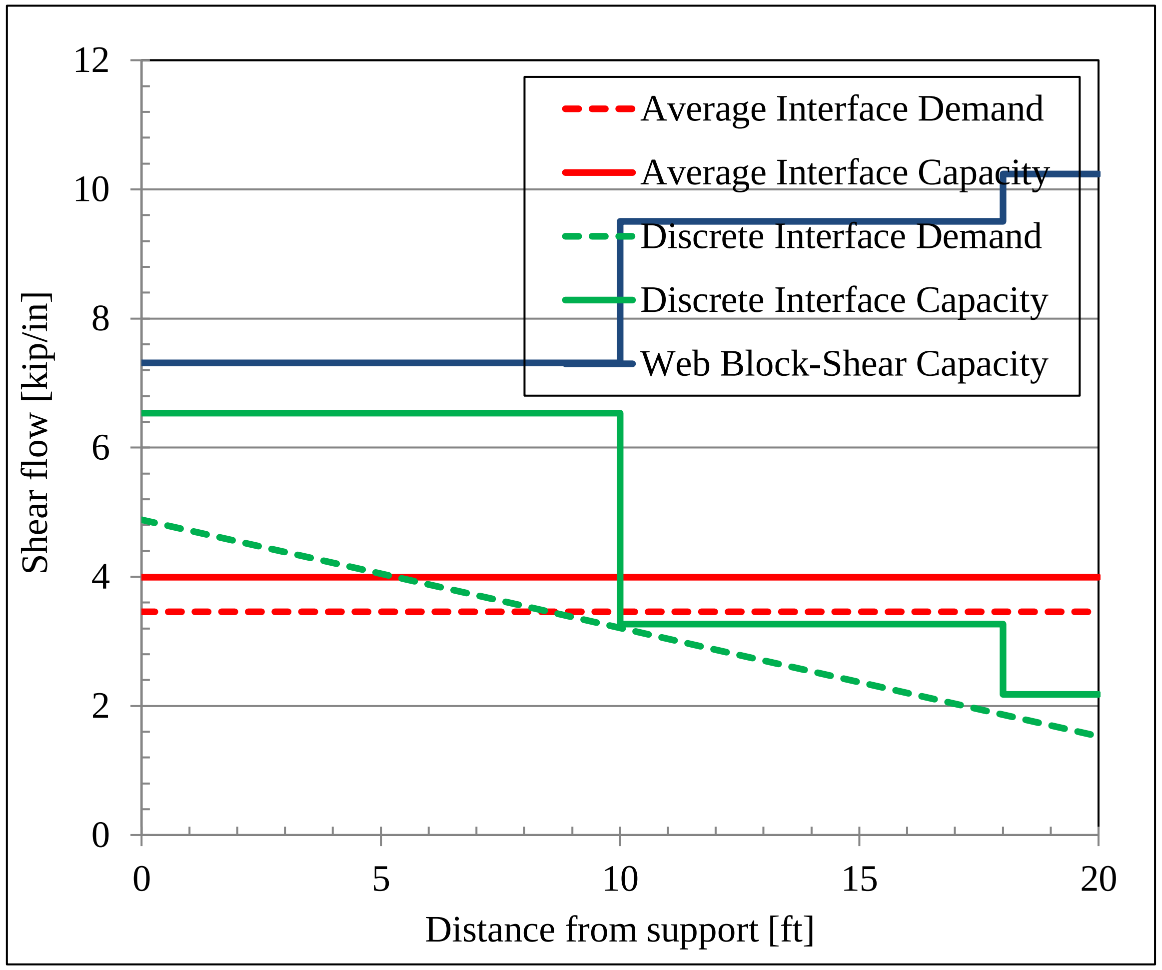
Capacity of Dowel		
Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	40 ft
Width of deck:	36 in
Q_{short}:	147 in^3
I_{short}:	3020 in^4
Area of steel:	16.594 in^2
Web thickness:	0.375 in
Web tensile:	65 ksi
Area of deck:	270 in^2
Avg. interface shear:	3.46 kip/in
Block shear ϕ:	0.80

Calculated Shear Demand Envelope from Matlab

Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	100.3	4.88	3.46
4	86.645	4.22	3.46
8	72.817	3.54	3.46
12	58.989	2.87	3.46
16	45.162	2.20	3.46
20	31.421	1.53	3.46



Shear Dowel Spacing Design

Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	30.0	6.534	7.3125	3.993
120	10.0	4		6.534	7.3125	3.993
120	10.0	8	12.0	3.267	9.50625	3.993
216	18.0	8		3.267	9.50625	3.993
216	18.0	12	2.0	2.178	10.2375	3.993
240	20.0	12		2.178	10.2375	3.993
		Total Number of Holes	44	holes on half span		
		Total Force Capacity	958	kip		
		Average Shear Capacity	3.99	kip/in		

Capacity of Dowel

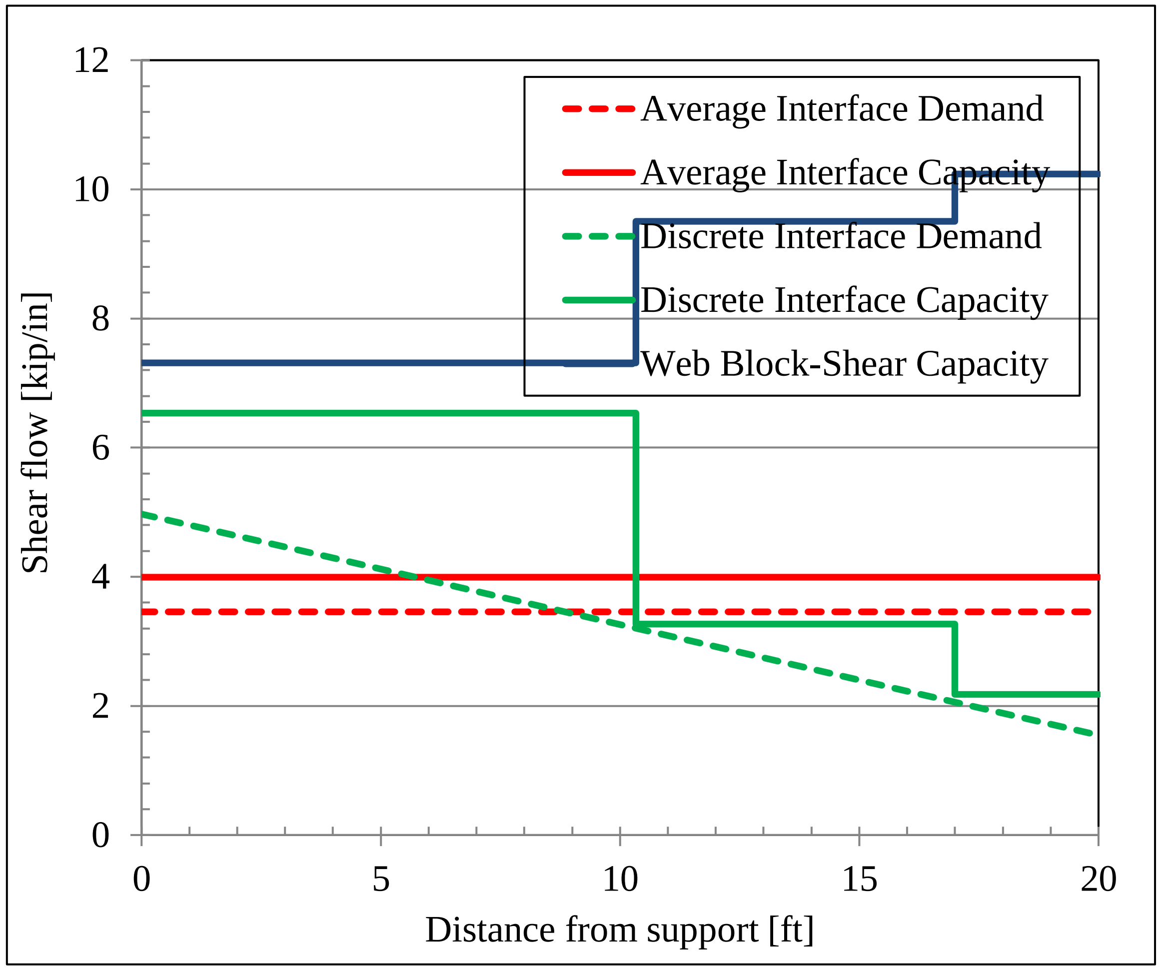
Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	40 ft
Width of deck:	40 in
Q_{short}:	184.86 in^3
I_{short}:	3860.5 in^4
Area of steel:	16.594 in^2
Web thickness:	0.375 in
Web tensile:	65 ksi
Area of deck:	300 in^2
Avg. interface shear:	3.46 kip/in
Block shear ϕ:	0.80

Calculated Shear Demand Envelope from Matlab

Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	103.8	4.97	3.46
4	89.623	4.29	3.46
8	75.269	3.60	3.46
12	60.914	2.92	3.46
16	46.559	2.23	3.46
20	32.294	1.55	3.46



Shear Dowel Spacing Design

Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	31.0	6.534	7.3125	3.993
124	10.3	4		6.534	7.3125	3.993
124	10.3	8	10.0	3.267	9.50625	3.993
204	17.0	8		3.267	9.50625	3.993
204	17.0	12	3.0	2.178	10.2375	3.993
240	20.0	12		2.178	10.2375	3.993

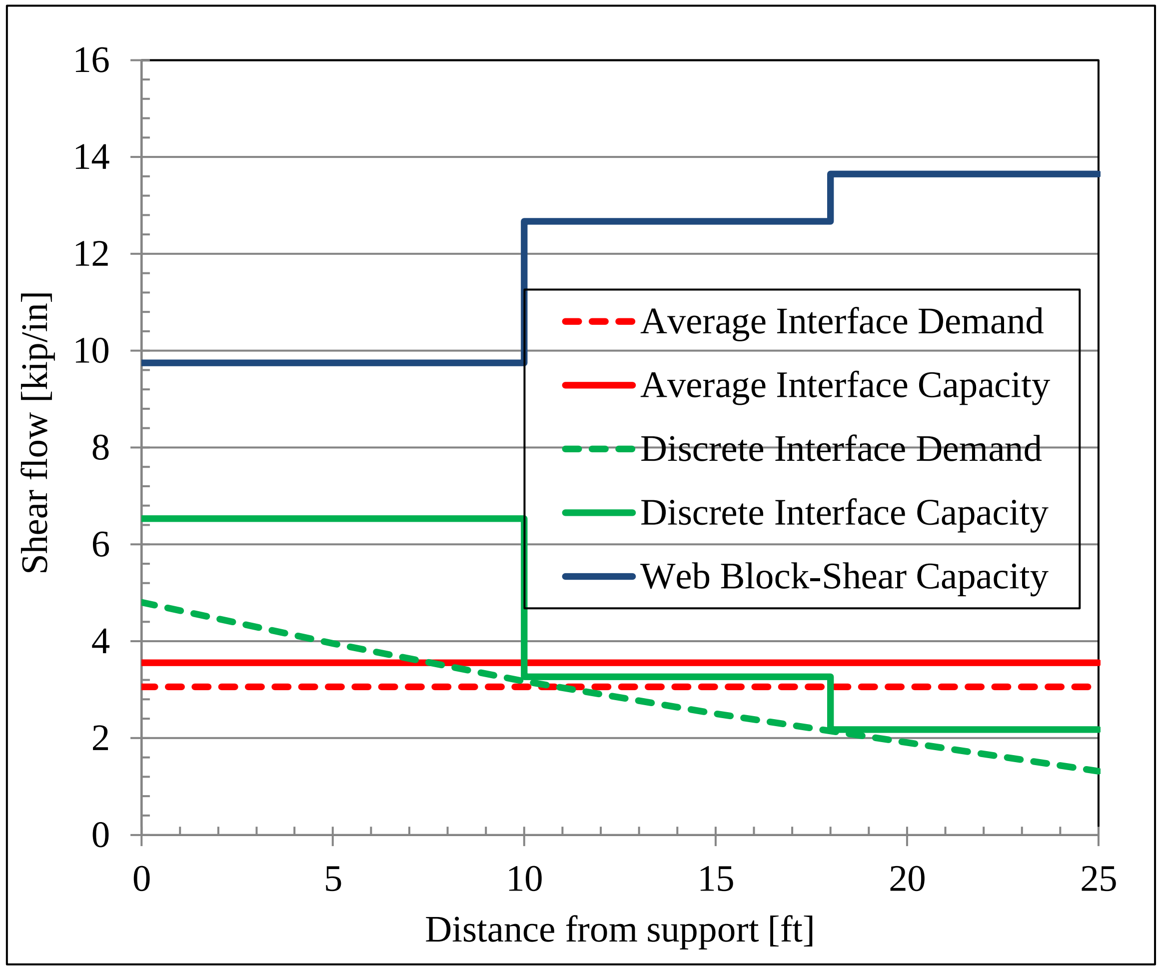
Total Number of Holes	44	holes on half span
Total Force Capacity	958	kip
Average Shear Capacity	3.99	kip/in

Capacity of Dowel

Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	50 ft
Width of deck:	36 in
Q_{short}:	213.8 in^3
I_{short}:	5166 in^4
Area of steel:	19.687 in^2
Web thickness:	0.5 in
Web tensile:	65 ksi
Area of deck:	270 in^2
Avg. interface shear:	3.06 kip/in
Block shear ϕ:	0.80



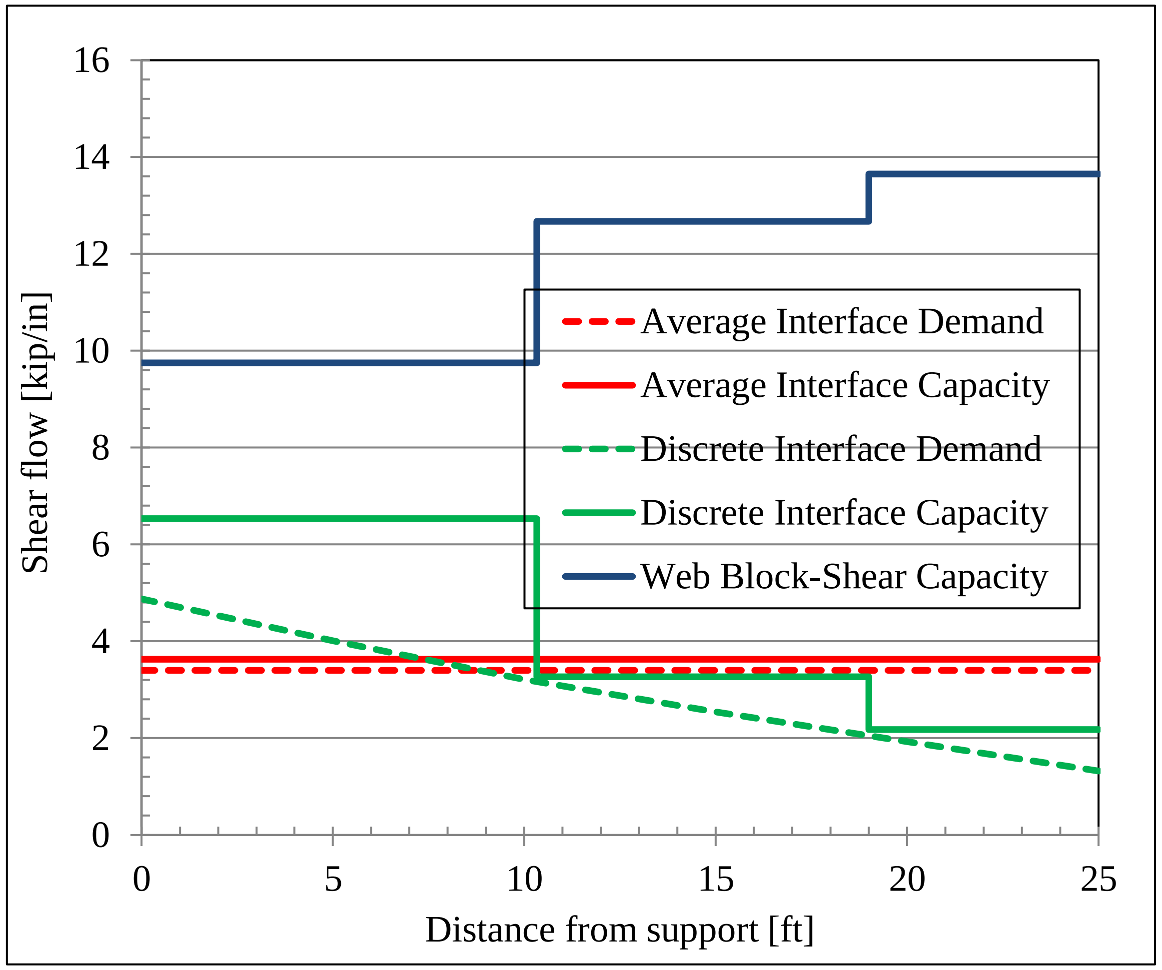
Calculated Shear Demand Envelope from Matlab			
Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	116.11	4.81	3.06
5	95.631	3.96	3.06
10	76.662	3.17	3.06
15	60.612	2.51	3.06
20	46.22	1.91	3.06
25	31.829	1.32	3.06

Shear Dowel Spacing Design						
Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	30.0	6.534	9.75	3.557
120	10.0	4		6.534	9.75	3.557
120	10.0	8	12.0	3.267	12.675	3.557
216	18.0	8		3.267	12.675	3.557
216	18.0	12	7.0	2.178	13.65	3.557
300	25.0	12		2.178	13.65	3.557
	Total Number of Holes		49	holes on half span		
	Total Force Capacity		1067	kip		
	Average Shear Capacity		3.56	kip/in		

Capacity of Dowel		
Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	50 ft
Width of deck:	40 in
Q_{short}:	270.7 in^3
I_{short}:	6699 in^4
Area of steel:	24 in^2
Web thickness:	0.5 in
Web tensile:	65 ksi
Area of deck:	300 in^2
Avg. interface shear:	3.40 kip/in
Block shear ϕ:	0.80



Calculated Shear Demand Envelope from Matlab			
Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	120.68	4.88	3.40
5	99.363	4.02	3.40
10	79.598	3.22	3.40
15	62.833	2.54	3.40
20	47.773	1.93	3.40
25	32.713	1.32	3.40

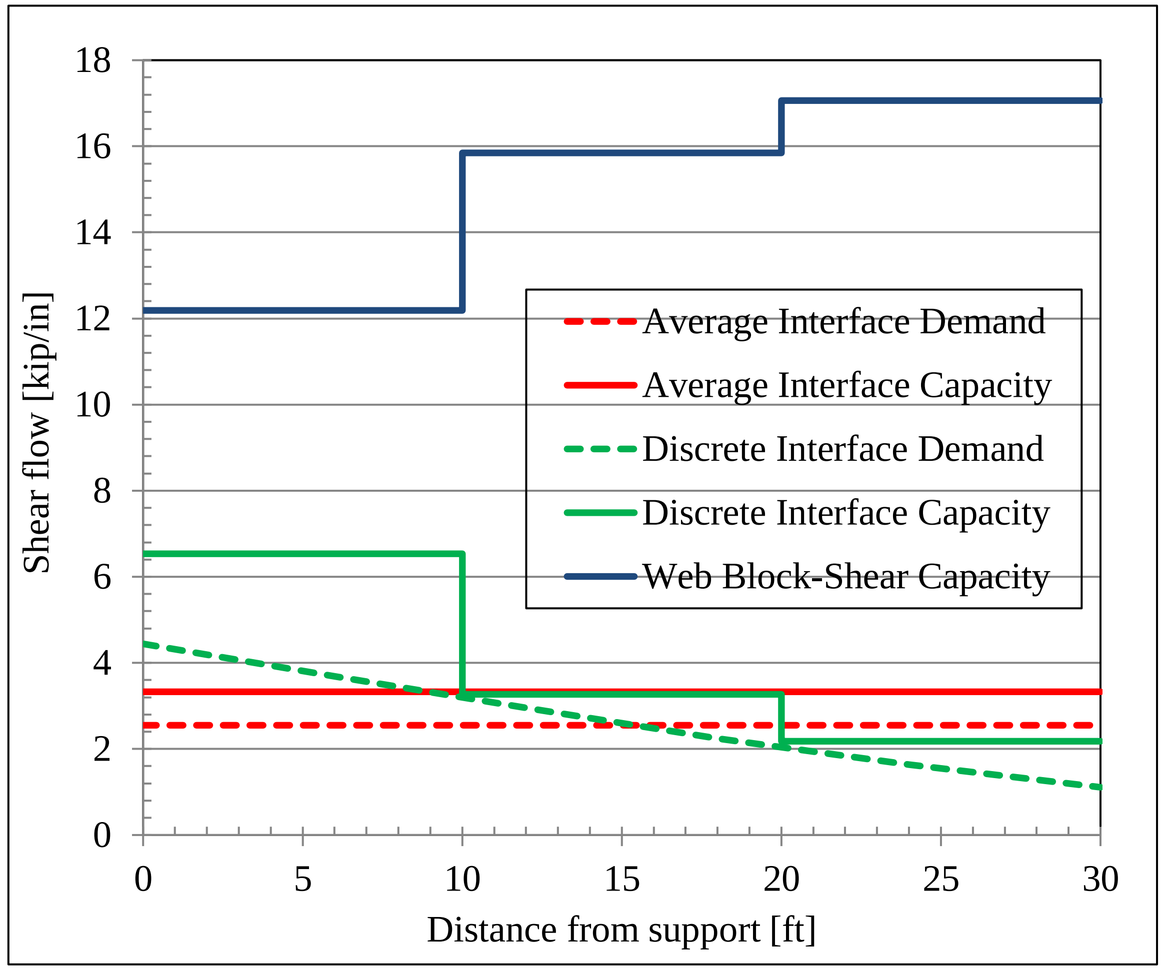
Shear Dowel Spacing Design						
Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	31.0	6.534	9.75	3.630
124	10.3	4		6.534	9.75	3.630
124	10.3	8	13.0	3.267	12.675	3.630
228	19.0	8		3.267	12.675	3.630
228	19.0	12	6.0	2.178	13.65	3.630
300	25.0	12		2.178	13.65	3.630
		Total Number of Holes	50	holes on half span		
		Total Force Capacity	1089	kip		
		Average Shear Capacity	3.63	kip/in		

Capacity of Dowel		
Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties		
Length:	60	ft
Width of deck:	36	in
Q_{short}:	311.2	in^3
I_{short}:	9406	in^4
Area of steel:	27.906	in^2
Web thickness:	0.625	in
Web tensile:	65	ksi
Area of deck:	270	in^2
Avg. interface shear:	2.55	kip/in
Block shear ϕ:	0.80	

Calculated Shear Demand Envelope from Matlab			
Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	134.23	4.44	2.55
6	111.37	3.68	2.55
12	89.269	2.95	2.55
18	67.887	2.25	2.55
24	49.323	1.63	2.55
30	33.522	1.11	2.55

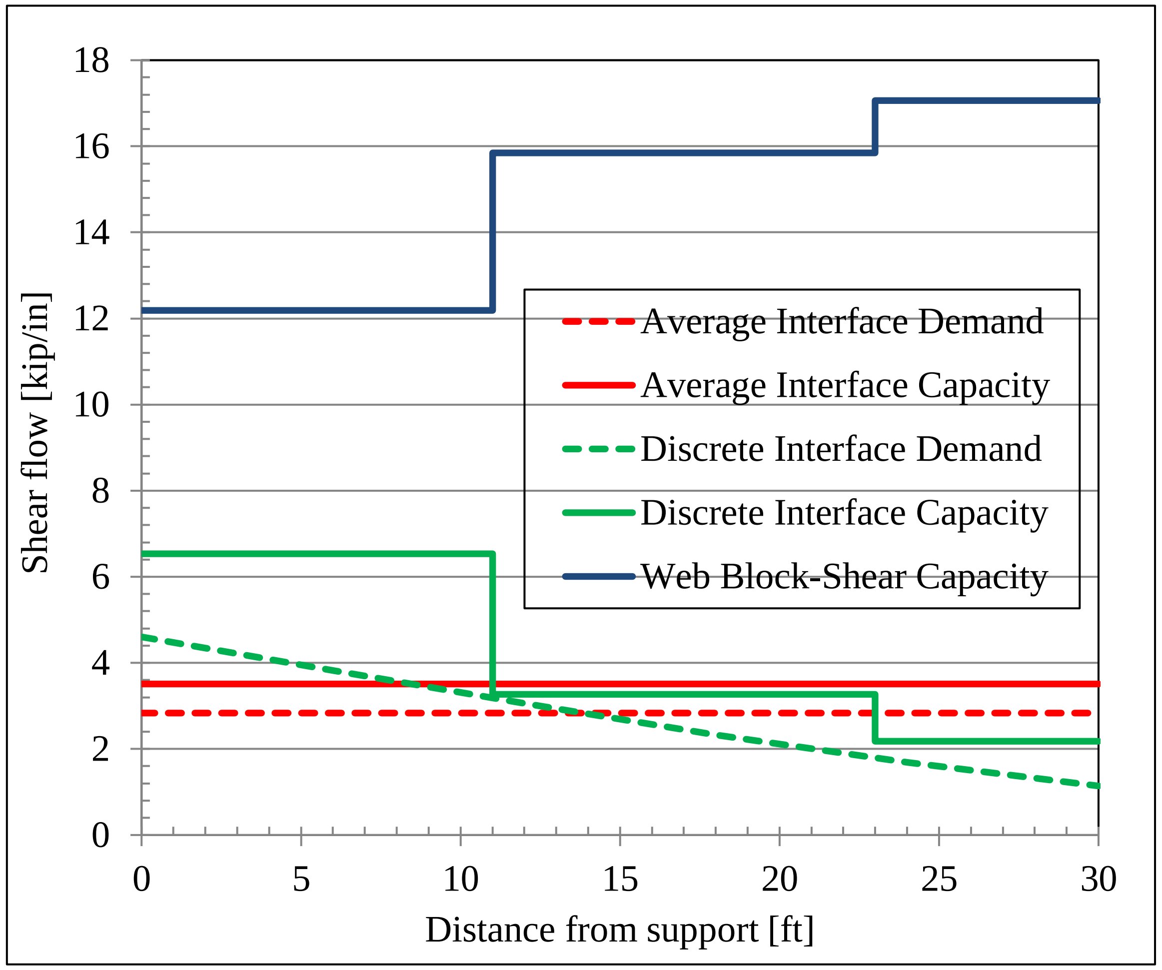


Shear Dowel Spacing Design						
Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	30.0	6.534	12.1875	3.328
120	10.0	4		6.534	12.1875	3.328
120	10.0	8	15.0	3.267	15.84375	3.328
240	20.0	8		3.267	15.84375	3.328
240	20.0	12	10.0	2.178	17.0625	3.328
360	30.0	12		2.178	17.0625	3.328
		Total Number of Holes	55	holes on half span		
		Total Force Capacity	1198	kip		
		Average Shear Capacity	3.33	kip/in		

Capacity of Dowel		
Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties		
Length:	60	ft
Width of deck:	40	in
Q_{short}:	343.32	in^3
I_{short}:	10380	in^4
Area of steel:	29.328	in^2
Web thickness:	0.625	in
Web tensile:	65	ksi
Area of deck:	300	in^2
Avg. interface shear:	2.83	kip/in
Block shear ϕ:	0.80	



Calculated Shear Demand Envelope from Matlab

Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	139.17	4.60	2.83
6	115.43	3.82	2.83
12	92.474	3.06	2.83
18	70.257	2.32	2.83
24	50.935	1.68	2.83
30	34.454	1.14	2.83

Shear Dowel Spacing Design

Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	33.0	6.534	12.1875	3.509
132	11.0	4		6.534	12.1875	3.509
132	11.0	8	18.0	3.267	15.84375	3.509
276	23.0	8		3.267	15.84375	3.509
276	23.0	12	7.0	2.178	17.0625	3.509
360	30.0	12		2.178	17.0625	3.509
	Total Number of Holes		58	holes on half span		
	Total Force Capacity		1263	kip		
	Average Shear Capacity		3.51	kip/in		

Capacity of Dowel

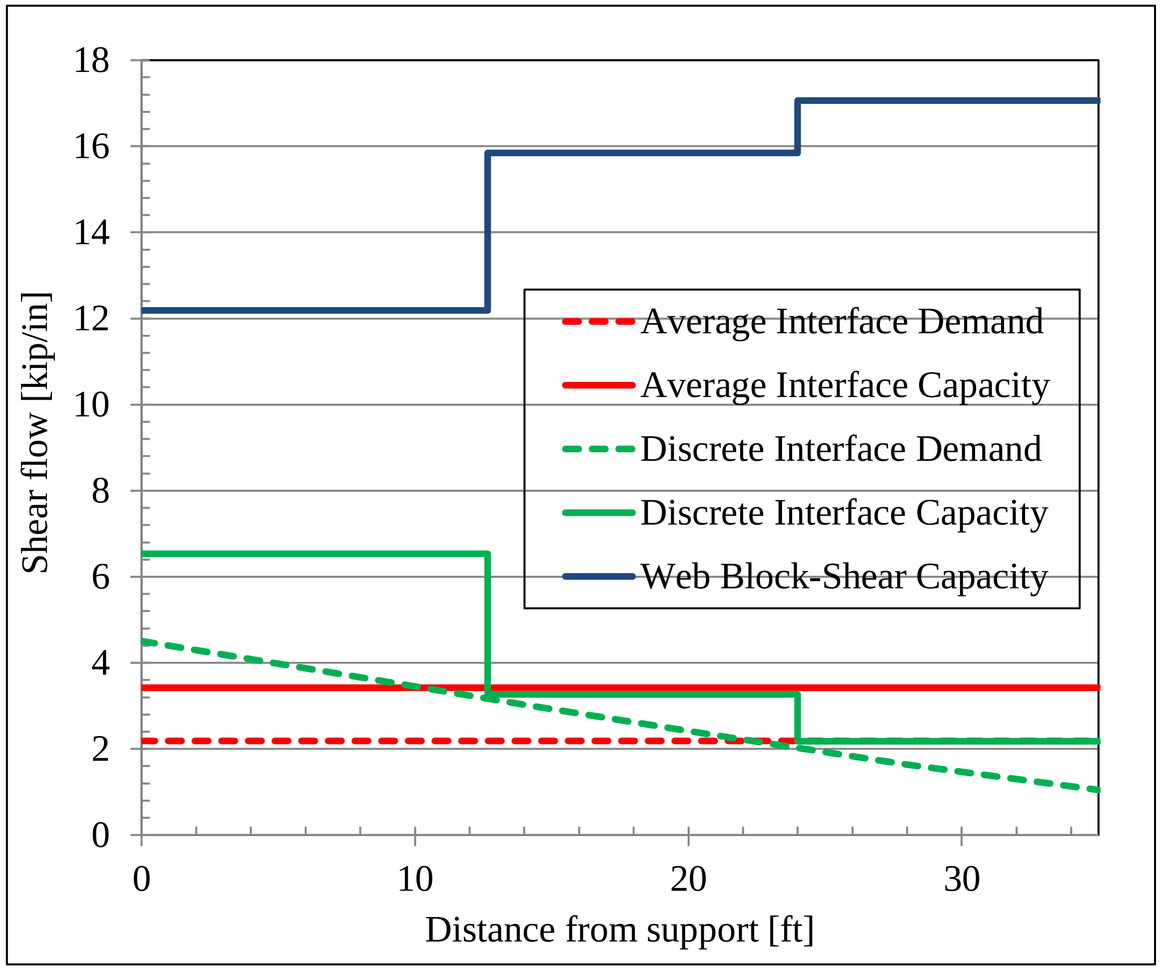
Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	70 ft
Width of deck:	36 in
Q_{short}:	348.25 in^3
I_{short}:	11550 in^4
Area of steel:	29.781 in^2
Web thickness:	0.625 in
Web tensile:	65 ksi
Area of deck:	270 in^2
Avg. interface shear:	2.19 kip/in
Block shear ϕ:	0.80

Calculated Shear Demand Envelope from Matlab

Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	149.43	4.51	2.19
7	125.05	3.77	2.19
14	100.5	3.03	2.19
21	76.958	2.32	2.19
28	54.188	1.63	2.19
35	34.732	1.05	2.19



Shear Dowel Spacing Design

Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	38.0	6.534	12.1875	3.423
152	12.7	4		6.534	12.1875	3.423
152	12.7	8	17.0	3.267	15.84375	3.423
288	24.0	8		3.267	15.84375	3.423
288	24.0	12	11.0	2.178	17.0625	3.423
420	35.0	12		2.178	17.0625	3.423

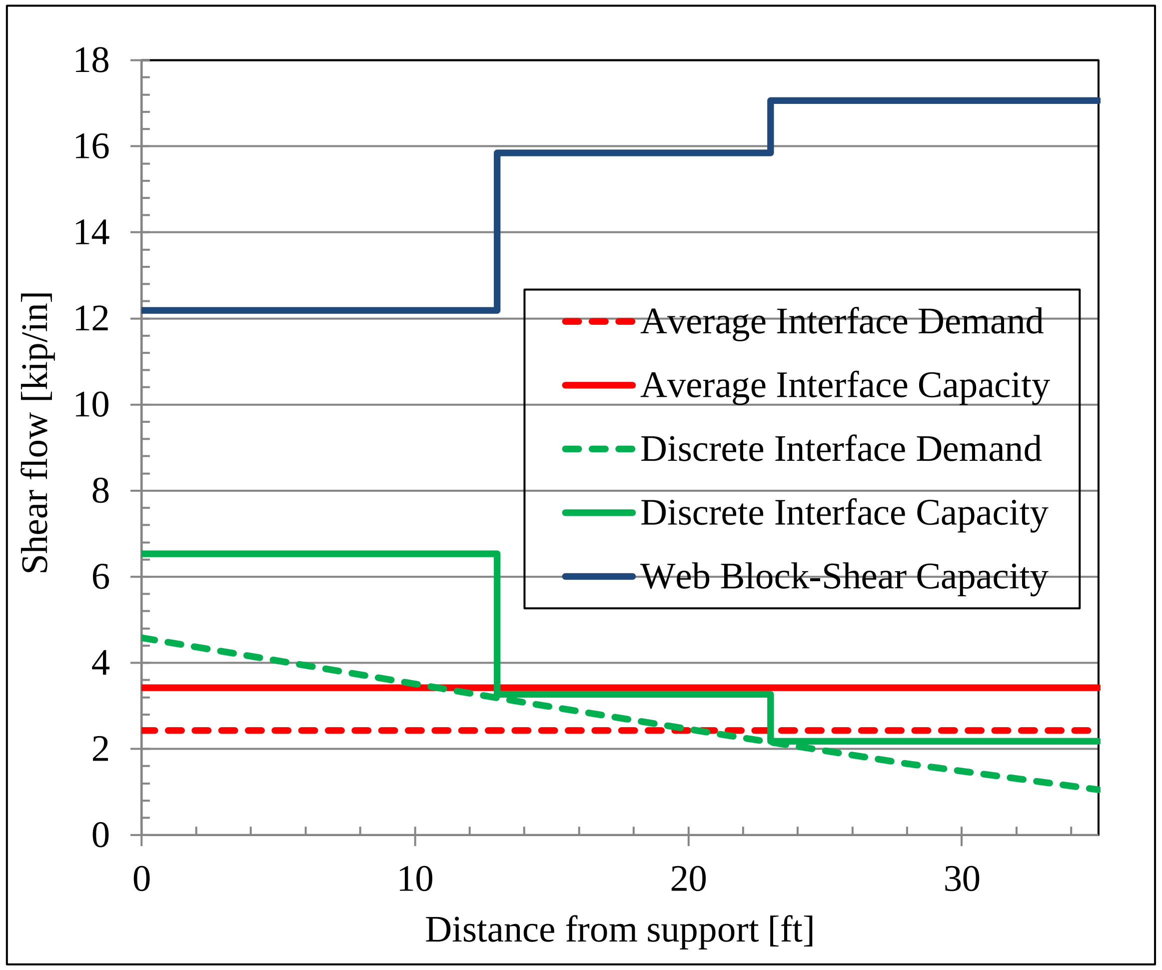
Total Number of Holes	66	holes on half span
Total Force Capacity	1437	kip
Average Shear Capacity	3.42	kip/in

Capacity of Dowel

Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties		
Length:	70	ft
Width of deck:	40	in
Q_{short}:	418.1	in^3
I_{short}:	14180	in^4
Area of steel:	34.047	in^2
Web thickness:	0.625	in
Web tensile:	65	ksi
Area of deck:	300	in^2
Avg. interface shear:	2.43	kip/in
Block shear ϕ:	0.80	



Calculated Shear Demand Envelope from Matlab

Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	155.42	4.58	2.43
7	130	3.83	2.43
14	104.4	3.08	2.43
21	79.832	2.35	2.43
28	56.061	1.65	2.43
35	35.697	1.05	2.43

Shear Dowel Spacing Design

Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	39.0	6.534	12.1875	3.423
156	13.0	4		6.534	12.1875	3.423
156	13.0	8	15.0	3.267	15.84375	3.423
276	23.0	8		3.267	15.84375	3.423
276	23.0	12	12.0	2.178	17.0625	3.423
420	35.0	12		2.178	17.0625	3.423

Total Number of Holes	66	holes on half span
Total Force Capacity	1437	kip
Average Shear Capacity	3.42	kip/in

Capacity of Dowel

Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

4. Appendix C – Example Configuration – Deflection Calculations

Sample deflection calculations for dead load are included in this section. Three deflections are tabulated at 10% span increments: The dead load deflection due to the self weight of the concrete and steel girder/deck, the superimposed dead load due to the 140 lb/ft^3 1.25 in. thick overlay, and the superimposed dead load due to the 45 inch F-shape Barrier which weighs 700 plf. The barrier superimposed dead load is assumed to be evenly distributed over the outer two steel sections.

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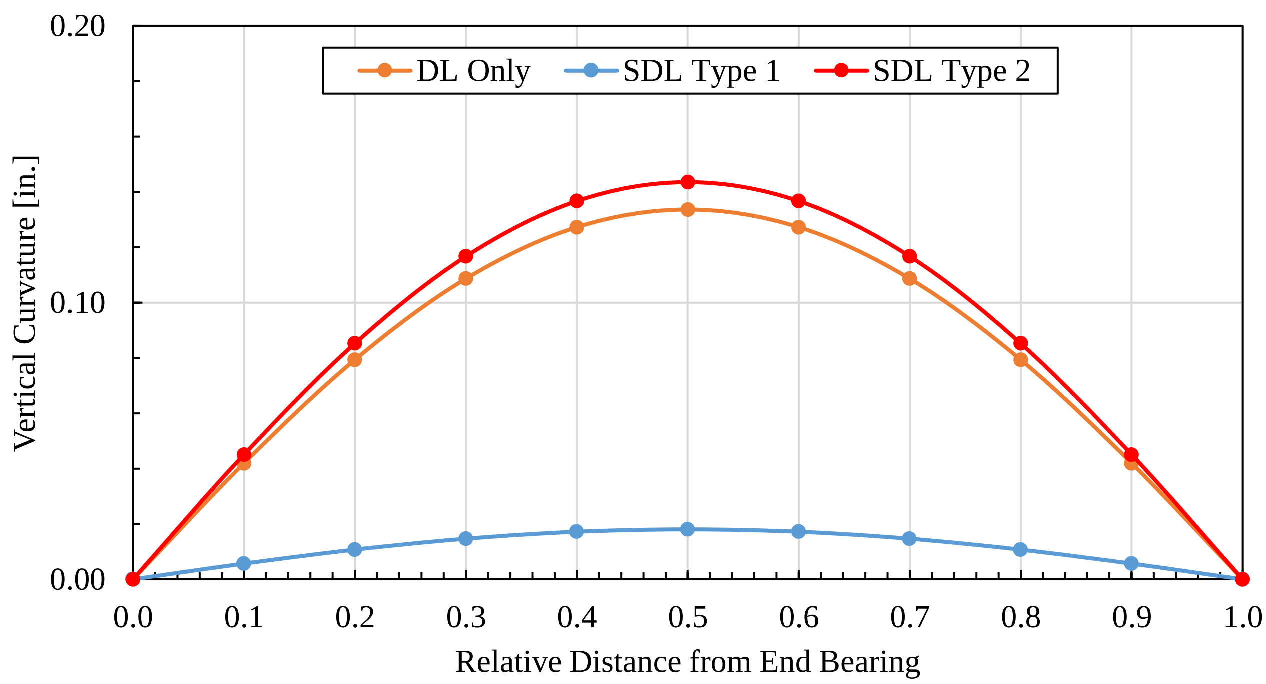
L	30	feet	Length of span in feet
S	36	inch	Girder spacing
DL	0.027	kip/in	Dead load of steel plus deck
SDL1	0.0036	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	1,524	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	360	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	36	0.042	0.006	0.045
0.2	72	0.079	0.011	0.085
0.3	108	0.109	0.015	0.117
0.4	144	0.127	0.017	0.137
0.5	180	0.134	0.018	0.144
0.6	216	0.127	0.017	0.137
0.7	252	0.109	0.015	0.117
0.8	288	0.079	0.011	0.085
0.9	324	0.042	0.006	0.045
1.0	360	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.042	0.079	0.109	0.127	0.134	0.127	0.109	0.079	0.042	0.000
SDL Deflection (Type 1)	0.000	0.006	0.011	0.015	0.017	0.018	0.017	0.015	0.011	0.006	0.000
SDL Deflection (Type 2)	0.000	0.045	0.085	0.117	0.137	0.144	0.137	0.117	0.085	0.045	0.000

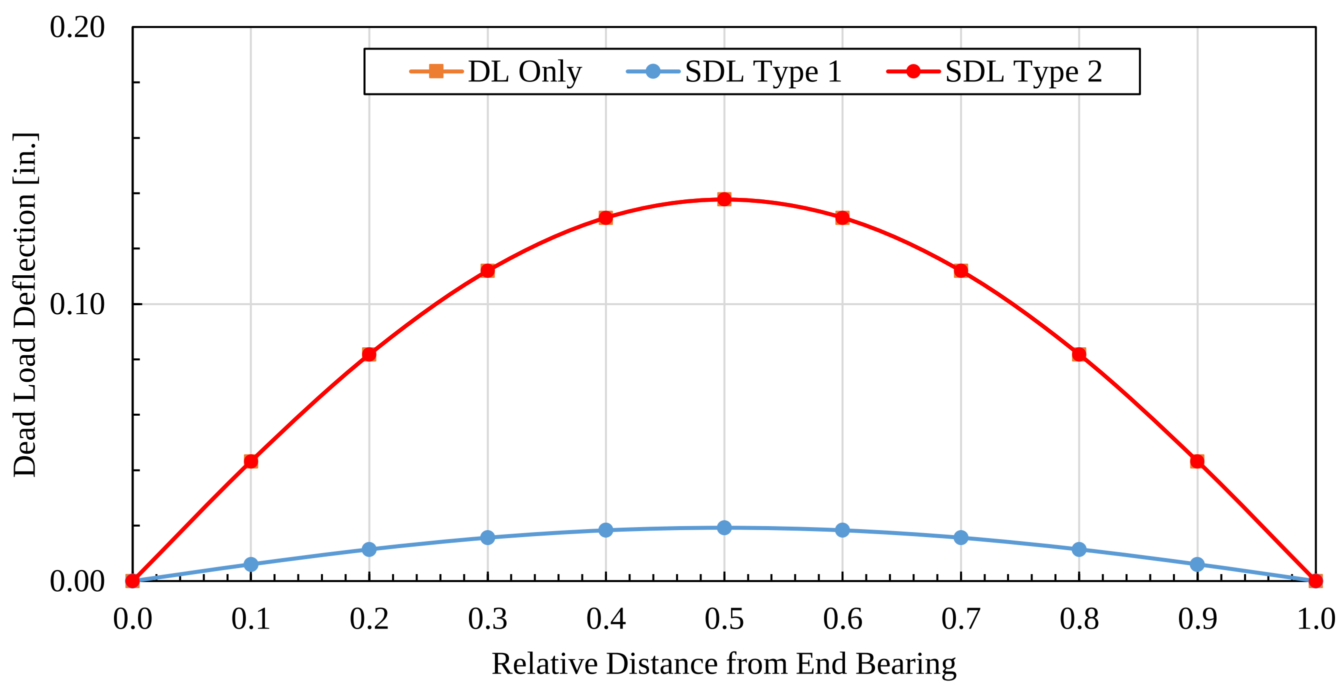


L	30	feet	Length of span in feet
S	40	inch	Girder spacing
DL	0.029	kip/in	Dead load of steel plus deck
SDL1	0.0041	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	1,588	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	360	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	36	0.043	0.006	0.043
0.2	72	0.082	0.011	0.082
0.3	108	0.112	0.016	0.112
0.4	144	0.131	0.018	0.131
0.5	180	0.138	0.019	0.138
0.6	216	0.131	0.018	0.131
0.7	252	0.112	0.016	0.112
0.8	288	0.082	0.011	0.082
0.9	324	0.043	0.006	0.043
1.0	360	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.043	0.082	0.112	0.131	0.138	0.131	0.112	0.082	0.043	0.000
SDL Deflection (Type 1)	0.000	0.006	0.011	0.016	0.018	0.019	0.018	0.016	0.011	0.006	0.000
SDL Deflection (Type 2)	0.000	0.043	0.082	0.112	0.131	0.138	0.131	0.112	0.082	0.043	0.000

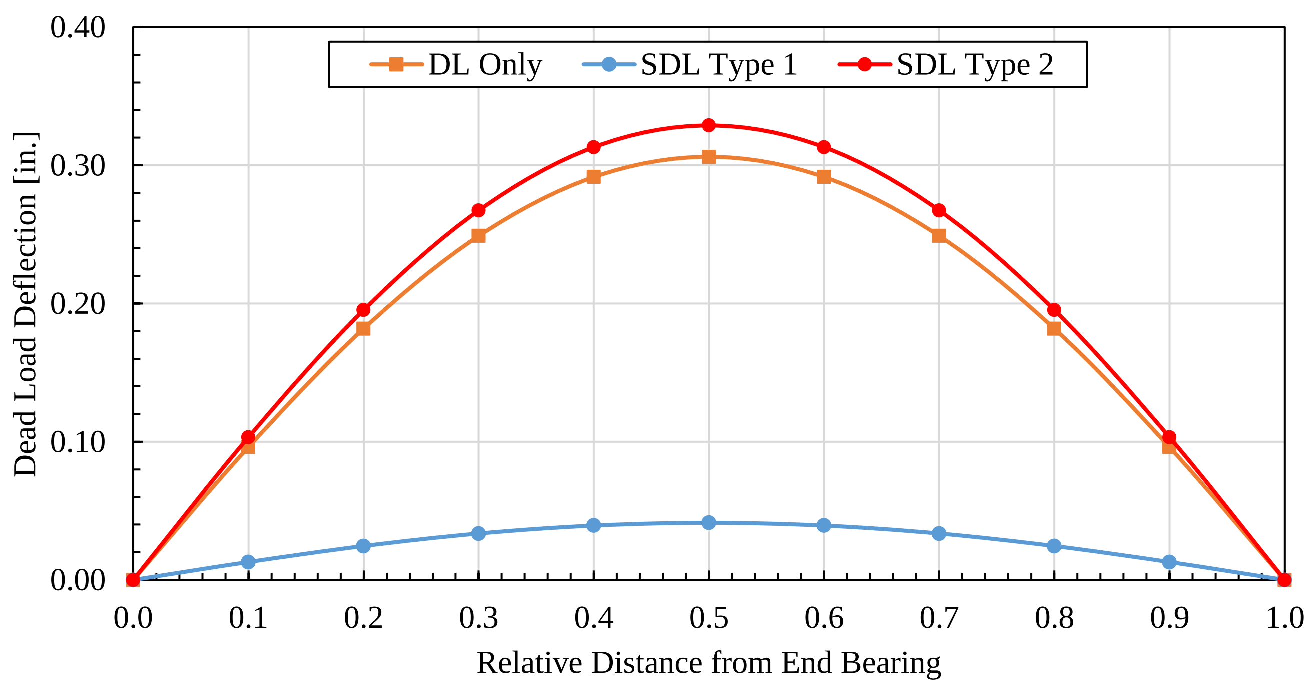


L	40	feet	Length of span in feet
S	36	inch	Girder spacing
DL	0.027	kip/in	Dead load of steel plus deck
SDL1	0.0036	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	2,102	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	480	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	48	0.096	0.013	0.103
0.2	96	0.182	0.025	0.195
0.3	144	0.249	0.034	0.267
0.4	192	0.292	0.039	0.313
0.5	240	0.306	0.041	0.329
0.6	288	0.292	0.039	0.313
0.7	336	0.249	0.034	0.267
0.8	384	0.182	0.025	0.195
0.9	432	0.096	0.013	0.103
1.0	480	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.096	0.182	0.249	0.292	0.306	0.292	0.249	0.182	0.096	0.000
SDL Deflection (Type 1)	0.000	0.013	0.025	0.034	0.039	0.041	0.039	0.034	0.025	0.013	0.000
SDL Deflection (Type 2)	0.000	0.103	0.195	0.267	0.313	0.329	0.313	0.267	0.195	0.103	0.000

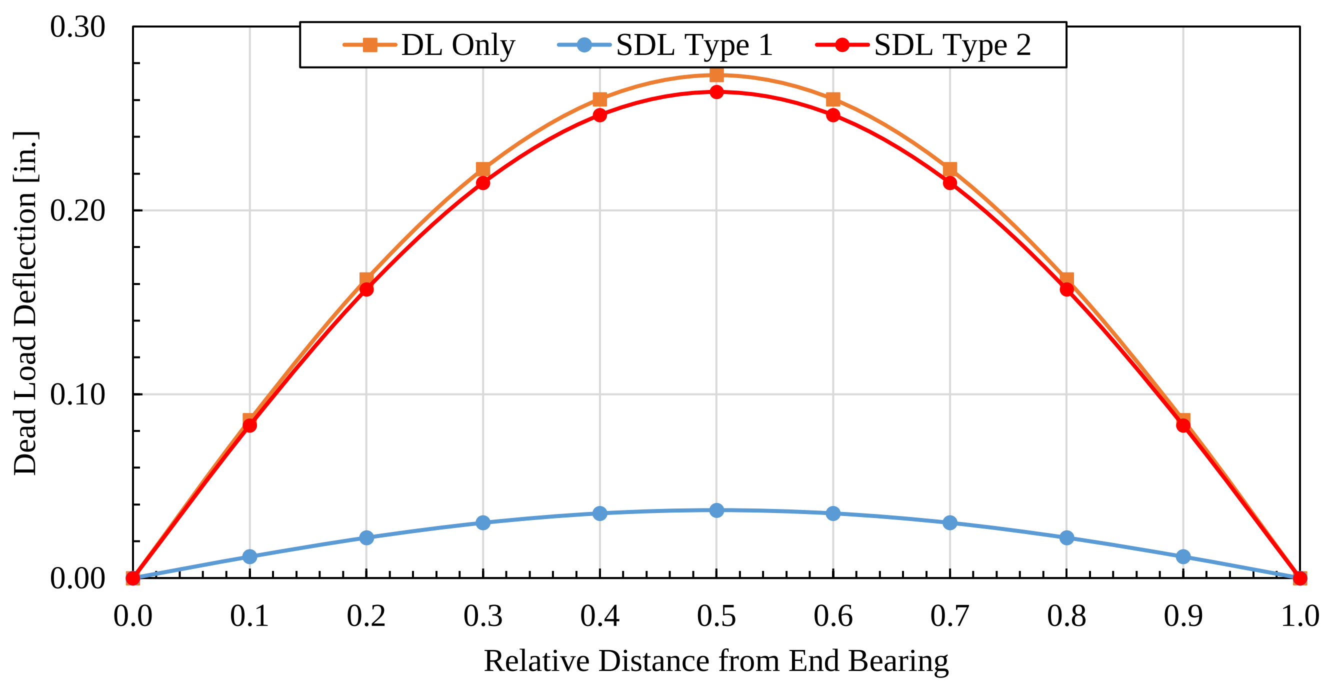


L	40	feet	Length of span in feet
S	40	inch	Girder spacing
DL	0.030	kip/in	Dead load of steel plus deck
SDL1	0.0041	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	2,614	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	480	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	48	0.086	0.012	0.083
0.2	96	0.162	0.022	0.157
0.3	144	0.222	0.030	0.215
0.4	192	0.260	0.035	0.252
0.5	240	0.274	0.037	0.264
0.6	288	0.260	0.035	0.252
0.7	336	0.222	0.030	0.215
0.8	384	0.162	0.022	0.157
0.9	432	0.086	0.012	0.083
1.0	480	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.086	0.162	0.222	0.260	0.274	0.260	0.222	0.162	0.086	0.000
SDL Deflection (Type 1)	0.000	0.012	0.022	0.030	0.035	0.037	0.035	0.030	0.022	0.012	0.000
SDL Deflection (Type 2)	0.000	0.083	0.157	0.215	0.252	0.264	0.252	0.215	0.157	0.083	0.000

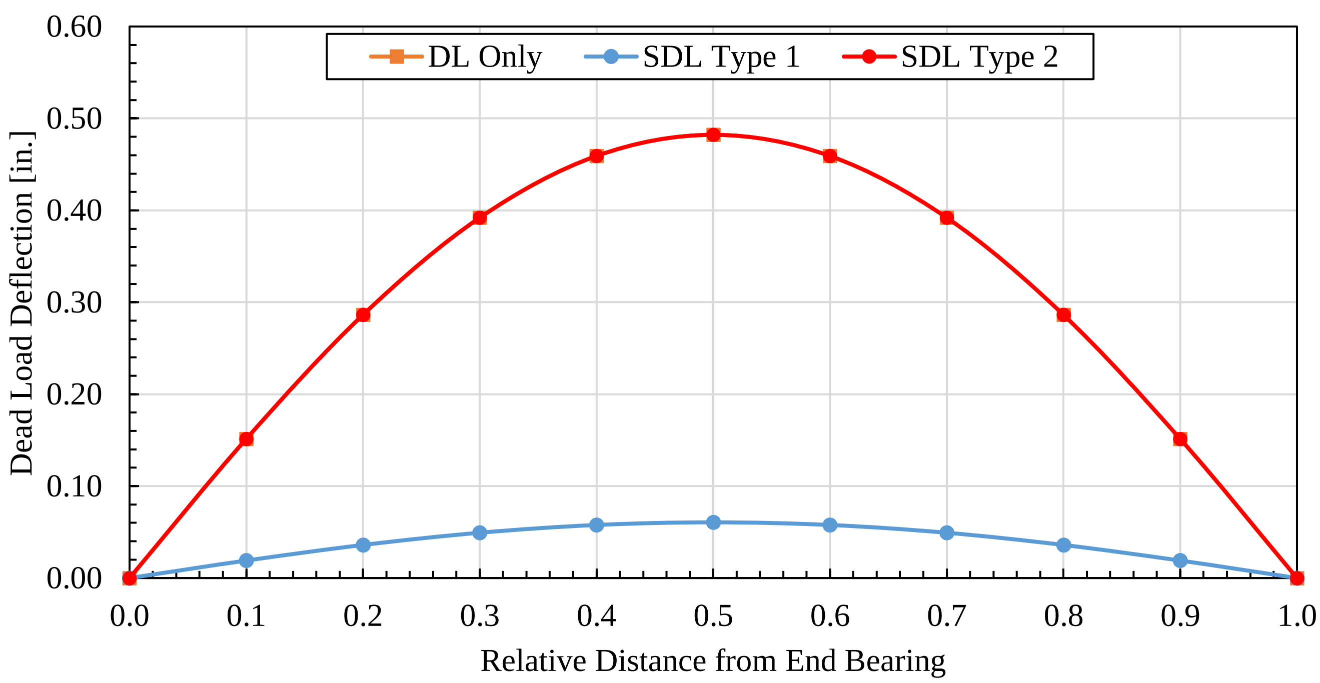


L	50	feet	Length of span in feet
S	36	inch	Girder spacing
DL	0.029	kip/in	Dead load of steel plus deck
SDL1	0.0036	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	3,500	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	600	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	60	0.151	0.019	0.151
0.2	120	0.286	0.036	0.286
0.3	180	0.392	0.049	0.392
0.4	240	0.459	0.058	0.459
0.5	300	0.482	0.061	0.482
0.6	360	0.459	0.058	0.459
0.7	420	0.392	0.049	0.392
0.8	480	0.286	0.036	0.286
0.9	540	0.151	0.019	0.151
1.0	600	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.151	0.286	0.392	0.459	0.482	0.459	0.392	0.286	0.151	0.000
SDL Deflection (Type 1)	0.000	0.019	0.036	0.049	0.058	0.061	0.058	0.049	0.036	0.019	0.000
SDL Deflection (Type 2)	0.000	0.151	0.286	0.392	0.459	0.482	0.459	0.392	0.286	0.151	0.000

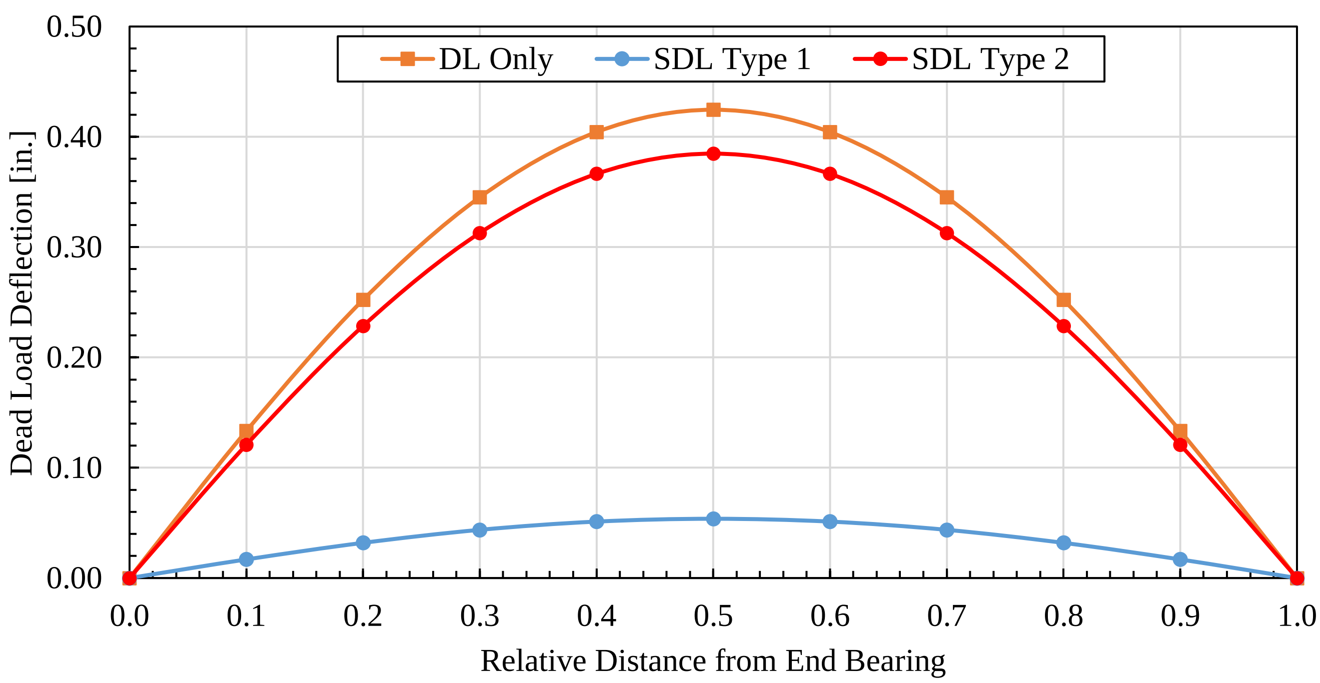


L	50	feet	Length of span in feet
S	40	inch	Girder spacing
DL	0.032	kip/in	Dead load of steel plus deck
SDL1	0.0041	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	4,386	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	600	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	60	0.133	0.017	0.121
0.2	120	0.252	0.032	0.229
0.3	180	0.345	0.044	0.313
0.4	240	0.404	0.051	0.366
0.5	300	0.425	0.054	0.385
0.6	360	0.404	0.051	0.366
0.7	420	0.345	0.044	0.313
0.8	480	0.252	0.032	0.229
0.9	540	0.133	0.017	0.121
1.0	600	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.133	0.252	0.345	0.404	0.425	0.404	0.345	0.252	0.133	0.000
SDL Deflection (Type 1)	0.000	0.017	0.032	0.044	0.051	0.054	0.051	0.044	0.032	0.017	0.000
SDL Deflection (Type 2)	0.000	0.121	0.229	0.313	0.366	0.385	0.366	0.313	0.229	0.121	0.000

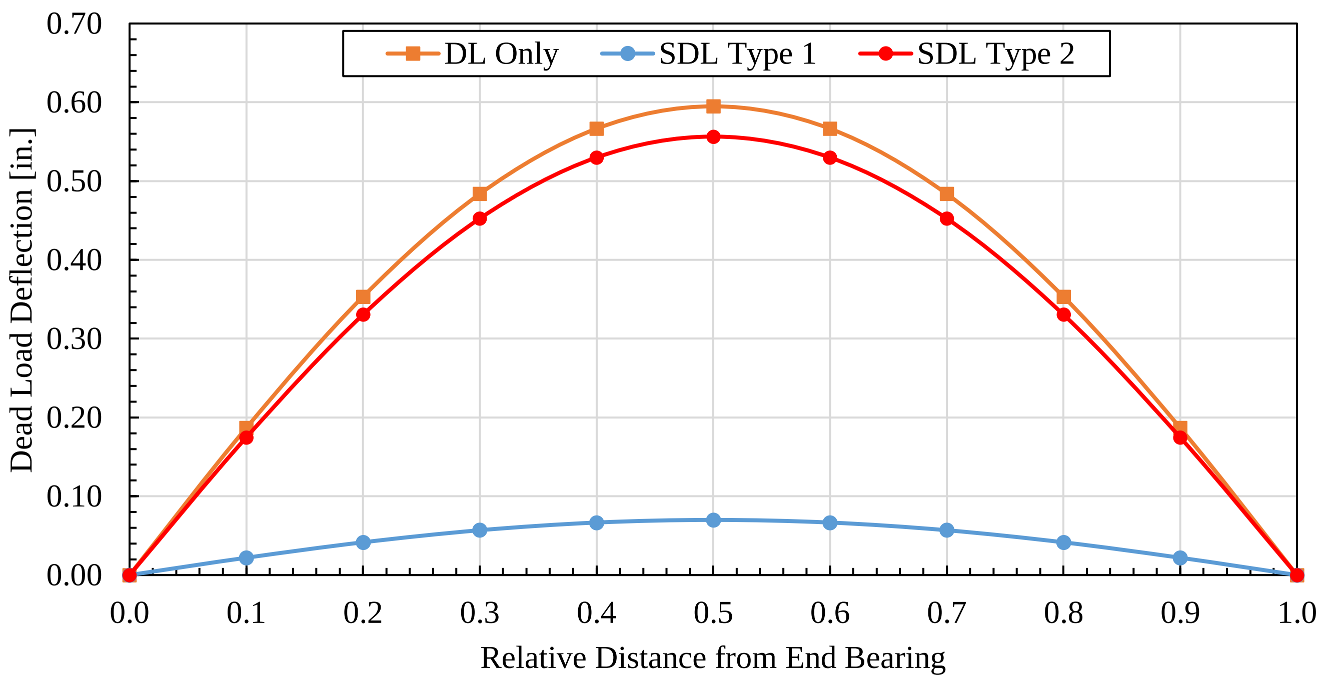


L	60	feet	Length of span in feet
S	36	inch	Girder spacing
DL	0.031	kip/in	Dead load of steel plus deck
SDL1	0.0036	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	6,288	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	720	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	72	0.187	0.022	0.175
0.2	144	0.353	0.042	0.331
0.3	216	0.484	0.057	0.452
0.4	288	0.567	0.067	0.530
0.5	360	0.595	0.070	0.556
0.6	432	0.567	0.067	0.530
0.7	504	0.484	0.057	0.452
0.8	576	0.353	0.042	0.331
0.9	648	0.187	0.022	0.175
1.0	720	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.187	0.353	0.484	0.567	0.595	0.567	0.484	0.353	0.187	0.000
SDL Deflection (Type 1)	0.000	0.022	0.042	0.057	0.067	0.070	0.067	0.057	0.042	0.022	0.000
SDL Deflection (Type 2)	0.000	0.175	0.331	0.452	0.530	0.556	0.530	0.452	0.331	0.175	0.000

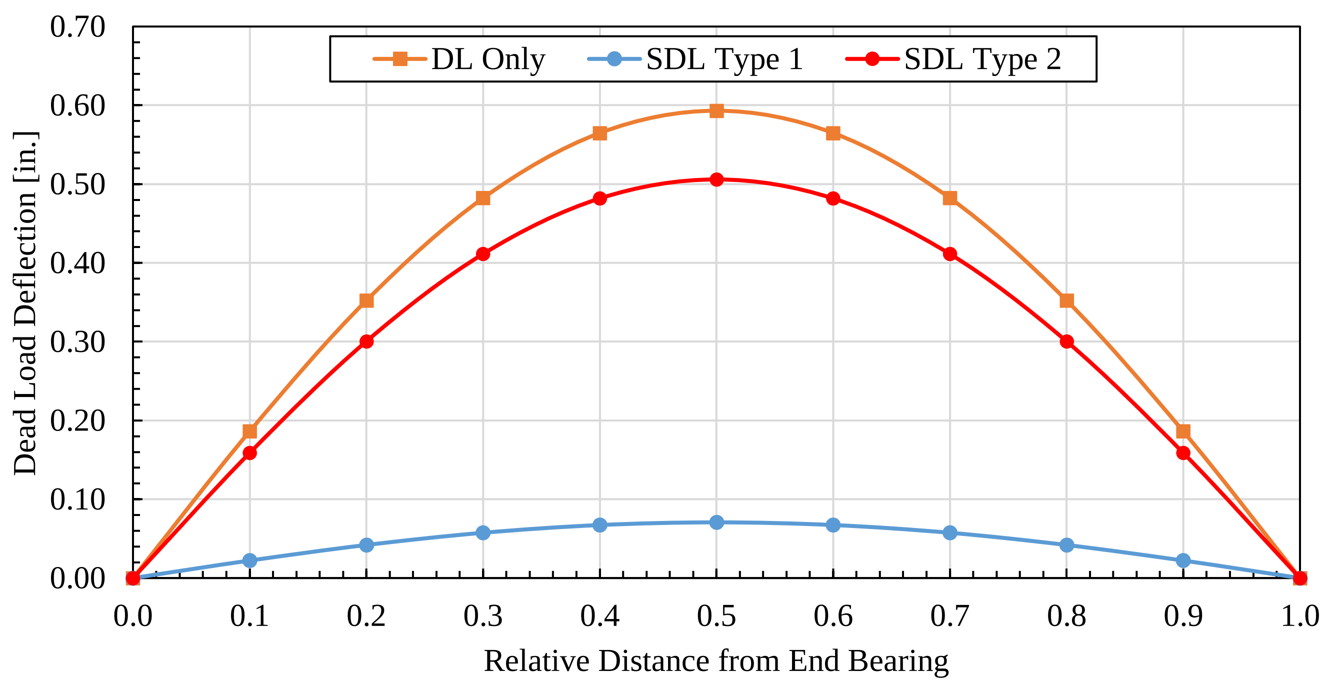


L	60	feet	Length of span in feet
S	40	inch	Girder spacing
DL	0.034	kip/in	Dead load of steel plus deck
SDL1	0.0041	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	6,917	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	720	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	72	0.186	0.022	0.159
0.2	144	0.352	0.042	0.300
0.3	216	0.482	0.057	0.411
0.4	288	0.565	0.067	0.482
0.5	360	0.593	0.071	0.506
0.6	432	0.565	0.067	0.482
0.7	504	0.482	0.057	0.411
0.8	576	0.352	0.042	0.300
0.9	648	0.186	0.022	0.159
1.0	720	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.186	0.352	0.482	0.565	0.593	0.565	0.482	0.352	0.186	0.000
SDL Deflection (Type 1)	0.000	0.022	0.042	0.057	0.067	0.071	0.067	0.057	0.042	0.022	0.000
SDL Deflection (Type 2)	0.000	0.159	0.300	0.411	0.482	0.506	0.482	0.411	0.300	0.159	0.000

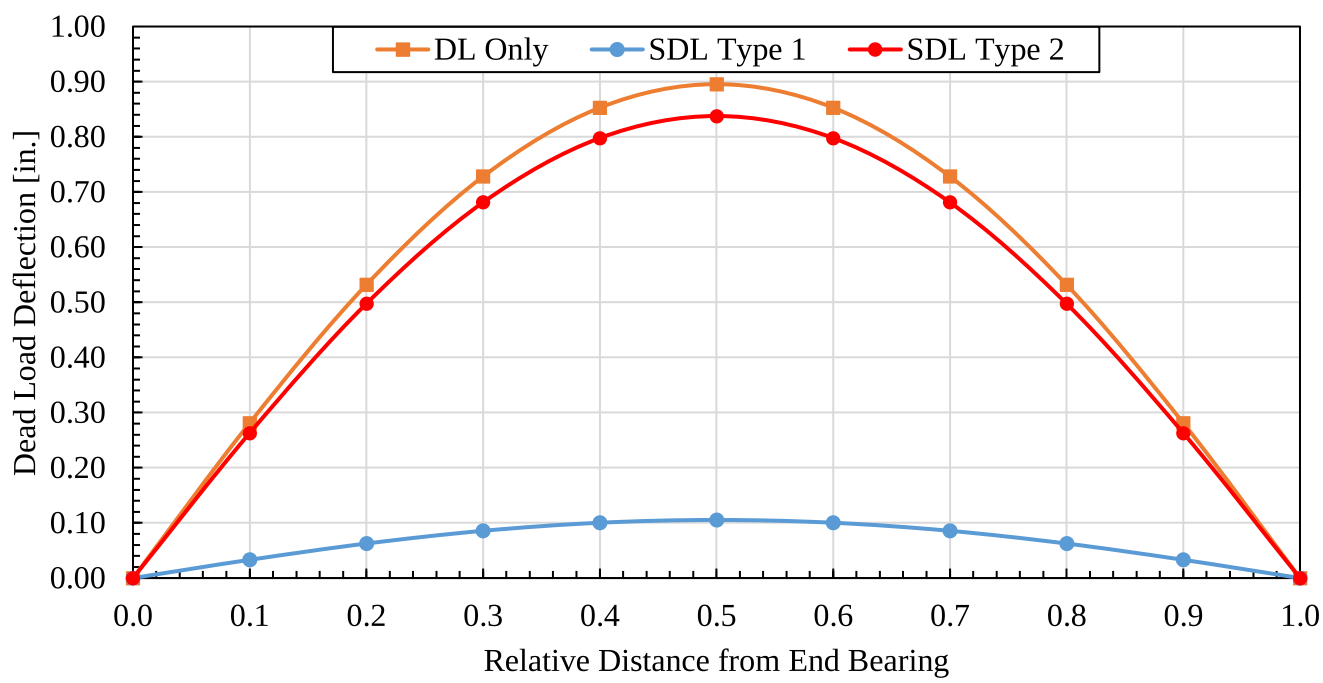


L	70	feet	Length of span in feet
S	36	inch	Girder spacing
DL	0.031	kip/in	Dead load of steel plus deck
SDL1	0.0036	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	7,739	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	840	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	84	0.281	0.033	0.263
0.2	168	0.532	0.063	0.498
0.3	252	0.728	0.086	0.681
0.4	336	0.853	0.100	0.798
0.5	420	0.895	0.105	0.838
0.6	504	0.853	0.100	0.798
0.7	588	0.728	0.086	0.681
0.8	672	0.532	0.063	0.498
0.9	756	0.281	0.033	0.263
1.0	840	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.281	0.532	0.728	0.853	0.895	0.853	0.728	0.532	0.281	0.000
SDL Deflection (Type 1)	0.000	0.033	0.063	0.086	0.100	0.105	0.100	0.086	0.063	0.033	0.000
SDL Deflection (Type 2)	0.000	0.263	0.498	0.681	0.798	0.838	0.798	0.681	0.498	0.263	0.000

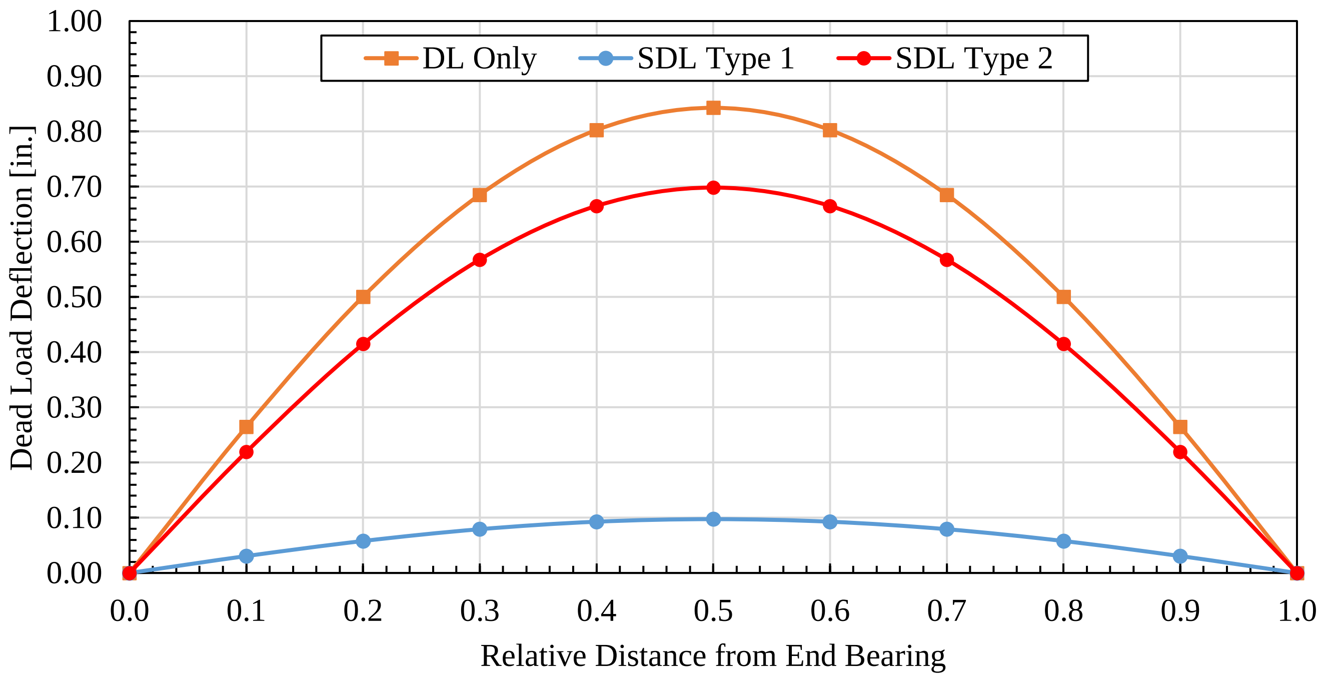


L	70	feet	Length of span in feet
S	40	inch	Girder spacing
DL	0.035	kip/in	Dead load of steel plus deck
SDL1	0.0041	kip/in	Superimposed dead load type 1 (140 lbf/ft^3 overlay 1.25" thick)
I_{cLong}	9,284	in^4	long term moment of inertia
SDL2	0.029	kip/in	Superimposed dead load type 2 (parapet, 0.7k/ft)
E	29,000	ksi	modulus of elasticity of steel
L	840	inch	length of span

Long Term Dead Load Deflections (in.)

% span	x	DL Only	SDL Type 1	SDL Type 2
0.0	0	0.000	0.000	0.000
0.1	84	0.265	0.031	0.219
0.2	168	0.501	0.058	0.415
0.3	252	0.685	0.079	0.568
0.4	336	0.803	0.093	0.665
0.5	420	0.843	0.098	0.698
0.6	504	0.803	0.093	0.665
0.7	588	0.685	0.079	0.568
0.8	672	0.501	0.058	0.415
0.9	756	0.265	0.031	0.219
1.0	840	0.000	0.000	0.000

	Long Term Dead Load Deflections [in.]										
Relative Distance from End	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DL Deflection	0.000	0.265	0.501	0.685	0.803	0.843	0.803	0.685	0.501	0.265	0.000
SDL Deflection (Type 1)	0.000	0.031	0.058	0.079	0.093	0.098	0.093	0.079	0.058	0.031	0.000
SDL Deflection (Type 2)	0.000	0.219	0.415	0.568	0.665	0.698	0.665	0.568	0.415	0.219	0.000



5. APPENDIX D – SUPERSTRUCTURE REPLACEMENT

The following Example Configuration Calculations are presented for a non-standard FlexBeam configuration. The example FlexBeam system consists of a 36 foot span superstructure replacement with an overall width of 26 feet and curb to curb interior width of 23 feet. The interior steel sections have a spacing of 31.5 in. and a 14.25 in. overhang is used on the exterior steel sections. The superstructure steel section size is based on the 40 ft span, 36 in. spacing example configuration. The shear connector spacing is revised for the change in span length and steel section spacing. Two analysis cases are considered: the first case is for a 31.5 in. tributary deck width and the second is for a 28.5 in. tributary deck width (twice the exterior overhang). In both cases, all dead loads are assumed to be distributed evenly over each steel section. The replacement superstructure is analyzed using a 10M barrier with a weight of 0.300 kip/ft. in place of the F-shape barrier considered in the other example configurations. The relevant steel section and shear connector properties are outlined in Table 2. The flexural and shear connector calculations are presented on subsequent pages.

Table 2: PA FlexBeam Superstructure Replacement Example Configuration

Span Length, L [ft]	Design Roadway [ft]	Superstructure Height, h [in.]	Number of Modules	Beam Spacing, b_e [in.]	Bottom Flange		Web		Stiffener Height [in.]	Revised Shear Dowel Spacing on Half Span from Support to Midspan		
					Thickness, t_f [in.]	Width, b_f [in.]	Depth, h_w [in.]	Thickness, t_w [in.]		Number of Spaces @ 4 in.	Number of Spaces @ 8 in.	Number of Spaces @ 12 in.
36	23	25	5	31-1/2 w/ 14-1/4 exterior overhang	0.500	12.000	20.500	0.375	16.400	24	9	4

Flexbeam Superstructure Replacement Calculations

Summary of Calculations:

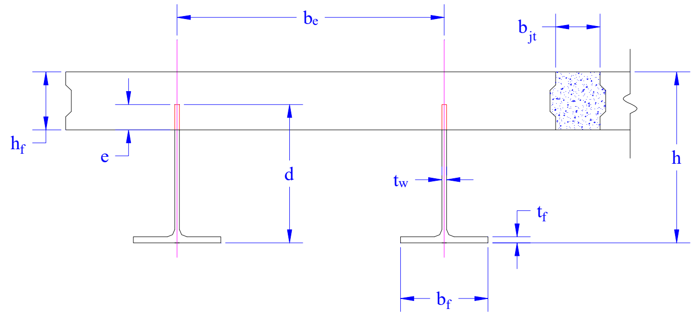
1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 10M barrier is used, Weight = 300 lb/ft (Distributed to exterior two beams).

Analysis of module with 28.5" tributary deck width

Variable Definitions:



Number of beams $N := 10$ $\quad$ $modules := \frac{N}{2} = 5$

$L := 36ft$ Span length

$b_e := 28.5in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad$ $t_f := 0.5in$ $\quad$ $b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 273.24 kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 89.7kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ $\quad$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ $\quad$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.147 \cdot in$$ Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 640.875 \cdot in^4$$ MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 13.687 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000\text{ksi}$

Yield Strength of Steel $F_y := 50\text{ksi}$

Yield Strength of Reinforcement $f_y := 60\text{ksi}$

Compressive Strength of Concrete $f_c := 4000\text{psi}$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150\,\dfrac{\text{kip}}{\text{ft}^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 8\text{in}$

Parapet width: $b_{par} := 18\text{in}$ **Parapet width (10M barrier)**

Width of the bridge $W := 312\text{in} = 312\cdot\text{in}$ $W = 26\,\text{ft}$ **fix bridge width at 26 ft**

Clear width of roadway $W_{lane} := W - 2\cdot b_{par} = 23\,\text{ft}$

$D_c := \left(w_c\cdot h_f\cdot W\right) = 2.437\cdot\dfrac{\text{kip}}{\text{ft}}$ DL due to slab

$D_b := .300\,\dfrac{\text{kip}}{\text{ft}}$ **DL due to each 10M barrier (BD-617M)**

$D_{FWS} := 30\,\dfrac{\text{lbf}}{\text{ft}^2}\cdot\left(W - 2\cdot b_{par}\right) = 0.69\cdot\dfrac{\text{kip}}{\text{ft}}$ DL due to future wearing surface

$D_s := A_{stl}\cdot 490\,\dfrac{\text{lbf}}{\text{ft}^3} = 0.047\cdot\dfrac{\text{kip}}{\text{ft}}$ Selfweight of the steel section

$DL_{self} := \dfrac{\left(D_c\right)}{N} + D_s = 0.29\cdot\dfrac{\text{kip}}{\text{ft}}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25\,\dfrac{\text{lbf}}{\text{ft}}\cdot 32\text{in} = 0.067\cdot\text{kip}$

$$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f\cdot w_c\right)\right] + 2D_s\right]\cdot L + 2\cdot W_{end\_diaphragms} = 17.268\cdot\text{kip}}$$

Determine number of design lane (AASHTO Section 3.6.1.1.1) **Fix number of lanes at 2**

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.259 \cdot \frac{kip}{ft}$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$DL_{steel} := D_s = 0.047 \cdot \frac{kip}{ft}$

$\boxed{DL := DL_{slab} + DL_{steel} = 0.305 \cdot \frac{kip}{ft}}$

STEP 2: *Compute Superimposed Dead Load*

$SDL_{WS} := \frac{D_{FWS}}{N} = 0.069 \cdot \frac{kip}{ft}$

$SDL_{parapet} := \frac{D_b}{2} = 0.15 \cdot \frac{kip}{ft}$ distributed equally on two outer beams

$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.055 \cdot \frac{kip}{ft}$ Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.274 \cdot \frac{kip}{ft}}$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 3.563 \cdot in$ Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.188 \cdot in$ Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

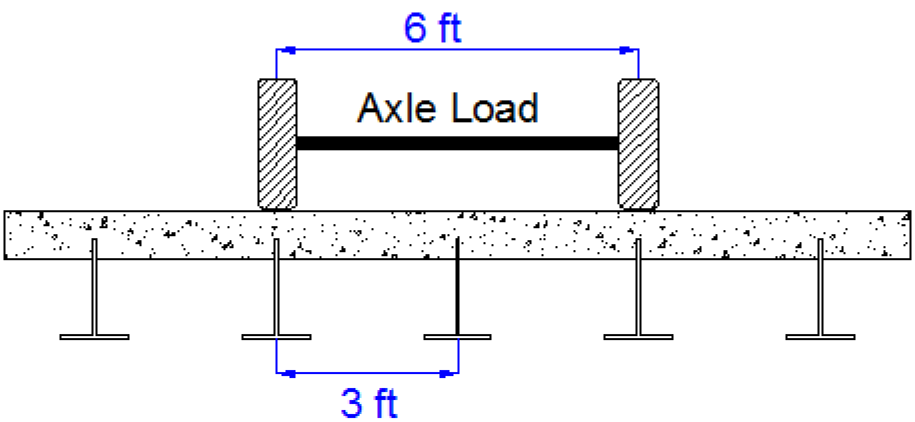
$$M_{DL} := \frac{DL \cdot L^2}{8} = 49.437 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 44.456 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 2.375$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 15.103 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^2\right) = 30103.16 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$P_z := 2.56 \cdot 10^{-6} \cdot (V_w)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$ Wind pressure on exterior girder

$h_{exp} := h - h_f = 17.5 \cdot in$ Exposed height of exterior girder

$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 5.032 \cdot kip \cdot ft$ Moment due to wind loading on exterior girder

$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$ Transverse moment of intertia

$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 5.032 \cdot ksi$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

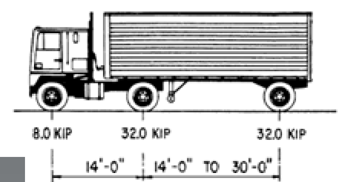
Design truck load in AASHTO (HS20 Truck) is used for this example.

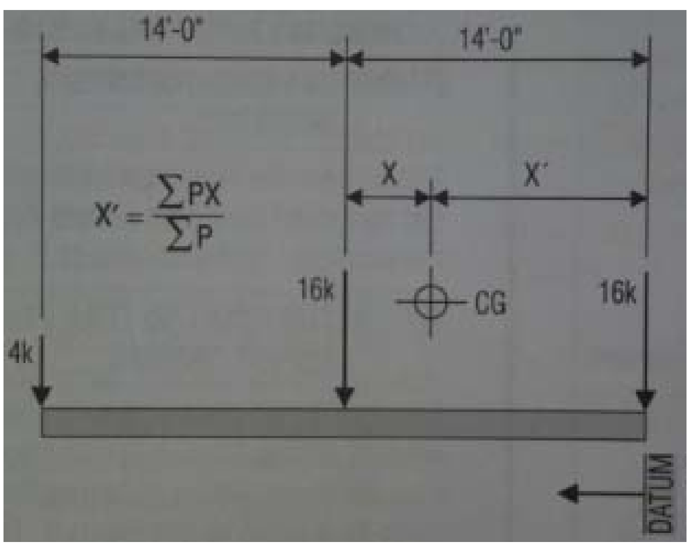
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$ Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$ Spacing btw CG and nearest load

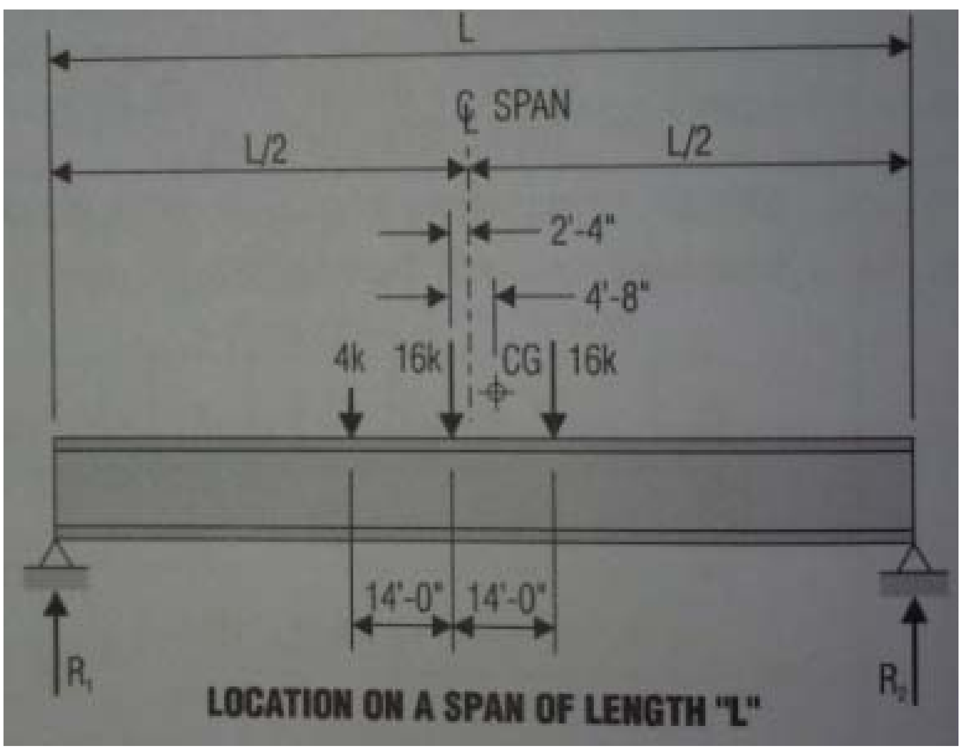
Find bridge reaction

$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.333 \cdot kip$$ Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 378.889 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \dfrac{X_{AS}}{2} = 2 \cdot \text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$$R_A := \frac{F_T \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + F_T \cdot \left(\frac{L}{2} - \frac{X}{2} - X\right)}{L} = 29.514 \cdot \text{kip}$$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$$M_{LL\_tandem} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) = 501.736 \cdot \text{kip} \cdot \text{ft}$$

Choose the max value:

$$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 501.736 \cdot \text{kip} \cdot \text{ft}}$$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh} \cdot DF \cdot (1 + I) = 200.193 \cdot \text{kip} \cdot \text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

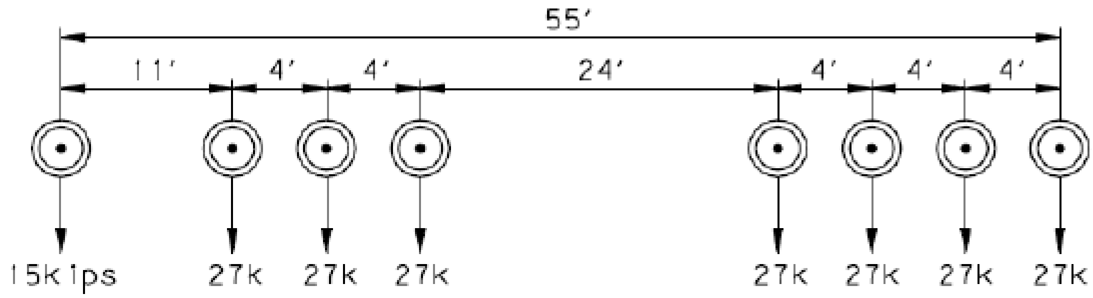
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64 \dfrac{\text{kip}}{\text{ft}} \cdot DF = 0.192 \cdot \dfrac{\text{kip}}{\text{ft}}$ Per beam in each lane

$$M_{LL\_IM\_lane} := \left(\frac{F_{DLL} \cdot L^2}{8}\right) = 31.104 \cdot \text{kip} \cdot \text{ft}$$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 231.297 \cdot \text{kip} \cdot \text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 640.875 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 26.719 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 125.244 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.134 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2831 \cdot in^4$$ Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 16.134 \cdot in$$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 8.906 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 41.748 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.101 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1913 \cdot in^4$$ Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 12.101 \cdot in$$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\quad$ $h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 7.5in$ $\qquad$ $0.42 \cdot h = 10.5 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 6.208 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.248}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h - d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1$ $\qquad$ $Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h - d)]] \cdot \frac{[Dp1 - (h - d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 10612.38 \cdot kip \cdot in}$ $\qquad$ $M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 9510.5 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h - d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h - d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 2.87 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.1148}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0}$ $\qquad$ $Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h - d)]] \cdot \frac{[Dp2 - (h - d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 10322.59 \cdot kip \cdot in}$ $\qquad$ $M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 10215.6 \cdot kip \cdot in$

$Mn := max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 792.5 \cdot kip \cdot ft$ $\qquad$ $Mn = 9510.5 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -1.158\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: *Load Combinations* (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

SERVICE II: DL+SDL+1.3(LL+IM)

STEP 8: *Compute and Check Stresses*

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.752\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 15.82\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 18.689\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.374\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.437\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 34.981\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.69\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.693\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.232\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.087\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.284\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 0.209\cdot ksi$$

STRENGTH I

$$f_{c\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 2.505\cdot ksi$$

$$M_{str1} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.75\cdot M_{LL\_IM} = 533.2\cdot kip\cdot ft \qquad M_{str1} < Mn = 1$$

STRENGTH II

$$f_{c\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 2.337\cdot ksi$$

$$M_{str2} := 1.25\cdot M_{DL} + 1.5\cdot M_{SDL} + 1.35\cdot M_{LL\_IM\_PL82} = 497\cdot kip\cdot ft \qquad M_{str2} < Mn = 1$$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$$f_{allow} := 0.6\cdot f_c = 2400\,psi$$

$$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1853.87\cdot psi$$

This assumes short term composite section for all calculations

$$f_{c\_2} < f_{allow} = 1$$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.167\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot\left(h - Y'_{c\_short}\right)}{k_1\cdot n\cdot I_{c\_short}} = 1.087\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot\left(h - Y'_{c\_long}\right)}{k_3\cdot n\cdot I_{c\_long}} = 0.15\cdot ksi$$

$$f_{c\_2} := 1.3f_{LL\_I} + f_{SDL} + f_{DL} = 1729.24\cdot psi \qquad f_{c\_2} < f_{allow} = 1$$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \dfrac{I_{c\_long}}{Y'_{c\_long}} = 158.115 \cdot in^3$ $\qquad S_{ST} := \dfrac{I_{c\_short}}{Y'_{c\_short}} = 175.441 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 128.48 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \dfrac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 588.446 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 716.926 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \dfrac{My}{F_y} = 172.062 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 440.731 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} = 464.781 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 792.543 \cdot kip \cdot ft \quad M_u + \dfrac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$ Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

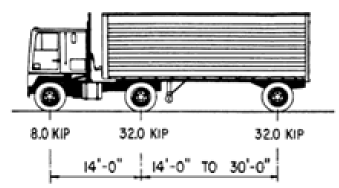
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8\text{kip}$ $P_2 := 32\text{kip}$ $P_3 := 32\text{kip}$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14\text{ft}}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14\text{ft}$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot \text{ft}$$ Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.473 \cdot \text{in}$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -0.667 \cdot \text{ft}$ distance btw load P1 and support A $b1check := b1 > 0 = 0$

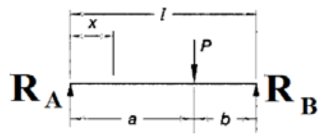
$b2 := \frac{L}{2} - (X_{ASr} - X') = 13.333 \cdot \text{ft}$ distance btw load P2 and support A $b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 8.667 \cdot \text{ft}$ distance btw load P3 and support B $b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.594 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

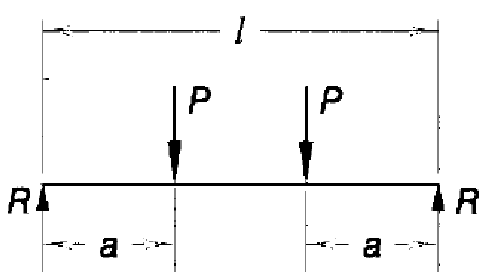
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot \left(E_s \cdot I_{c\_short}\right) \cdot L} \cdot \left(L^2 - b^2 - x^2\right) = 0.436 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.274 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.334 \cdot in$$ (Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.059 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.428 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.54 \cdot in$ $\frac{L}{1000} = 0.432 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 2.375$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.455}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 136.693 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.305 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.274 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - \left(X_{ASr} + X_{ASf}\right) = 8 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 22 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 36 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 53.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 32.275 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{\left(L - X_{AS}\right)}{L} = 59.028 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left[DF_s \cdot (1 + I)\right] = 35.721 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 11.52 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot \left(DF_s\right) = 5.242 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.493 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 4.94 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 85.96 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 48.976 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 80.393 \cdot kip$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 85.96 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \qquad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ Yield strength of stiffener

$t_p := 0.75in$ Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.4 \cdot in$ Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ Projected area of stiffeners

Check that effective width of stiffener meets requirements $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ Bearing capacity check

$Q_s := 1.0$ Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w^{\,3} + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.592 \times 10^4 \cdot kip$ Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.861 \cdot kip$ Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.718 \cdot kip$ Compressive capacity of bearing stiffener $Pr \ge Vu = 1$

Flexbeam Superstructure Replacement Calculations

Summary of Calculations:

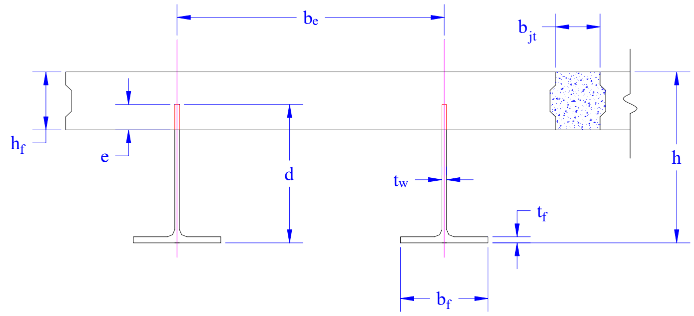
1. Input section dimensions and material properties for steel WT and the concrete deck
2. Calculate demands (dead load, superimposed dead, and Live load demands for Strength I and Strength II
3. Calculate composite section properties for the given section
4. Check Strength I, Strength II, and Service II stresses
5. Check Strength I, Strength II, and Strength V moment demands
6. Perform live load deflection check
7. Determine maximum shear demand and capacity
8. Note: Shear envelope data was obtained from Matlab script (FlexBeam_Moment_ShearEnvelope.m). Shear envelope calculations are summarized separately for each design (Shear Design *.xlsx).

Design Assumptions:

- Design Specifications Used:
  AASHTO LRFD Bridge Design Specifications 7th Edition with 2016 Interim Revisions
  PennDOT Design Manual, Part 4 (DM-4) June 30, 2015
- The deck thickness is 7.5" and is assumed to have a non-structural wearing surface added after installation. Section properties are calculated using the full 7.5" deck thickness.
- The WT sections are embedded 3.5" into the concrete deck.
- The steel section spacing used is outside the scope of the AASHTO and PennDOT distribution factor equations. The flexural distribution factor is assumed to be 0.3 which was shown to tbe conservative according to refined analysis
- The shear distribution factor is calculated using the DM-4 shear distribution factor equation which was shown to be conservative using refined analysis.
- Refined analysis is presented in: Zihan Ma, "Distribution Factors for Composite Steel Tee Concrete Deck Bridge System," M.S. Thesis, Deparement of Civil and Environmental Engineering, Lehigh University, Bethlehem, 2019
- Shear and moment demands for the PL82 permit load vehicle are determined by Matlab script (FlexBeam_Moment_ShearEnvelope.m) and are manually input into this calculation sheet.
- All fabrication will occur as double modules, i.e. each module contains two WT sections. The number of modules is half the number of beams, N.
- A 10M barrier is used, Weight = 300 lb/ft (Distributed to exterior two beams).

Analysis of module with 31.5" tributary deck width

Variable Definitions:



Number of beams $N := 10$ $\quad modules := \frac{N}{2} = 5$

$L := 36ft$ Span length

$b_e := 31.5in$ Effective width of Flexbeam assuming constructed bridge width (i.e. closure joints cast)

Built Up Section Properties

$t_w := 0.375in$ $\quad t_f := 0.5in$ $\quad b_f := 12in$

$h := 25in$ Overall Height of section

$h_f := 7.5in$ Total thickness of deck

$e := 3.5in$ Deck/Steel overlap

$d := h - h_f + e = 21 \cdot in$ depth of built up steel section

$h_w := d - t_f = 20.5 \cdot in$ height of web of built up steel

Moment and shear for permit load PL82 obtained from Matlab script: FlexBeam_Moment_ShearEnvelope

$M_{LL\_IM\_PL82} := 273.24kip \cdot ft$ Moment due to PL82 with distribution factor and impact factor!!

$V_{PL82span} := 89.7kip$ Shear due to PL82 WITHOUT distribution factor and impact factor!!

Check web and flange dimensions in sections 6.10.2.1 and 6.10.2.2

(Ignore 6.10.2.2-4 since no compression flange)

equation 6.10.2.1.1-1 $\frac{d}{t_w} < 150 = 1$ equation 6.10.2.2-1 $\frac{b_f}{2 \cdot t_f} < 12 = 1$

equation 6.10.2.2-2 $b_f > \frac{h}{6} = 1$ equation 6.10.2.2-3 $t_f > 1.1 \cdot t_w = 1$

$$y\_bar := \frac{t_f \cdot b_f \cdot \frac{t_f}{2} + h_w \cdot t_w \cdot \left(t_f + \frac{h_w}{2}\right)}{t_f \cdot b_f + h_w \cdot t_w} = 6.147 \cdot in$$

Center of gravity of steel section

$$I_{stl} := \frac{b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot \left(y\_bar - \frac{t_f}{2}\right)^2 + \frac{t_w \cdot h_w^3}{12} + t_w \cdot h_w \cdot \left(t_f + \frac{h_w}{2} - y\_bar\right)^2 = 640.875 \cdot in^4$$

MOI of steel

$A_{stl} := t_f \cdot b_f + h_w \cdot t_w = 13.687 \cdot in^2$ Area of the WT steel section

Material Properties

Elastic Modulus of Steel $E_s := 29000ksi$

Yield Strength of Steel $F_y := 50ksi$

Yield Strength of Reinforcement $f_y := 60ksi$

Compressive Strength of Concrete $f_c := 4000psi$ AAAP PennDOT Concrete

Concrete Modular Ratio $n := 8$ modular ratio for 4ksi concrete per AASHTO

Unit weight of concrete slab $w_c := 0.150 \frac{kip}{ft^3}$ Unit weight of concrete

Section Properties

Width of UHPC Joint: $b_{jt} := 8in$

Parapet width: $b_{par} := 18in$ **Parapet width (10M barrier)**

Width of the bridge $W := 312in = 312 \cdot in$ $W = 26\,ft$ **fix bridge width at 26 ft**

Clear width of roadway $W_{lane} := W - 2 \cdot b_{par} = 23\,ft$

$D_c := \left(w_c \cdot h_f \cdot W\right) = 2.437 \cdot \frac{kip}{ft}$ DL due to slab

$D_b := .300 \frac{kip}{ft}$ **DL due to each 10M barrier (BD-617M)**

$D_{FWS} := 30 \frac{lbf}{ft^2} \cdot \left(W - 2 \cdot b_{par}\right) = 0.69 \cdot \frac{kip}{ft}$ DL due to future wearing surface

$D_s := A_{stl} \cdot 490 \frac{lbf}{ft^3} = 0.047 \cdot \frac{kip}{ft}$ Selfweight of the steel section

$DL_{self} := \frac{\left(D_c\right)}{N} + D_s = 0.29 \cdot \frac{kip}{ft}$ Self-weight per WT

Total weight of each precast FlexBeam for transportation:

Weight of C12x25 Diaphragms $W_{end\_diaphragms} := 25 \frac{lbf}{ft} \cdot 32in = 0.067 \cdot kip$

$\boxed{\left[\left[\left(2b_e - b_{jt}\right)\left(h_f \cdot w_c\right)\right] + 2D_s\right] \cdot L + 2 \cdot W_{end\_diaphragms} = 18.956 \cdot kip}$

Determine number of design lane (AASHTO Section 3.6.1.1.1) **Fix number of lanes at 2**

Take the integer part of the ratio of W/12 ft. Number of lanes in bridge: $N_l := 2$

STEP 1: *Compute Dead Load for each Flex Beam*

$$DL_{slab} := \frac{D_c}{N} + w_c \cdot 1in \cdot \frac{b_e}{2} = 0.26 \cdot \frac{kip}{ft}$$

Dead load of slab assuming exterior overhang has 1" of additional concrete thickness

$$DL_{steel} := D_s = 0.047 \cdot \frac{kip}{ft}$$

$$\boxed{DL := DL_{slab} + DL_{steel} = 0.307 \cdot \frac{kip}{ft}}$$

STEP 2: *Compute Superimposed Dead Load*

$$SDL_{WS} := \frac{D_{FWS}}{N} = 0.069 \cdot \frac{kip}{ft}$$

$$SDL_{parapet} := \frac{D_b}{2} = 0.15 \cdot \frac{kip}{ft}$$

distributed equally on two outer beams

$$SDL_{OL} := 140 \frac{lbf}{ft^3} \cdot b_e \cdot 2in = 0.061 \cdot \frac{kip}{ft}$$

Overlay is 2" thick and is 140 lbs/cubic foot

NOTE: For this example barrier load is distributed equally among the two outer beams per conversation with PennDOT and M&M.

PennDOT DM4: "For composite adjacent and spread beams, the barrier (single barrier) load shall be equally distributed to the nearest three (adjacent) and two (spread) beams, respectively, when the barriers are placed after slab has hardened".

$$\boxed{SDL := SDL_{WS} + SDL_{parapet} + SDL_{OL} = 0.28 \cdot \frac{kip}{ft}}$$

STEP 3: *Compute Transformed Width of Slab*

For live load and dead load acting on the stringer $k_1 := 1$ For short-term section property evaluation

$$b_{tr1} := \frac{b_e}{k_1 \cdot n} = 3.938 \cdot in$$

Transformed width of slab for short-term

For superimposed dead loads $k_3 := 3$ For long-term section property evaluation

$$b_{tr3} := \frac{b_e}{k_3 \cdot n} = 1.313 \cdot in$$

Transformed width of slab for long-term

STEP 4: *Compute Dead and Superimposed Dead Load Moments*
(Based on first interior girder demands)

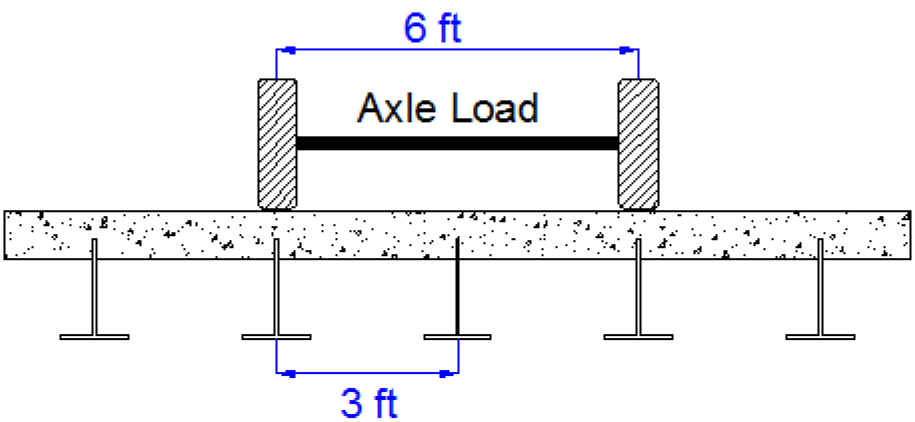
$$M_{DL} := \frac{DL \cdot L^2}{8} = 49.691 \cdot kip \cdot ft$$

$$M_{SDL} := \frac{SDL \cdot L^2}{8} = 45.401 \cdot kip \cdot ft$$

STEP 5: *Compute live load and Dynamic Load Allowance (IM)*

Live load wheel load distribution factor (DF):

$S := \frac{b_e}{12in} = 2.625$ transverse girder spacing



NOTE: AASHTO presents equations to define DF for different types of superstructures including Deck Composite wirh RC slab on steel girders (Type (a) in Table 4.6.2.2.1-1) for shear and moment in interior and exterior beams.

Using DM4 Table 4.6.2.2.2b-1:

$e_g := h - y\_bar - \frac{h_f}{2} = 15.103 \cdot in$ Distance between the centers of gravity of the basic beam and deck

$K_g := n \cdot \left(I_{stl} + A_{stl} \cdot e_g^{\,2}\right) = 30103.16 \cdot in^4$ Longitudinal stiffness parameter (in^4)

Distribution of Live Load ***Per Lane*** for Moment in Interior Beams:

to be conservative, use $DF := 0.3$

Dynamic Load Allowance (Section 3.6.2 DM4)

Static effects of the design truck or tandem, shall be increased by the impact factor.

NOTE: The dynamic load allowance shall not be applied to the design lane load (Section 3.6.2.1 DM4)

$I := 0.33$ Impact factor for moment

Note: PennDOT requires that the impact for decks be increased from 33% to 50%, the deck is not being designed here. Instrad the deck design is based on the BD recommendations.

Dynamic load allowance for the Permit Load P-82 shall be less than 20%. (Section 3.6.2.1 DM4)

$I_{PL} := 0.20$ Impact factor for moment on Strength II

Design Wind Loading for Exterior Girder

$V_w := 80$ Wind speed for Strength V load case

$C_D := 1.3$ Gust factor for windard side of I-Girder and Box Girder Superstructures (Table 3.8.1.2.1-2)

$G := 1.0$ Gus effect factor is 1.0 for all structures other than sound barriers (Table 3.8.1.2.1-1)

$K_z := 1.0$ Pressure coefficient taken as 1 for Strength V (3.8.1.2.1)

$$P_z := 2.56 \cdot 10^{-6} \cdot \left(V_w\right)^2 \cdot K_z \cdot G \cdot C_D \cdot \frac{kip}{ft^2} = 0.021 \cdot \frac{kip}{ft^2}$$ Wind pressure on exterior girder

$$h_{exp} := h - h_f = 17.5 \cdot in$$ Exposed height of exterior girder

$$M_{WL} := \frac{P_z \cdot h_{exp} \cdot L^2}{8} = 5.032 \cdot kip \cdot ft$$ Moment due to wind loading on exterior girder

$$I_{yy} := \frac{b_f^{\,3} \cdot t_f}{12} = 72 \cdot in^4$$ Transverse moment of intertia

$$f_l := \frac{M_{WL} \cdot \frac{b_f}{2}}{I_{yy}} = 5.032 \cdot ksi$$ Flange lateral bending stress

Design Vehicular Live Load

PHL-93 and P-82 are examined herein.

PHL-93 (A3.6.1.2.1)

PHL-93 consists of a combination of (design truck or design tandem) + (design lane load)

Design Truck

PennDOT defines the design truck in accordance with AASTHO 3.6.1.2

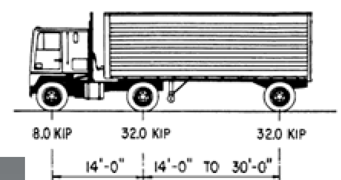
Design truck load in AASHTO (HS20 Truck) is used for this example.

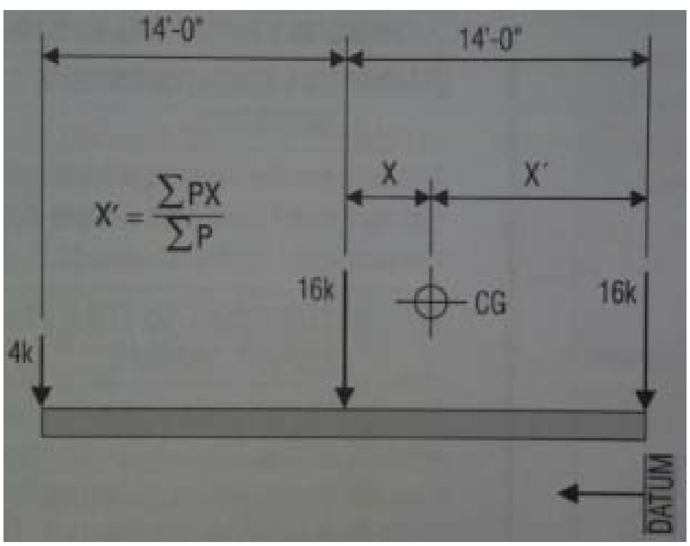
NOTE : MAX moment occurs when the center line of the span is midway between the center of gravity of the loads and the nearest concentrated load.

$P_1 := 8kip$ $P_2 := 32kip$ $P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck





Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

$$X := X_{ASr} - X' = 4.667 \cdot ft$$

Spacing btw CG and nearest load

Find bridge reaction

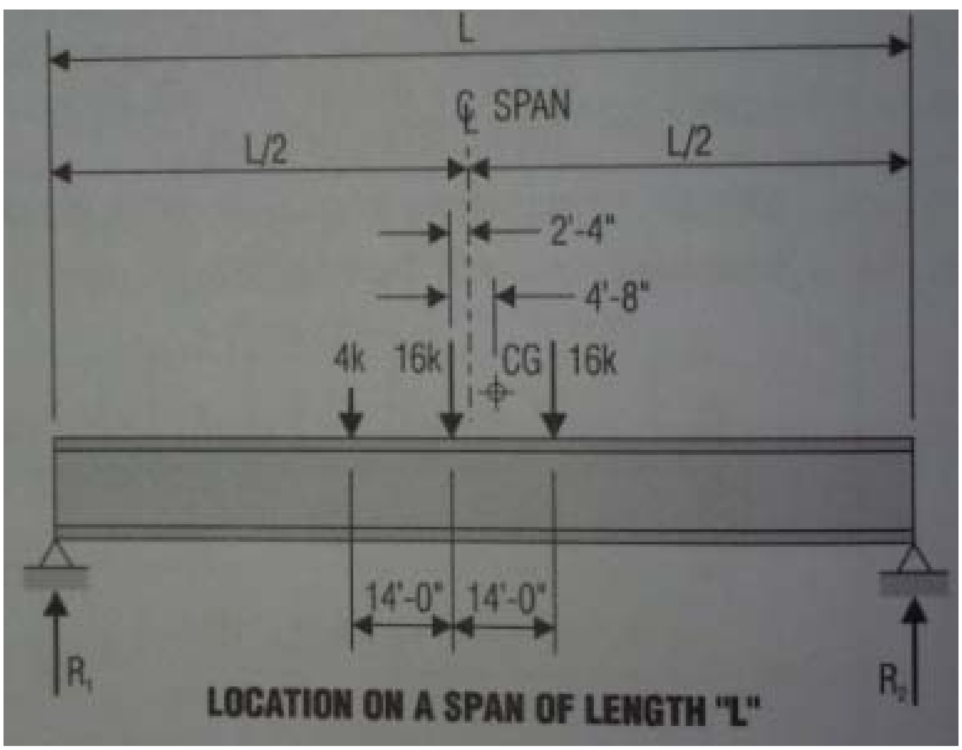
$$R_A := \frac{P_1 \cdot \left(\frac{L}{2} + X_{ASf} + \frac{X}{2}\right) + P_2 \cdot \left(\frac{L}{2} + \frac{X}{2}\right) + P_3 \cdot \left(\frac{L}{2} - X' - \frac{X}{2}\right)}{L} = 31.333 \cdot kip$$

Truck moves toward Support A

Max live load moment (at the location of P2):

$$M_{LL\_truck} := R_A \cdot \left(\frac{L}{2} - \frac{X}{2}\right) - P_1 \cdot X_{ASf} = 378.889 \cdot kip \cdot ft$$

NOTE: Find the worst case by varying X_{ASr}



Design Tandem

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25\text{kip}$

$X_{AS} := 4\text{ft}$ Axle spacing

Center of gravity (CG) of loads:

$X := \frac{X_{AS}}{2} = 2\cdot\text{ft}$ Spacing btw CG and nearest load

Find bridge reaction

$R_A := \frac{F_T\cdot\left(\frac{L}{2}+\frac{X}{2}\right)+F_T\cdot\left(\frac{L}{2}-\frac{X}{2}-X\right)}{L} = 29.514\cdot\text{kip}$ Truck moves toward Support A

Max live load moment (at the location of front wheel):

$M_{LL\_tandem} := R_A\cdot\left(\frac{L}{2}-\frac{X}{2}\right) = 501.736\cdot\text{kip}\cdot\text{ft}$

Choose the max value:

$\boxed{M_{LL\_veh} := \max\left(M_{LL\_tandem}, M_{LL\_truck}\right) = 501.736\cdot\text{kip}\cdot\text{ft}}$

Apply the impact and wheel load distribution:

$M_{LL\_IM\_veh} := M_{LL\_veh}\cdot DF\cdot(1+I) = 200.193\cdot\text{kip}\cdot\text{ft}$ Due to tandem or truck

Design Lane Load (distributed over a 10-ft width for each lane)

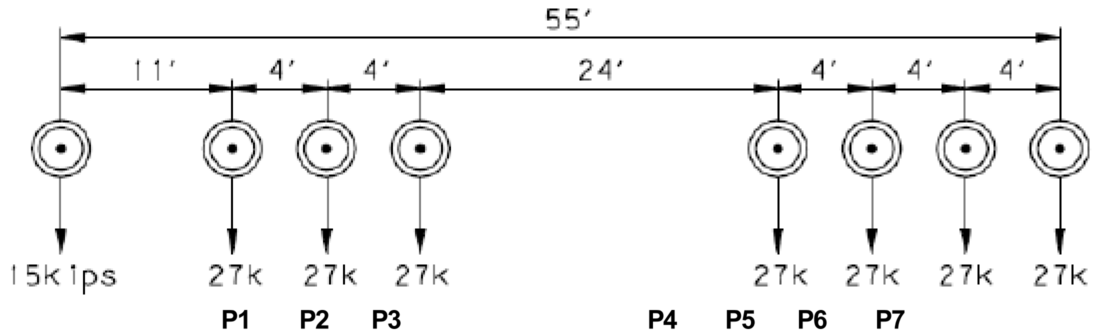
NOTE: The uniform load may be continuous or discontinuous as necessary to produce the maximum force effect. For this example continuous uniform distribution assumed.

$F_{DLL} := 0.64\frac{\text{kip}}{\text{ft}}\cdot DF = 0.192\cdot\frac{\text{kip}}{\text{ft}}$ Per beam in each lane

$M_{LL\_IM\_lane} := \left(\frac{F_{DLL}\cdot L^2}{8}\right) = 31.104\cdot\text{kip}\cdot\text{ft}$ Due to lane load

Moment due to Total Design Vehicular Live Load: $\boxed{M_{LL\_IM} := M_{LL\_IM\_lane} + M_{LL\_IM\_veh} = 231.297\cdot\text{kip}\cdot\text{ft}}$

Design Permit Load P-82 (Section 3.6.1.2.7 DM4)



NOTE: The moment for the permit load is defined at the beginning of the sheet and is obtained from MATLab File FlexBeam_Moment_ShearEnvelope.m

STEP 6: *Compute Moment of Inertia* (Note: rebar neglected!)

1) Moment of Inertia resisting dead load only:

a) Acting on the non-composite section (before the deck has hardened - DC1)

Use steel section only

$I_{nc} := I_{stl} = 640.875 \cdot in^4$ Non-composite section modulus

$Y'_{nc} := y\_bar$ S.G. of the steel section of Flex Beam (Datum is bottom)

b) Acting on the composite section (after the deck has hardened - DC2)

Use I_{c_short} as defined below.

2) Moment of Inertia resisting live load plus impact:

Using transformed slab with k_1=1 (short-term composite).

$$A_{ctr1} := h_f \cdot b_{tr1} = 29.531 \cdot \quad I_{tr1} := \frac{b_{tr1} \cdot h_f^{\,3}}{12} = 138.428 \cdot in^4$$

$$Y' := \frac{A_{ctr1} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr1}} = 16.467 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_short} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr1} + A_{ctr1} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 2913 \cdot in^4$$ Composite section modulus (short-term)

$$Y'_{c\_short} := Y' = 16.467 \cdot in$$ N.A. of the short-term composite section

3) Moment of Inertia resisting superimposed dead load:

Using transformed slab with k_3=3.

$$A_{ctr3} := h_f \cdot b_{tr3} = 9.844 \cdot in^2 \qquad I_{tr3} := \frac{b_{tr3} \cdot h_f^{\,3}}{12} = 46.143 \cdot in^4$$

$$Y' := \frac{A_{ctr3} \cdot \left(h - \frac{h_f}{2}\right) + A_{stl} \cdot Y'_{nc}}{A_{stl} + A_{ctr3}} = 12.465 \cdot in$$ S.G. of the total flex beam (Datum is bottom)

$$I_{c\_long} := I_{stl} + A_{stl} \cdot \left(Y'_{nc} - Y'\right)^2 + I_{tr3} + A_{ctr3} \cdot \left[\left(h - \frac{h_f}{2}\right) - Y'\right]^2 = 1993 \cdot in^4$$ Composite section modulus (long-term)

$$Y'_{c\_long} := Y' = 12.465 \cdot in$$ N.A. of the long-term composite section

STEP 6.5 Check Ductility Requirement

In accordance with AASHTO Article 6.10.7.3 check ductility requirement. Assume compression block of concrete is at 0.85 f'c and that the concrete supports no tension. Steel section is assumed to be fully plastic at Fy.

***Case 1**: Assuming PNA is in Concrete*

Assuming (h-d) < D_p < h_f $\qquad h - d = 4 \cdot in$

$$Fp(D_p) := 0.85 \cdot f_c \cdot b_e \cdot D_p + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 7.5in \qquad 0.42 \cdot h = 10.5 \cdot in$

$\boxed{Dp1 := root(Fp(D_p), D_p) = 5.77 \cdot in}$ Must be less than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp1}{h} = 0.231}$ must be < 0.42

$M_{n1check} := (Dp1 < h_f) \cdot [(h-d) < Dp1] \cdot (Dp1 < 0.42 \cdot h) = 1 \qquad Dp1 \le 0.42 \cdot h = 1$

$$Mp1 := 0.85 \cdot f_c \cdot b_e \cdot Dp1 \cdot \left(\frac{Dp1}{2}\right) + F_y \cdot [t_w \cdot [Dp1 - (h-d)]] \cdot \frac{[Dp1 - (h-d)]}{2} \ldots$$
$$+ \left[F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp1 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp1 - t_f)] \cdot \frac{(h - Dp1 - t_f)}{2}\right]$$

$\boxed{Mp1 = 10795.08 \cdot kip \cdot in}$ $\qquad M_{n1} := Mp1 \cdot \left(1.07 - 0.7 \cdot \frac{Dp1}{h}\right) = 9806.6 \cdot kip \cdot in$

***Case 2**: Assuming PNA is NOT in Concrete*

Assuming h_f < D_p

$$Fp2(D_p) := 0.85 \cdot f_c \cdot b_e \cdot h_f + F_y \cdot [t_w \cdot [D_p - (h-d)]] - F_y \cdot [A_{stl} - [t_w \cdot [D_p - (h-d)]]]$$

$D_p := 2.5in$

$\boxed{Dp2 := root(Fp2(D_p), D_p) = 0.83 \cdot in}$ Must be more than $h_f = 7.5 \cdot in$ $\boxed{\frac{Dp2}{h} = 0.0332}$ must be < 0.42

$\boxed{M_{n2check} := (Dp2 > h_f) \cdot (Dp2 < h) \cdot (Dp2 < 0.42 \cdot h) = 0} \qquad Dp2 \le 0.42 \cdot h = 1$

$$Mp2 := 0.85 \cdot f_c \cdot b_e \cdot h_f \cdot \left(Dp2 - \frac{h_f}{2}\right) + F_y \cdot [t_w \cdot [Dp2 - (h-d)]] \cdot \frac{[Dp2 - (h-d)]}{2} \ldots$$
$$+ F_y \cdot (b_f \cdot t_f) \cdot \left(h - Dp2 - \frac{t_f}{2}\right) + F_y \cdot [t_w \cdot (h - Dp2 - t_f)] \cdot \frac{(h - Dp2 - t_f)}{2}$$

$\boxed{Mp2 = 10177.24 \cdot kip \cdot in}$ $\qquad M_{n2} := Mp2 \cdot \left(1.07 - 0.7 \cdot \frac{Dp2}{h}\right) = 10653.1 \cdot kip \cdot in$

$Mn := \max(M_{n1} \cdot M_{n1check}, M_{n2} \cdot M_{n2check}) = 817.2 \cdot kip \cdot ft \qquad Mn = 9806.6 \cdot kip \cdot in$

Check Composite Compact Section Criteria (6.10.6.2)

$$D_{cp} := \frac{d}{2}\cdot\left(\frac{F_y\cdot t_f\cdot b_f - 0.85\cdot f_c\cdot b_e\cdot h_f}{F_y\cdot h_w\cdot t_w} + 1\right) = -3.247\cdot in \qquad D_{cp0} := D_{cp} > 0 = 0$$

$$D_{cp} := D_{cp}\cdot D_{cp0} = 0\cdot in \qquad \frac{2\cdot D_{cp}}{t_w} < 3.76\cdot\sqrt{\frac{E_s}{F_y}} = 1$$

Check if section is compact

STEP 7: Load Combinations (Table 3.4.1-1)

The following load combinations will be considered herein. Only the maximum load factors for permanent-loads are used in the following load combinations (uplift is not an issue for this type of bridge).

STRENGTH I	1.25DL + 1.5SDL + 1.75 (LL+IM)
SERVICE II:	DL+SDL+1.3(LL+IM)

STEP 8: Compute and Check Stresses

Stress at bottom fiber of steel section

$$f_{DL} := \frac{M_{DL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.729\cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM}\cdot Y'_{c\_short}}{I_{c\_short}} = 15.692\cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82}\cdot Y'_{c\_short}}{I_{c\_short}} = 18.538\cdot ksi$$

$$f_{SDL} := \frac{M_{SDL}\cdot Y'_{c\_long}}{I_{c\_long}} = 3.407\cdot ksi$$

STRENGTH I $f_{bot\_str1} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.75\cdot f_{LL\_I} = 37.234\cdot ksi$

STRENGTH II $f_{bot\_str2} := 1.25f_{DL} + 1.5\cdot f_{SDL} + 1.35\cdot f_{PL} = 34.799\cdot ksi$

SERVICE II $f_{bot\_svc2} := f_{DL} + f_{SDL} + 1.3f_{LL\_I} = 27.54\cdot ksi$

Allowable Stress is based on Section 6.10.4.2.2-2

Flange stress due to service load II without consideration of flange lateral bending: $f_f := f_{bot\_svc2}$

Flange lateral bending stress due to service II: $f_{lSII} := 0ksi$

Hybrid Factor: $R_h := 1.0$ For rolled and homogenous built-up sections

$$f_f + \frac{f_{lSII}}{2} = 27.537\cdot ksi \qquad 0.95\cdot R_h\cdot F_y = 47.5\cdot ksi \qquad f_f + \frac{f_l}{2} < 0.95R_h\cdot F_y = 1$$

Stress at top fiber of concrete

Note: For calculating longitudinal flexural stresses in the concrete deck due to all permanent and transient loads, the short-term modular ratio shall be used.

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.218 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.016 \cdot ksi \qquad f_{PL} := \frac{M_{LL\_IM\_PL82} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.201 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 0.2 \cdot ksi$$

STRENGTH I $f_{c\_str1} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.75 \cdot f_{LL\_I} = 2.351 \cdot ksi$

$M_{str1} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.75 \cdot M_{LL\_IM} = 535 \cdot kip \cdot ft$ $\quad M_{str1} < Mn = 1$

STRENGTH II $f_{c\_str2} := 1.25 f_{DL} + 1.5 \cdot f_{SDL} + 1.35 \cdot f_{PL} = 2.193 \cdot ksi$

$M_{str2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} + 1.35 \cdot M_{LL\_IM\_PL82} = 499 \cdot kip \cdot ft$ $\quad M_{str2} < Mn = 1$

SERVICE II

Allowable Stress in concrete - AASHTO 6.10.4.2.2
For compact composite sections in positive flexure utilized in shored construction, the longitudinal compressive stress in the concrete deck due to the Service II loads, determined as specified in Article 6.10.1.1.1d, shall not exceed 0.6f´c.

$f_{allow} := 0.6 \cdot f_c = 2400\, psi$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1739.29 \cdot psi$ This assumes short term composite section for all calculations $\quad f_{c\_2} < f_{allow} = 1$

SERVICE II with Long Term Composite for dead loads

$$f_{DL} := \frac{M_{DL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.156 \cdot ksi$$

$$f_{LL\_I} := \frac{M_{LL\_IM} \cdot (h - Y'_{c\_short})}{k_1 \cdot n \cdot I_{c\_short}} = 1.016 \cdot ksi$$

$$f_{SDL} := \frac{M_{SDL} \cdot (h - Y'_{c\_long})}{k_3 \cdot n \cdot I_{c\_long}} = 0.143 \cdot ksi$$

$f_{c\_2} := 1.3 f_{LL\_I} + f_{SDL} + f_{DL} = 1620.43 \cdot psi$ $\quad f_{c\_2} < f_{allow} = 1$

Strength V Moment Check

Strength V is related to normal vehicular use of the bridge with an 80 mph wind gust

Note: that Strength V is checked for an exterior girder under wind load with the interior girder distribution factor for flexure used. If Strength V controls design, then this approach should be modified

Design vertical moment is: 1.25DL + 1.5SDL + 1.35 (LL+IM)

$S_{LT} := \frac{I_{c\_long}}{Y'_{c\_long}} = 159.889 \cdot in^3$ $\quad S_{ST} := \frac{I_{c\_short}}{Y'_{c\_short}} = 176.874 \cdot in^3$ short and long term section modulus

Section D6.2.2 Find the moment required to cause yieldling of the flange. The approach is laid out in D6.2.2

$M_{D2} := 1.25 \cdot M_{DL} + 1.5 \cdot M_{SDL} = 130.214 \cdot kip \cdot ft$ From D6.2.2-1 (all dead loads are in MD2 due to shored construction

$M_{AD} := \left(F_y - \frac{M_{D2}}{S_{LT}}\right) \cdot S_{ST} = 592.929 \cdot kip \cdot ft$ Additional moment to cause nominal yielding of steel flange

$My := M_{D2} + M_{AD} = 723.143 \cdot kip \cdot ft$ D6.2.2-2: Moment to cause yielding of flange

6.10.7.1 Compact Section Check at Strength V limit state

$S_{xt} := \frac{My}{F_y} = 173.554 \cdot in^3$ Section Modulus for yielding of the flange (My/Fy) for equation 6.10.7.1.1-1

$M_u := 1.25 \cdot M_{DL} + 1.5M_{SDL} + 1.35M_{LL\_IM} = 442.464 \cdot kip \cdot ft$ Moment due to Strength V limit state

$M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} = 466.723 \cdot kip \cdot ft$ Equation 6.10.7.1.7-1 $\quad Mn = 817.218 \cdot kip \cdot ft$ $\quad M_u + \frac{1}{3} \cdot f_l \cdot S_{xt} < Mn = 1$

STEP 9: *Compute Deflection*

Optional Live Load Deflection Evaluation (AASHTO 3.6.1.3.2)

$DF_d := \frac{N_l}{N} = 0.2$

Deflection distribution factor for vehicular loads: For a straight multibeam bridge, this is equivalent to saying that the distribution factor for deflection is equal to the number of lanes divided by the number of beams. 2.5.2.6.2

Deflection should be taken as 125% the larger of:
- Design truck
- 25% of design truck + design lane

The live load portion of Load Combination Service I of Table 3.4.1-1 should be used, including the dynamic load allowance, IM;

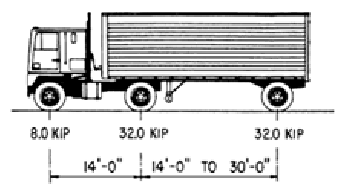
Deflection due to design truck load

NOTE : Assuming maximum deflection occurs when the center of gravity of the loads is at the center line of the span.

$P_1 := 8kip$ $\quad P_2 := 32kip$ $\quad P_3 := 32kip$ Axle loads (AASHTO 3.6.1.2.2)

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck



Center of gravity (CG) of loads:

$$X' := \frac{P_1 \cdot (X_{ASr} + X_{ASf}) + P_2 \cdot X_{ASr}}{P_1 + P_2 + P_3} = 9.333 \cdot ft$$

Datum is the rear axle

Method1: Assuming single load = P1+P2+P3 at the midspan (Conservative approach)

$$\frac{(P_1 + P_2 + P_3) \cdot L^3}{48 \cdot (E_s \cdot I_{c\_short})} = 1.432 \cdot in$$

Method2: Calculation of Midspan deflection using superposition

$b1 := \frac{L}{2} - (X_{ASr} - X') - X_{ASf} = -0.667 \cdot ft$ distance btw load P1 and support A $\quad b1check := b1 > 0 = 0$

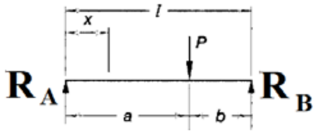
$b2 := \frac{L}{2} - (X_{ASr} - X') = 13.333 \cdot ft$ distance btw load P2 and support A $\quad b2check := b2 > 0 = 1$

$b3 := \frac{L}{2} - X' = 8.667 \cdot ft$ distance btw load P3 and support B $\quad b3check := b3 > 0 = 1$

Deflection of the beam at midspan due to P_1: $P := P_1$

$x := \frac{L}{2}$ at midspan $b := b1 \cdot b1check$ The smallest distance btw load P1 and support

$$\Delta 1 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0 \cdot in$$



Deflection of the beam at midspan due to P_2: $P := P_2$

$x := \frac{L}{2}$ at midspan $b := b2 \cdot b2check$ The smallest distance btw load P2 and support

$$\Delta 2 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.578 \cdot in$$

Deflection of the beam at midspan due to P_3: $P := P_3$

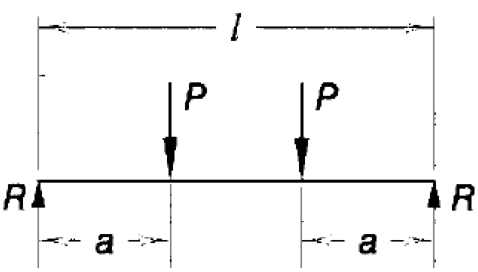
$x := \frac{L}{2}$ at midspan $b := b3 \cdot b3check$ The smallest distance btw load P3 and support

$$\Delta 3 := \frac{P \cdot b \cdot x}{6 \cdot (E_s \cdot I_{c\_short}) \cdot L} \cdot (L^2 - b^2 - x^2) = 0.424 \cdot in$$

Total Deflection at midspan:

$$\boxed{\Delta_{truck} := (\Delta 1 + \Delta 2 + \Delta 3) \cdot DF_d \cdot (1 + I) = 0.266 \cdot in}$$ (Considering IM)

Deflection due to tandem (Note: this calculation is not required according to Article 3.6.1.3.2 which denotes only the design truck for deflection calculation)



$$\Delta_{max} = \frac{Pa}{24\,EI}\left(3\,l^2 - 4\,a^2\right)$$

$F_T = 31.25 \cdot kip$ Per axle

$X_{AS} := 4ft$ Axle spacing

$$\Delta_{tandem} := \frac{F_T \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)}{24 \cdot E_s \cdot I_{c\_short}} \cdot \left[3 \cdot L^2 - 4 \cdot \left(\frac{L}{2} - \frac{X_{AS}}{2}\right)^2\right] \cdot DF_d \cdot (1 + I) = 0.325 \cdot in$$

(Considering IM)

Deflection due to lane load

$$F_{DLLd} := 640 \cdot \frac{lbf}{ft} \cdot DF_d = 128 \cdot \frac{lbf}{ft}$$

$$\boxed{\Delta_{lane} := \frac{\left(5 \cdot F_{DLLd} \cdot L^4\right)}{384 \cdot E_s \cdot I_{c\_short}} = 0.057 \cdot in}$$

Live Load Deflection:

$$\Delta_{LL} := 1.25\,\max\left(\Delta_{truck}, 0.25\Delta_{truck} + \Delta_{lane}\right) \cdot 1.25 = 0.416 \cdot in$$

$$\Delta_{LL} \le \frac{L}{800} = 1$$

Deflection Limit

For steel or concrete vehicular bridges:

Vehicular load = Span/800 $\frac{L}{800} = 0.54 \cdot in$ $\frac{L}{1000} = 0.432 \cdot in$

Evaluation of Deflection at the Service Limit State (AASHTO 3.4.2.2)

The associated permitted deflections shall be included in the contract documents.

Shear demands:

Determination of the distribution factor

Using DM4 Table 4.6.2.2.3a-1

$S := \frac{b_e}{12in} = 2.625$ transverse girder spacing (ft)

Distribution of live loads per lane for shear in interior beams:

$\boxed{DF_s := 0.36 + \frac{S}{25} = 0.465}$ For two design lanes loaded, 3.5' <= S =< 16', number of beam >= 4

$Q := A_{ctr1} \cdot \left(h - Y'_{c\_short} - \frac{h_f}{2} \right) = 141.25 \cdot in^3$ first moment of deck slab for interface shear

Method 1: Range of shear based on Strength I Limit

STRENGTH I 1.25DL + 1.5SDL + 1.75 (LL+IM)

Computation of the range of shear in the beam

Dead Load: $DL = 0.307 \cdot \frac{kip}{ft}$

Superimposed Dead Load: $SDL = 0.28 \cdot \frac{kip}{ft}$

Design Lane Load: $0.64 \frac{kip}{ft}$

Design Truck:

$\boxed{X_{ASr} := 14ft}$ Rear Axle spacing for truck (14ft - 30ft)

$X_{ASf} := 14ft$ Front Axle spacing for truck

$P_1 = 8 \cdot kip$ $P_2 = 32 \cdot kip$ $P_3 = 32 \cdot kip$ Wheel loads (AASHTO 3.6.1.2.2)

Design Tandem:

Consist of a pair of 25-kip axles spaced 4-ft apart. (AASHTO 3.6.1.2.3)

NOTE: PennDOT DM4 increases 25-kip to 31.25-kip.

$F_T := 31.25kip$

$X_{AS} := 4ft$ Axle spacing

Maximum positive shear at various points along the FlexBeam are examined.

At x=0 (at left support) (Truck is moving from left to right)

Truck

Max Positive Shear (when P_3 is at x = 0):

$x_1 := L - (X_{ASr} + X_{ASf}) = 8 \cdot ft$ Distance between P_1 and right support

$x_2 := L - X_{ASr} = 22 \cdot ft$ Distance between P_2 and right support

$x_3 := L = 36 \cdot ft$ Distance between P_3 and right support

$$V_1 := P_3 \cdot \frac{x_3}{L} + P_2 \cdot \frac{x_2}{L} + P_1 \cdot \frac{x_1}{L} = 53.333 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{truck1} := V_1 \cdot \left\lfloor DF_s \cdot (1 + I) \right\rfloor = 32.984 \cdot kip}$$

Tandem:

$$V_1 := F_T + F_T \cdot \frac{(L - X_{AS})}{L} = 59.028 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{tandem1} := V_1 \cdot \left\lfloor DF_s \cdot (1 + I) \right\rfloor = 36.506 \cdot kip}$$

Lane Load

$$V_1 := 0.64 \frac{kip}{ft} \cdot \frac{L}{2} = 11.52 \cdot kip$$

Total Vertical Shear Force for each FlexBeam (considering dynamic loads):

$$\boxed{V_{lane1} := V_1 \cdot (DF_s) = 5.357 \cdot kip}$$

Dead Load

$$\boxed{V_{dl1} := DL \cdot \frac{L}{2} = 5.521 \cdot kip}$$

Superimposed Dead Load

$$\boxed{V_{sdl1} := SDL \cdot \frac{L}{2} = 5.045 \cdot kip}$$

Strength I Max Positive Shear:

$$\boxed{V_{str1} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.75 \cdot \left(V_{lane1} + \max\left(V_{truck1}, V_{tandem1}\right)\right) = 87.728 \cdot kip}$$

Design Permit Load P-82

Shear at x=0

Strength II Max Positive Shear:

$V_{PL82} := V_{PL82span} \cdot DF_s \cdot \left(1 + I_{PL}\right) = 50.053 \cdot kip$

STRENGTH II 1.25DL + 1.5SDL + 1.35 (LL+IM)

$V_{strII\_PL} := 1.25 \cdot V_{dl1} + 1.5 \cdot V_{sdl1} + 1.35 \cdot \left(V_{PL82}\right) = 82.039 \cdot kip$

Max Shear Demand $Vu := \max\left(V_{str1}, V_{strII\_PL}\right) = 87.728 \cdot kip$

Determine the Shear Strength of the Web

Shear Strength of Unstiffened web from AASHTO section 6.10.9.2

Depth of Web $D := d = 21 \cdot in$ Thickness of web $t_w = 0.375 \cdot in$

Transverse Stiffener Spacing $d_0 := 0in$ Shear Buckling Coefficient $k := 5$ 6.10.9.3.2-7

Plastic Shear Force $V_p := 0.58 \cdot F_y \cdot D \cdot t_w = 228.375 \cdot kip$ Equation 6.10.9.3.2-3

Ratio of Shear-Buckling resistance to shear yield, C

$\frac{D}{t_w} = 56 \quad 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 60.314 \quad C_{check} := \frac{D}{t_w} \le 1.12 \cdot \sqrt{\frac{E_s \cdot k}{F_y}} = 1$ Equation 6.10.9.3.2-4

$C := 1.0 \cdot C_{check} = 1$ C=1 for most cases, if the criteria are not met, then C will go to zero and the actual coefficients should be determined

Nominal Shear Resistance of Web $V_n := C \cdot V_p = 228.375 \cdot kip$ equation 6.10.9.3.3-1

Shear Resistance Factor $\varphi_v := 1.0$

Reduced Shear Strength $\varphi_v \cdot V_n = 228.375 \cdot kip$ $\varphi_v \cdot V_n > Vu = 1$

Check Bearing Stiffeners

$F_{Ys} := 50ksi$ | Yield strength of stiffener

$t_p := 0.75in$ | Thickness of stiffener

$b_t := \frac{b_f - t_w}{2} - 1.5in = 4.312 \cdot in$ | Effective width of stiffener

$D_{st} := h_w - 4.1in = 16.4 \cdot in$ | Depth of stiffeners

$A_{pn} := 2 \cdot t_p \cdot b_t = 6.469 \cdot in^2$ | Projected area of stiffeners

Check that effective width of stiffener meets requirements | $b_t \le 0.48 \cdot t_p \cdot \sqrt{\frac{E_s}{F_{Ys}}} = 1$

$R_{sbn} := 1.4 \cdot A_{pn} \cdot F_{Ys} = 452.812 \cdot kip$ | Bearing capacity of stiffener (phi=1 for bearing per AASHTO 6.5.4.2)

$R_{sbn} \ge Vu = 1$ | Bearing capacity check

$Q_s := 1.0$ | Q factor for bearing stiffner, taken as 1 (AASHTO 6.9.4.1.1)

$I_s := \frac{18 \cdot t_w \cdot t_w{}^3 + t_p \cdot (2 \cdot b_t)^3}{12} = 40.131 \cdot in^4$ | Moment of inertia of stiffener and web (9*tw is used on each side) AASHTO 6.10.11.2.4b

$A_g := 18 \cdot t_w \cdot t_w + 2 \cdot b_t \cdot t_p = 9 \cdot in^2$ | Gross area of stiffener and web (AASHTO 6.10.11.2.4b)

$r_s := \sqrt{\frac{I_s}{A_g}} = 2.112 \cdot in$ | Radius of gyration of stiffener/web

$Po := Q_s \cdot F_{Ys} \cdot A_{pn} = 323.437 \cdot kip$ | Nominal yeild resistance of stiffener (AASHTO 6.9.4.1.1)

$Pe := \frac{\pi^2 \cdot E_s}{\left(\frac{0.75 \cdot D_{st}}{r_s}\right)^2} \cdot A_g = 7.592 \times 10^4 \cdot kip$ | Elastic critical buckling stress of stiffener (AASHTO 6.9.4.1.2-1)

Check that Pe/Po is great than 0.44 (AASHTO 6.9.4.1.1) | $PePo := \frac{Pe}{Po} \ge 0.44 = 1$

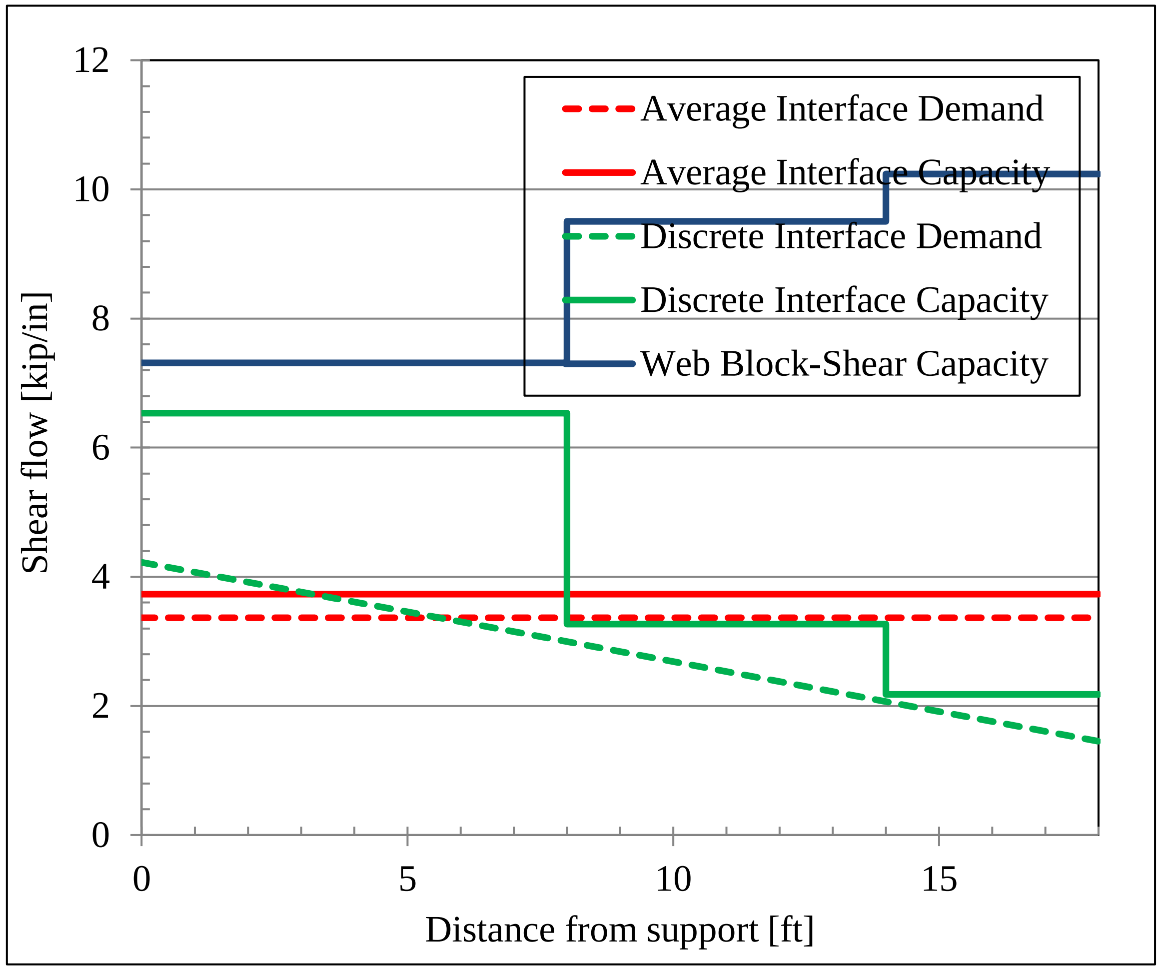
$Pn := PePo \cdot 0.658^{\frac{Po}{Pe}} \cdot Po = 322.861 \cdot kip$ | Nominal compressive resistance of stiffener for Pe/Po>0.44 (AASHTO 6.9.4.1.1-1)

$\varphi cr := 0.95$ | phi factor for axial compression (AASHTO 6.5.4.2)

$Pr := \varphi cr \cdot Pn = 306.718 \cdot kip$ | Compressive capacity of bearing stiffener | $Pr \ge Vu = 1$

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties	
Length:	36 ft
Width of deck:	28.5 in
Q_{short}:	136.7 in^3
I_{short}:	2831 in^4
Area of steel:	16.594 in^2
Web thickness:	0.375 in
Web tensile:	65 ksi
Area of deck:	213.75 in^2
Avg. interface shear:	3.36 kip/in
Block shear ϕ:	0.80



Calculated Shear Demand Envelope from Matlab

Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	87.467	4.22	3.36
3.6	76.096	3.67	3.36
7.2	64.564	3.12	3.36
10.8	53.033	2.56	3.36
14.4	41.501	2.00	3.36
18	30.064	1.45	3.36

Shear Dowel Spacing Design

Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	24.0	6.534	7.3125	3.731
96	8.0	4		6.534	7.3125	3.731
96	8.0	8	9.0	3.267	9.50625	3.731
168	14.0	8		3.267	9.50625	3.731
168	14.0	12	4.0	2.178	10.2375	3.731
216	18.0	12		2.178	10.2375	3.731

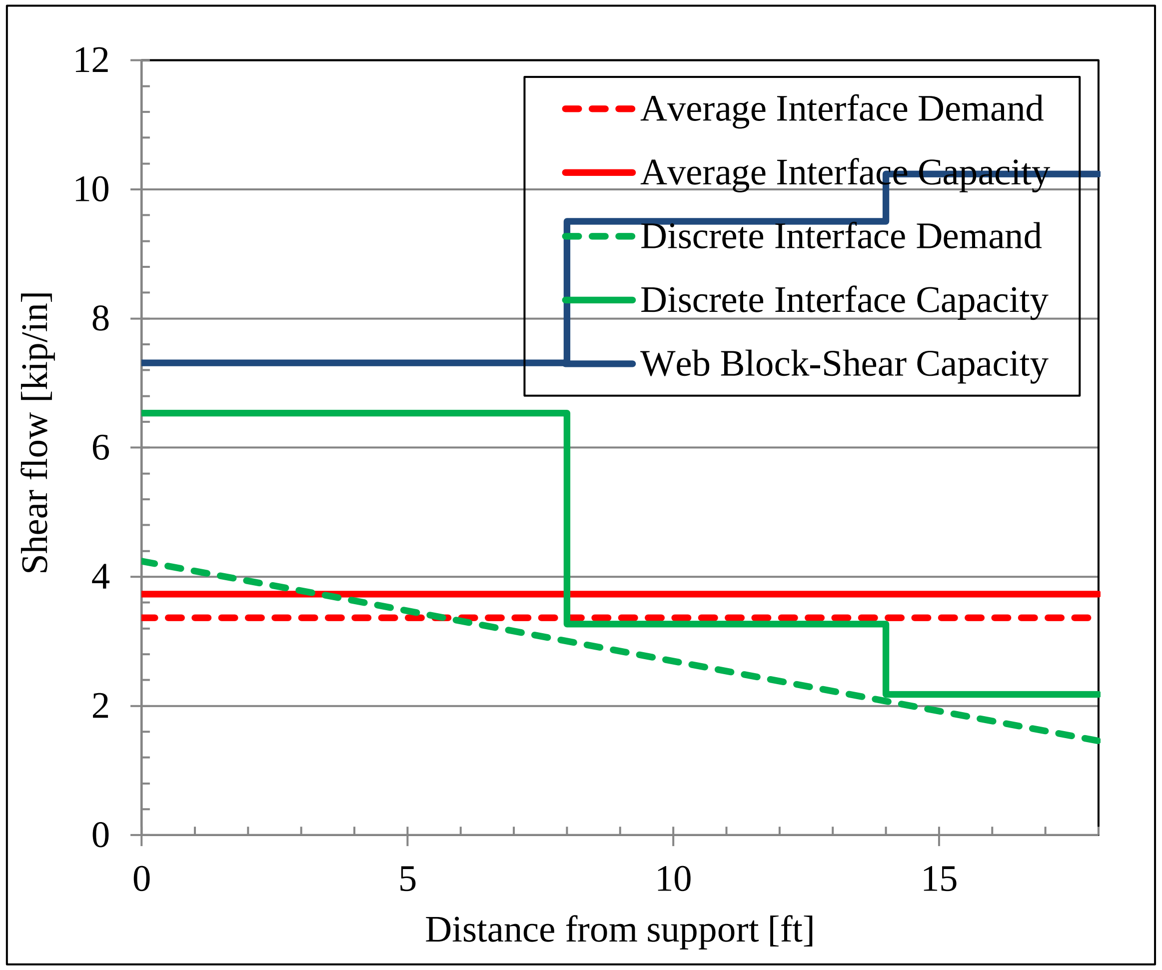
Total Number of Holes	37	holes on half span
Total Force Capacity	806	kip
Average Shear Capacity	3.73	kip/in

Capacity of Dowel

Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

Shear Spacing Analysis for PA Flex Beam Example Configuration

Section Properties		
Length:	36	ft
Width of deck:	28.5	in
Q_{short}:	141.25	in^3
I_{short}:	2913	in^4
Area of steel:	16.594	in^2
Web thickness:	0.375	in
Web tensile:	65	ksi
Area of deck:	213.75	in^2
Avg. interface shear:	3.36	kip/in
Block shear ϕ:	0.80	



Calculated Shear Demand Envelope from Matlab			
Distance from support (ft)	Shear force (kip)	Shear Flow (kip/in)	Avg Shear Flow (kip/in)
0	87.467	4.24	3.36
3.6	76.096	3.69	3.36
7.2	64.564	3.13	3.36
10.8	53.033	2.57	3.36
14.4	41.501	2.01	3.36
18	30.064	1.46	3.36

Shear Dowel Spacing Design						
Distance from support [in.]	Distance from support [ft]	Hole spacing (in.)	Number of holes	Dowel shear strength [kip/in.]	Web block shear strength [kip/in.]	Average Shear Capacity [kip/in.]
0	0.0	4	24.0	6.534	7.3125	3.731
96	8.0	4		6.534	7.3125	3.731
96	8.0	8	9.0	3.267	9.50625	3.731
168	14.0	8		3.267	9.50625	3.731
168	14.0	12	4.0	2.178	10.2375	3.731
216	18.0	12		2.178	10.2375	3.731
	Total Number of Holes		37	holes on half span		
	Total Force Capacity		806	kip		
	Average Shear Capacity		3.73	kip/in		

Capacity of Dowel		
Hole Dia.	1.5	in
Yield Strength	60	ksi
Bar Area	0.44	sq.in.
Shear Factor	0.55	
Discrete ϕ	0.9	
Average ϕ	0.75	
Discrete Cap	26.136	kip/dowel
Avg. Capacity	21.78	kip/dowel

6. Appendix E – Matlab Load Case Script

The load cases are examined for the various bridge span using a Matlab m-file script. The program computes the maximum moment distribution and shear envelope for the simply supported FlexBeam configurations in accordance with PennDOT live load combinations and vehicles.

```
clearvars;
close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% VERSION 01/15/2020 Updated to Include Overlay and additional concrete on
% exterior overhang
%
% Flexbeam analysis for shear and moment due to vehicular live loads,
% distributed live loads, distributed superimposed dead loads(wearing
% surface and parapet) and distributed dead loads (weight of members)
%
% ASSUMPTIONS
%   - The entired width of the bridge is paved
%   - The barrier dead load is applied evenly over two outside WT sections
%   - The wearing surface is distributed evenly over each WT section
%   - Overlay is distributed evenly over each WT section
%   - The lane load and vehicle loads have a dist factor of 0.3 for flexure
%     and is calculated for shear
%   - The worst case loading occurs at 1st interior WT section (that
%     location sees 0.3 of lane/vehicle load + 0.5 barrier load
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Enter in the section properties and weights to compute the dead load and
% superimposed dead loads, lane live loads and generate the shear and
% moment diagrams for the superimposed dead and lane live loads
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=70;           % Length of bridge in ft.
S=40/12;        % Spacing of WT sections in ft.
N=10;           % Total number of WT beams in bridge
W=S*N-6/12;     % Width of whole bridge in ft. (subract half joint thickness on each↙
side)
wwt=0.116;      % Weight of WT section in kip/ft
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

t=7.5/12;       %  Thickness of deck in ft.
wd=72/12;       % Width of 72 inch deck in ft.

DFm=0.3;                % Distribution factor for moment
DFv=0.36+S/25;          % !!Distribution factor for shear (Single Lane !!CONTROLS!!)
% DFv=0.2+S/12-(S/35)^2;  % Distribution factor for shear (Two or more lanes loaded)


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Define the live loads, impact factors and distribution factors
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

LoadCase=2;         % LoadCase=1 is Strength I, LoadCase=2 Is Strength II, LoadCase=3 is↙
Service II
```

```
% P=[8,0; 32,14; 32,14]; Title='Design Truck'; I=0.33;
P=[31.25 0; 31.25 4]; Title='Design Tandem'; I=0.33;
% P=[15 0; 27 11; 27 4; 27 4; 27 24; 27 4; 27 4; 27 4]; Title='Permit Load'; I=0.2;


switch LoadCase
    case 1

        Name='Strength I';  % Name of case
        Fd=1.25;            % Load factor for dead load
        Fsd=1.5;            % Load factor for superimposed dead load
        Fl=1.75;            % Load factor for live loads
        P=[31.25 0; 31.25 4]; Title='Design Tandem'; I=0.33;

    case 2

        Name='Strength II';
        Fd=1.25;          % Load factor for dead load
        Fsd=1.5;          % Load factor for superimposed dead load
        Fl=1.35;          % Load factor for live loads
        P=[15 0; 27 11; 27 4; 27 4; 27 24; 27 4; 27 4; 27 4]; Title='Permit Load'; I=0.2;

    case 3

        Name='Service II';
        Fd=1.0;             % Load factor for dead load
        Fsd=1.0;            % Load factor for superimposed dead load
        Fl=1.30;            % Load factor for live loads
end

%%% Dead Loads (Self weight of steel and concrete) %%%%%%%%%%%%%%%%%%%%%%%%

wc=0.150;                          % Weight of concrete is 150 lbs/ft^3 or 0.150↙
kips/ft^3
wdl=W/N*t*wc+wwt+1/12*S/2*wc;      % Dead load of each beam in kips/ft (includes↙
additional 1" concrete on exterior overhang
twdl=(t*wd*wc+2*wwt)/2;            % Dead load per beam of the test specimen in kips/ft

%%% Live Loads %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

wll=0.64;   % wll is Lane Load 0.64 kip/ft

%%% Superimposed Dead Loads %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

tol=2/12;               % thickness of overlay in ft (2" overlay assumed)
wol=0.140*W*tol;        % Weight of overlay is 140 lbs/ft^3 over entire width of bridge↙
and 2" thick
Wws=W-2*1.6875;         % Width of future wearing surface in ft (Width of bridge-↙
2*barrier width)
wws=0.03*Wws;           % Weight of wearing surface is 30 lbs/ft^2 times width of bridge
```

```
wb=0.700;               % Weight of 45" F-Shape Barrier is 0.7 kips/ft
DFwb=0.5;               % weight of barrier is distributed to two outside beams

wsd=wol/N+wws/N+wb*DFwb;      % Total weight of superimposed dead load (kip/ft)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Discretization increments
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

dx=.05;         % Length discretization increments in feet
x=dx:dx:L;      % Discretization of length in feet
np=length(x);   % Number of discretized points

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Create the shear and moment influence lines: rows are points where
% influence line is calculated, while columns are load position.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

VI=zeros(np,np);    % Preallocate for shear influnce lines
MI=zeros(np,np);    % Preallocate for moment influnce lines

for i=1:np          % Loop through the rows of the VI matrix (rows are influence line↙
locations)
    pI=x(i);        % pI is the location where the influence line is calculated
    r1=pI/L;        % r1 is ratio of force for shear line 1
    r2=(1-r1);      % r2 is ratio of force for shear line 2
    rm=(L-pI)/L*pI; % rm is max moment at pI

    for j=1:np      % Loop through the columns of the VI matrix (columns are load↙
positions)
        p=x(j);     % p is the location where the load is applied

        if p<=pI
            VI(i,j)=-(r1/pI)*p; % Calculate each column of the of VI(i) for p<=pI
            MI(i,j)=rm/pI*p;    % Calculate each column of the of MI(i) for p<=pI
        else
            VI(i,j)=-(r2/(L-pI)*p-r2-r2*pI/(L-pI)); % Calculate each column of VI(i) for↙
p>pI
            MI(i,j)=-rm/(L-pI)*p+rm+rm*pI/(L-pI);   % Calculate each column of MI(i) for↙
p>pI
        end
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Loop through and move the truck along the bridge with the postition
% indexed to the front wheel. j loop covers all vehicle positions. i loop
% calculates the shear at all locations on bridge for each vehicle
% position. The results are stored in V, which has vectors of shear forces
% for each vehicle position.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
p=dx:dx:L+sum(P(:,2));          % Discretized positions of front wheel
V=zeros(np,length(p));          % Preallocate V
M=zeros(np,length(p));          % Preallocate M

for j=1:length(p)

    for i=1:np

        d=P(1,2);   % Initialize distance from front wheel to wheel being checked at↙
front wheel spacing (usually 0)

        for k=1:size(P,1)

            d=d+P(k,2);                 % Add the spacing of the wheel being added to the↙
wheel distance
            fw=p(j);                    % Location of front wheel
            wheel=fw-d;                 % Location of wheel being considered
            index=int16(wheel/dx);      % Index of wheel being considered in VI, MI
            if wheel<dx^2               % Check if wheel is very close to zero and assign↙
wheel=0 to prevent indexing issues
                wheel=0;
            end

            if wheel>0 && wheel<=L  % Check if wheel is on the bridge

                V(i,j)=V(i,j)+P(k,1)*VI(i,index);  % If x1 is on the bridge, find↙
corresponding shear and write to V
                M(i,j)=M(i,j)+P(k,1)*MI(i,index);  % If x1 is on the bridge, find↙
corresponding moment and write to M

            end

        end
    end

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate the unfactored dead load and live load moments and add them
% together to find the worst case
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


Mdl=(wdl*L/2.*x-wdl.*x.^2/2)';  % Moment due to dead load
Vdl=(wdl*L/2-wdl.*x)';          % Shear due to dead load

Mdlt=(twdl*L/2.*x-twdl.*x.^2/2)';   % Moment due to dead load of test specimen
Vdlt=(twdl*L/2-twdl.*x)';           % Shear due to dead load of test specimen

Mll=(wll*L/2.*x-wll.*x.^2/2)';  % Moment due to lane load
```

```
Vll=(wll*L/2-wll.*x)';          % Shear due to lane load

Msd=(wsd*L/2.*x-wsd.*x.^2/2)';  % Moment due to Superimposed Dead Load
Vsd=(wsd*L/2-wsd.*x)';          % Shear due to Superimposed Dead Load

Mtot=Mdl+Msd+Mll+M;                 % Total moment for max moment (live + dead)
Vtot=Vdl+Vsd+Vll+V;                 % Total shear for max moment (live + dead)

MMvl=max(max(M));
VVvl=max(max(V));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find the factored dead, superimposed dead, lane and vehicle loads
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Find factored moment and shear

fMdl=Mdl*Fd;                 % Multiply dead load by load factor
fMsd=Msd*Fsd;                % Multiply superimposed dead by load factor
fMll=Mll*Fl*DFm;             % Multiply live load by load factor
if LoadCase==2
    fMll=0;
end
fMMvl=MMvl*Fl*DFm*(1+I);     % Multiply vehicle load by load factor and impact factor
fMtot=fMdl+fMsd+fMll+fMMvl;  % Find the total moment

fVdl=Vdl*Fd;                 % Same as above but for shear
fVsd=Vsd*Fsd;
fVll=Vll*Fl*DFv;
fVMvl=VVvl*Fl*DFv*(1+I);
fVMtot=Vdl+Vsd+Vll+VVvl;     % Find total shear for max shear

fVllenv=Vll*Fl*DFv;          % Mulitply lane lane by shear distribution factor for shear ↙
envelope

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Combine the load cases based on which case(Strength I, Strength II,
% Service II) are being plotted
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

switch LoadCase

    case 1  % Case 1 is Strength I

        Venv(:,1)=min(fVdl+fVsd+fVllenv+V*Fl*DFv*(1+I),[],2);  % Create shear envelopes
        Venv(:,2)=max(fVdl+fVsd+fVllenv+V*Fl*DFv*(1+I),[],2);


    case 2  % Case 2 is Strength II (No lane load is included)

        Venv(:,1)=min(fVdl+fVsd+V*Fl*DFv*(1+I),[],2);   % Create shear envelopes
```

```
        Venv(:,2)=max(fVdl+fVsd+V*Fl*DFv*(1+I),[],2);

    case 3  % Case 3 is Service II

        Venv(:,1)=min(fVdl+fVsd+fVllenv+V*Fl*DFv*(1+I),[],2);  % Create shear envelopes
        Venv(:,2)=max(fVdl+fVsd+fVllenv+V*Fl*DFv*(1+I),[],2);

end

M2p=zeros(np,1);
V2p=zeros(np,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot the shear envelope
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mid=(L/2)/dx;               % Point for midspan of the bridge
Venvp=zeros(1,mid);         % Preallocate Venvp
xVenvp=x(1:mid);            % vector of x from start to midspan

for i=1:mid
    Venvp(i)=max(abs(Venv(i,1)),abs(Venv(i,2)));
end


figure;
hold on;
grid on;
box on;
plot(xVenvp,Venvp,'LineWidth',2,'color','blue');
xlabel('Distance Along Beam (ft)');
ylabel('Shear Force (kip)');
title(horzcat(Name,' ',Title,' Shear Envelope'));


envpoints=0:L/10:L/2;
dist=zeros(6,1);
shear=zeros(6,1);
shear(1)=Venvp(1);
xVenvp=round(xVenvp,2);

for i=2:length(envpoints)
    vindex=find(xVenvp==envpoints(i));
    dist(i)=xVenvp(vindex);
    shear(i)=Venvp(vindex);
end

disp(horzcat('Load Case: ',Name));
disp(horzcat('Vehicle: ',Title));
disp('');
disp(horzcat('Maximum Vehicle Moment: ',num2str(max(max(M))*DFm*(1+I)),' kip-ft','↙
```

```
includes distribution and impact'));
disp(horzcat('Maximum Vehicle Shear: ',num2str(max(max(V))),' kip',' does not include↙
distribution and impact'));
disp(table(dist,shear));
```

NOTES:

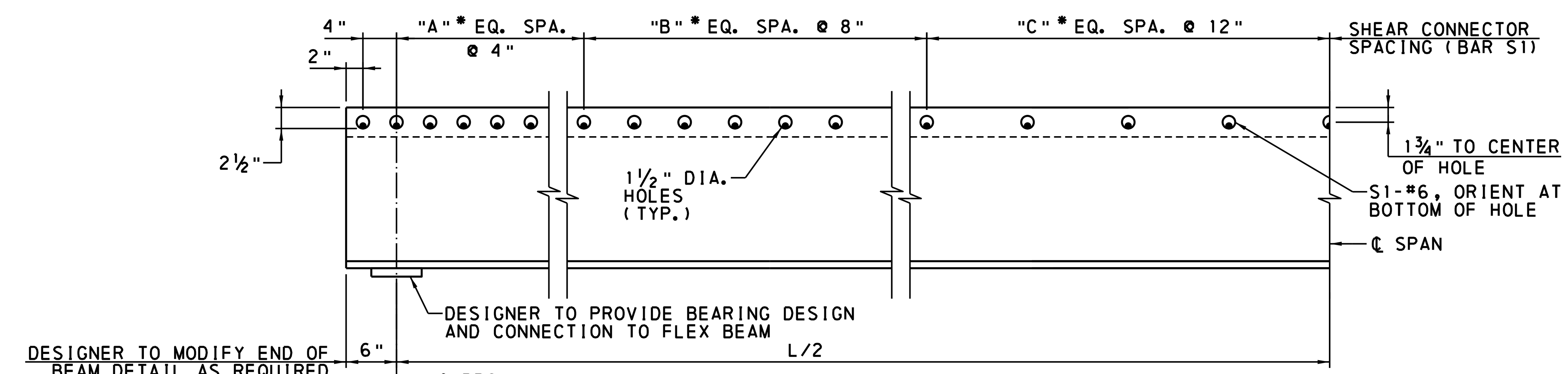
1. DESIGN SPECIFICATIONS:
   - -AASHTO LRFD BRIDGE DESIGN SPECIFICATIONS AND COMMENTARY 7TH EDITION
   - -PENNDOT DESIGN MANUAL, PART 4 (DM-4) APRIL 29, 2015.
   - -AASHTO/AWS D1.5M/D1.5: 2008-BRIDGE WELDING CODE.
2. MATERIAL STRENGTH:
   - -REINFORCEMENT STEEL: GRADE 60, EPOXY COATED
   - -STRUCTURAL STEEL: ASTM A709 GRADE 50
   - -CONCRETE $f'c$ = 4 KSI (CLASS AAAP CONCRETE) FOR DECK SLABS AND CONCRETE END DIAPHRAGMS AND $f'c$ = 3.5 KSI (CLASS AA CONCRETE) FOR BARRIERS
   - -MODULAR RATIO (Es/Ec) N = 8
   - -ALL BOLTS TO BE ASTM F3125 GRADE A325, HAVING AN UNTHREADED SHANK OF SUFFICIENT LENGTH TO NOT ALLOW ANY THREADS TO EXIST IN THE PLANE BETWEEN THE CONNECTED PARTS (SHEAR PLANE)
3. LIVE LOAD: -AS PER DESIGN MANUAL PART 4.
4. DESIGN LOAD:
   - -NORMAL WEIGHT CONCRETE = 150 LB./FT³
   - -F-SHAPE BARRIER, MODIFIED = 700 LB./FT.
   - -FUTURE WEARING SURFACE = 30 LB./FT²
   - -MAXIMUM OVERLAY/WEARING SURFACE PLACED AT TIME OF INITIAL CONSTRUCTION. = 23 LB./FT²
   - -DECK SLAB AND BARRIER LOADS ARE ASSUMED TO BE PLACED USING SHORED CONSTRUCTION.
5. DESIGN CONTROLS:
   - -CONCRETE COVER: DECK TOTAL TOP COVER = 2"; DECK BOTTOM COVER = 1"; BARRIER = 2"
   - -MIN. CLEAR DISTANCE BETWEEN REINFORCEMENT MATS = 2"
   - -BAR SIZE: MAXIMUM BAR SIZE: #6; MINIMUM BAR SIZE: #4; S1 AND S2, BARS: #6; S3 AND S4 BARS: #4
   - -BAR SPACINGS: MAXIMUM SPACING = 12" SLAB; = 12" BARRIER
   - -PLACE A 1¼" LATEX MODIFIED CONCRETE OR POLYESTER POLYMER CONCRETE (PPC) WEARING SURFACE ON THE 7½" PRECAST DECK PRIOR TO OPENING BRIDGE TO TRAFFIC. AN EPOXY OVERLAY, IF EQUIVALENT IN WEIGHT, IS ALSO ALLOWED AS AN ALTERNATIVE.
6. USE ONLY FUSION BONDED EPOXY COATED REINFORCEMENT IN ACCORDANCE WITH PUBLICATION 408, SECTION 709.
7. REINFORCEMENT IN SOME SECTIONS NOT SHOWN FOR CLARITY.
8. DYNAMIC LOAD ALLOWANCE FOR DECK SLAB DESIGN (IM) = 50%
   DYNAMIC LOAD ALLOWANCE STEEL BEAM DESIGN (IM) = 33%
9. DRAWINGS ARE NOT TO SCALE.
10. DIMENSIONS SHOWN ARE FOR A NORMAL TEMPERATURE OF 68°F.
11. ALL DIMENSIONS SHOWN ARE HORIZONTAL, UNLESS OTHERWISE NOTED.
12. FOR STANDARD TYPICAL WATERPROOFING AND EXPANSION DETAILS SEE BC-788M.
13. SUBMIT SHOP DRAWINGS FOR FLEX BEAM PANELS AND ASSOCIATED COMPONENTS.
14. NO-LOAD CAMBER TOLERANCE AT MIDSPAN SHALL BE 0 IN. TO +0.5 IN. NO-LOAD CAMBER OF GIRDERS SHALL MEET THE REQUIREMENTS SPECIFIED IN THE DESIGN DRAWINGS. CAMBER PRIOR TO WELDING AND PRIMING.
15. HOLES SHALL BE DRILLED. PUNCHING MAY BE ALLOWED BUT WILL BE SUBJECT TO APPROVAL. IF PUNCHING IS USED, CUT HOLES CLEAN WITHOUT TORN OR RAGGED EDGES.
16. LIFTING DEVICES SHALL BE DESIGNED BY THE CONTRACTOR AND SUBMITTED AS PART OF THE SHOP DRAWING SUBMISSION.
17. DO NOT MAKE WELDS BY MANUAL SHIELDED METAL ARC PROCESS FOR PRIMARY GIRDER WELDS, SUCH AS FLANGE-TO-WEB WELDS OR FOR SHOP SPLICES OF WEBS AND FLANGES.
18. PAINT STRUCTURAL STEEL IN ACCORDANCE WITH PUBLICATION 408, SECTION 1060.
19. CASTING OF THE DECK SHALL BE FULLY SUPPORTED.
20. STEEL T REQUIRES MIDSPAN SUPPORT TO NO LOAD CONDITION DURING CASTING.
21. INTERFACE OF PRECAST PANELS ALONG THE TRANSVERSE AND LONGINTUDINAL JOINTS SHALL BE BLAST CLEANED TO CREATE AN EXPOSED AGGREGATE FINISH.
22. PRE-WET PRECAST INTERFACE OF JOINT WITH WATER TO CREATE A SATURATED SURFACE CONDITION.
23. PLACE THE FOLLOWING NOTE ON THE CONTRACT DRAWINGS - "THE STEEL SUPERSTRUCTURE SHALL BE DETAILED AND FABRICATED FOR TOTAL DEAD LOAD FIT (TDLF). GIRDER WEBS SHALL BE PLUMB UNDER THE FULL DEAD LOAD EXISTING AT THE END OF CONSTRUCTION."
24. PROVIDE CHARPY V-NOTCH (CVN) TESTING FOR ZONE 2 FOR ALL MEMBERS SUBJECTED TO TENSION PER PUB. 408, SECTION 1105.02(a)5.
25. FILLET WELD SIZES ARE GOVERNED BY MATERIAL THICKNESS IN ACCORDANCE WITH AASHTO/AWS EXCEPT AS NOTED.
26. SLIGHTLY STAGGER LONGITUDINAL REBARS IN THE BRIDGE DECK SUCH THAT NO REBAR IN THE TOP MAT IS DIRECTLY ABOVE A REBAR IN THE BOTTOM MAT.

LEGEND:

DIA.	-	DIAMETER
⌀	-	DIAMETER
U.H.P.C.	-	ULTRA-HIGH PERFORMANCE CONCRETE
EQ.	-	EQUAL
SPA.	-	SPACING

DESIGN NOTES:

1. IF NEEDED DETAILS ARE NOT FOUND IN THIS STANDARD, A SPECIAL SUBMISSION REQUESTING APPROVAL FOR SPECIFIC DETAILS MUST BE MADE TO THE CHIEF BRIDGE ENGINEER.
2. THE INFORMATION SHOWN IN THIS STANDARD IS PROVIDED FOR USE IN THE DEVELOPMENT OF THE CONTRACT DOCUMENTS. THE DESIGNER IS RESPONSIBLE FOR THE PRESENTATION OF ALL DESIGN INFORMATION.
3. USE THIS STANDARD FOR FLEXBEAM STRUCTURES SUBJECT TO LIMITATIONS AS SHOWN. USE THIS STANDARD AS A BASIS FOR THE PREPARATION OF STRUCTURE LAYOUTS AND CONTRACT DRAWINGS.
4. THE DESIGNER MUST PROVIDE THE INFORMATION OBTAINED FROM THE DESIGN TABLES AND DETAILS ON THE CONTRACT DRAWINGS.
5. DESIGN COMPUTATIONS ARE REQUIRED FOR ANY PORTION OF THE STRUCTURE FOR WHICH THE INFORMATION IS NOT TAKEN DIRECTLY FROM THE DESIGN TABLES CONTAINED IN THIS STANDARD. DO NOT VIOLATE CRITERIA USED FOR THE DEVELOPMENT OF THESE DESIGN TABLES.
6. DESIGN TABLES INCLUDED IN THIS STANDARD ARE BASED ON THE DESIGN CRITERIA SHOWN ON THIS SHEET.
7. THE SPAN RANGE INCLUDED IN THIS STANDARD IS AS FOLLOWS:
   BD-663M: FLEXBEAM STRUCTURES FROM 30' TO 70'
8. THE DESIGN TABLES INCLUDE ACCEPTABLE MEMBER AND MODULE SIZES AND SPACINGS FOR VARIOUS STRUCTURE WIDTHS.
9. CAMBERS ARE GIVEN ASSUMING PRECAST BARRIERS ARE USED AND ARE PLACED USING SHORED CONSTRUCTION. IF CAST-IN-PLACE BARRIERS ARE USED, THE DESIGNER IS TO MODIFY DEAD LOAD CAMBERS ACCORDINGLY.
10. FLEX BEAMS ARE DESIGNED TO SATISFY LIVE LOAD DEFLECTION REQUIREMENT:
    $\Delta MAX \leq L/800$
11. BEARINGS ARE TO BE DESIGNED BY THE ENGINEER AND ARE NOT INCLUDED AS PART OF THIS STANDARD.
12. SUBSTRUCTURE UNITS ARE TO BE DESIGNED BY THE ENGINEER AND ARE NOT INCLUDED AS PART OF THIS STANDARD.
13. THIS STANDARD APPLIES TO STRAIGHT BRIDGES WITH A SKEW OF 90 DEGREES.
14. DETAIL ALL BARS ON THE CONTRACT DRAWINGS.
15. FOR STANDARD DOUBLE MODULE DESIGN, SEE SHEET NO. 2.
16. STEEL END DIAPHRAGM DESIGN AVAILABLE. FOR STEEL END DIAPHRAGM DETAILS, SEE SHEET NO. 4.
17. CONCRETE END DIAPHRAGM DESIGN AVAILABLE. FOR CONCRETE DIAPHRAGM DETAILS, SEE SHEET NO. 5.
18. FOR BEAM AND SHEAR CONNECTOR DESIGN TABLES, SEE SHEET NO. 3.
19. FOR CAMBER DESIGN TABLES, SEE SHEET NO. 6.
20. ULTRA HIGH PERFORMANCE CONCRETE STRENGTH VARIES WITH TIME, DESIGNER SHOULD CHECK WITH MANUFACTURER TO DETERMINE APPROPRIATE MATERIAL FOR PROJECT SCHEDULE.
21. MASH COMPLIANT F-SHAPED BARRIER SHOWN. OTHER MASH COMPLIANT BARRIERS MAY BE SUBSTITUTED AT THE DISCRETION OF THE DISTRICT BRIDGE ENGINEER.
22. LATEX MODIFIED CONCRETE, PPC OVERLAY, OR EPOXY OVERLAY ARE ALL ACCEPTABLE AS A FINAL RIDING SURFACE AT THE DISCRETION OF THE DISTRICT BRIDGE ENGINEER.



TYPICAL BEAM ELEVATION

* FOR NUMBER OF SPACES, "A", "B", AND "C", SEE DESIGN TABLES ON SHEET NO. 3.

BD-601M	CONCRETE DECK SLAB
BD-660M	DECK SLAB, FORMS AND STEEL REINFORCEMENT PLACEMENT
BC-739M	BRIDGE BARRIER TO GUIDE RAIL TRANSITION
BC-752M	CONCRETE DECK SLAB DETAILS
BC-754M	STEEL DIAPHRAGMS FOR STEEL BEAM/GIRDER STRUCTURES
BC-788M	TYPICAL WATERPROOFING AND EXPANSION DETAILS

REFERENCE DRAWINGS

COMMONWEALTH OF PENNSYLVANIA
DEPARTMENT OF TRANSPORTATION
BUREAU OF PROJECT DELIVERY

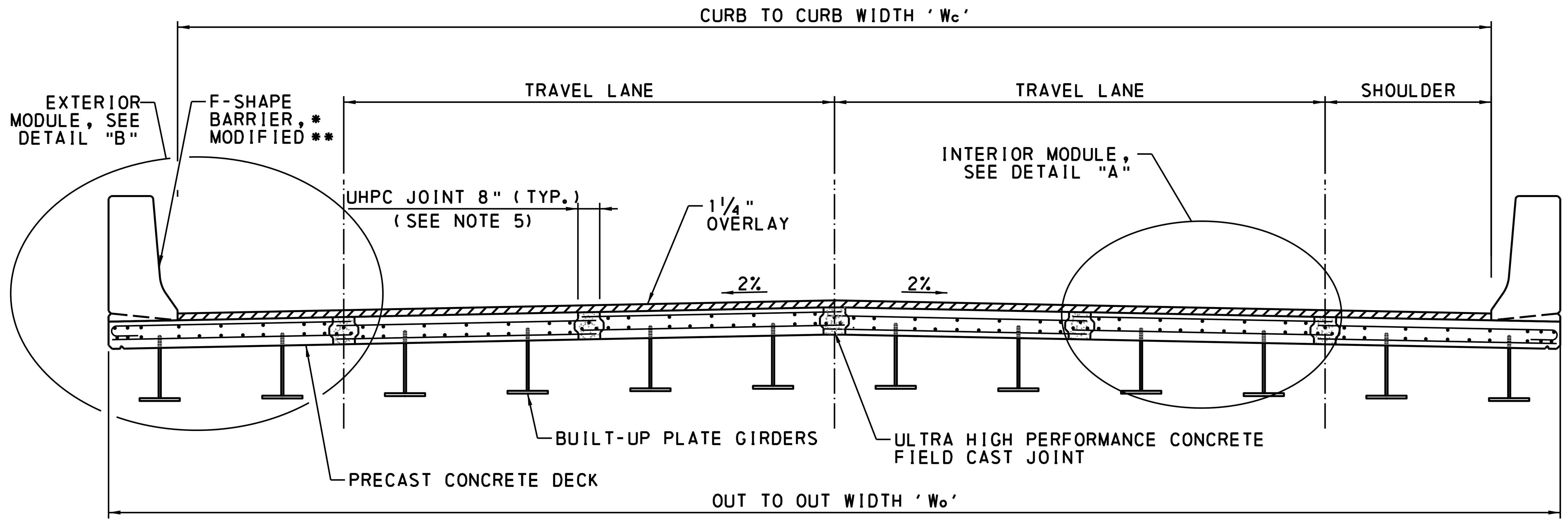
STANDARD
CONCRETE DECK SLAB DESIGN & DETAILS FOR FLEX BEAM BRIDGES
GENERAL NOTES

RECOMMENDED ________ CHIEF BRIDGE ENGINEER

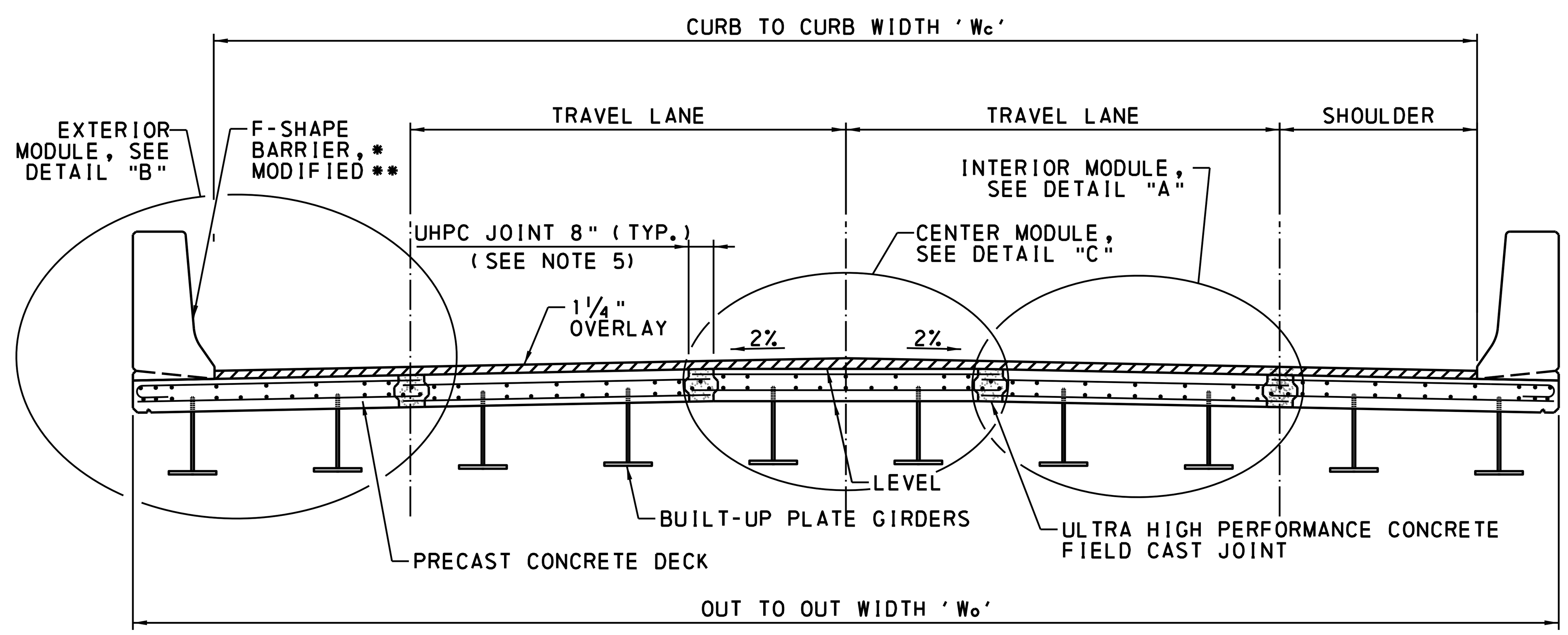
RECOMMENDED ________ DIRECTOR, BUR. OF PROJECT DELIVERY

SHEET 1 OF 6

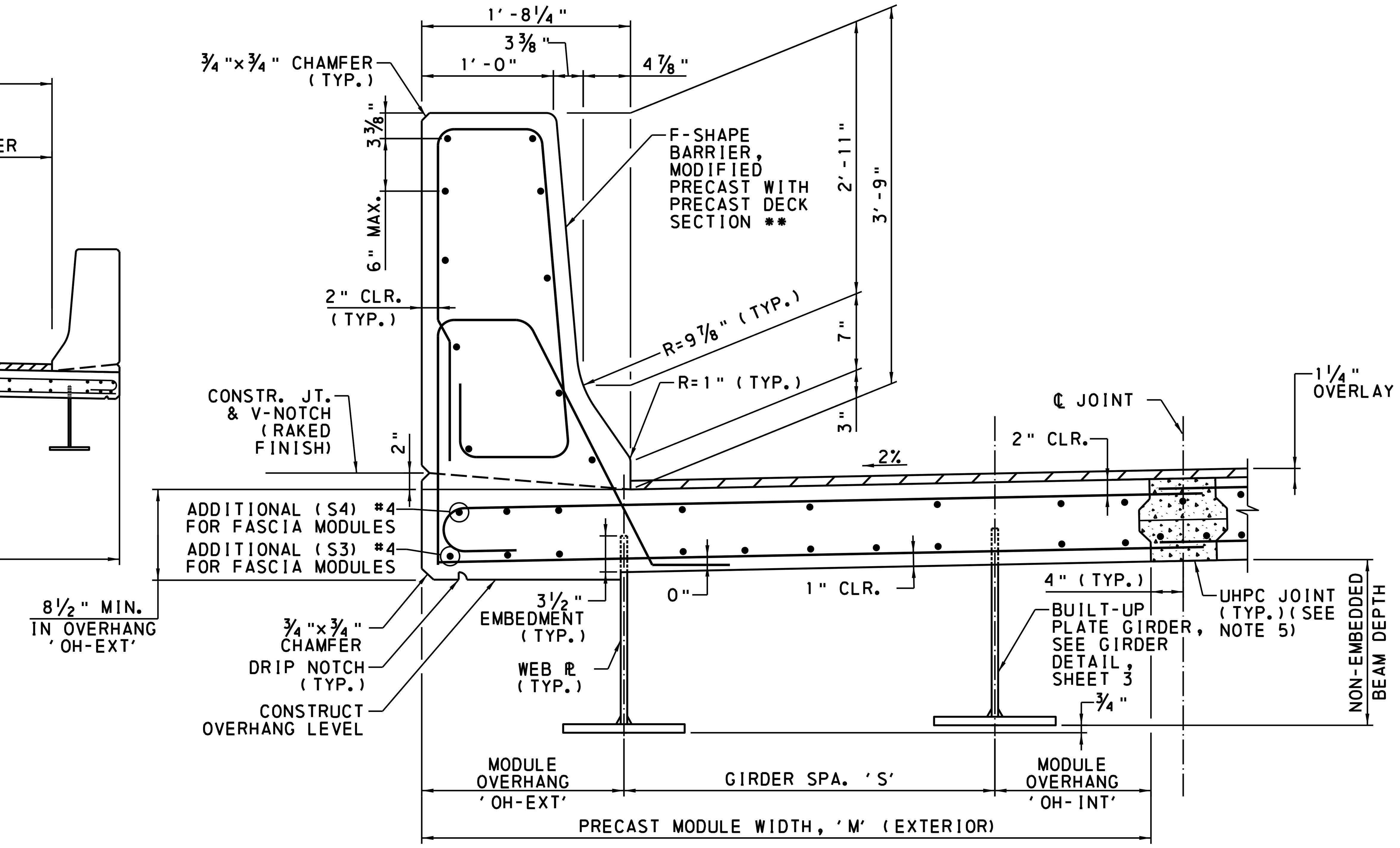
BD-663M



TYPICAL DECK SECTION - DOUBLE MODULE (EVEN NUMBER OF MODULES)

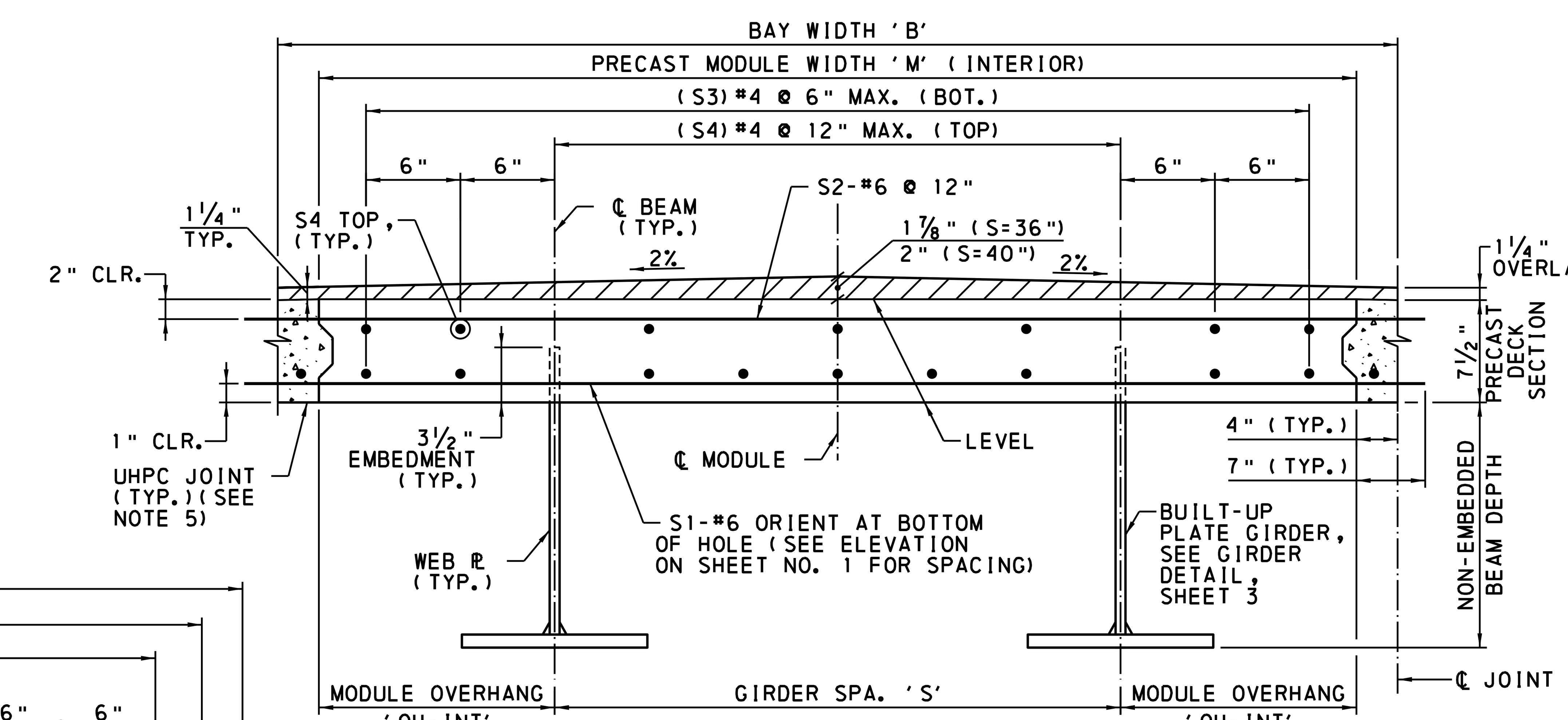


TYPICAL DECK SECTION - DOUBLE MODULE (ODD NUMBER OF MODULES)



DETAIL B

(UNLESS OTHERWISE NOTED, SEE DETAIL A, THIS SHEET FOR DECK REINFORCEMENT)



DETAIL C

LEGEND:

ULTRA HIGH PERFORMANCE CONCRETE

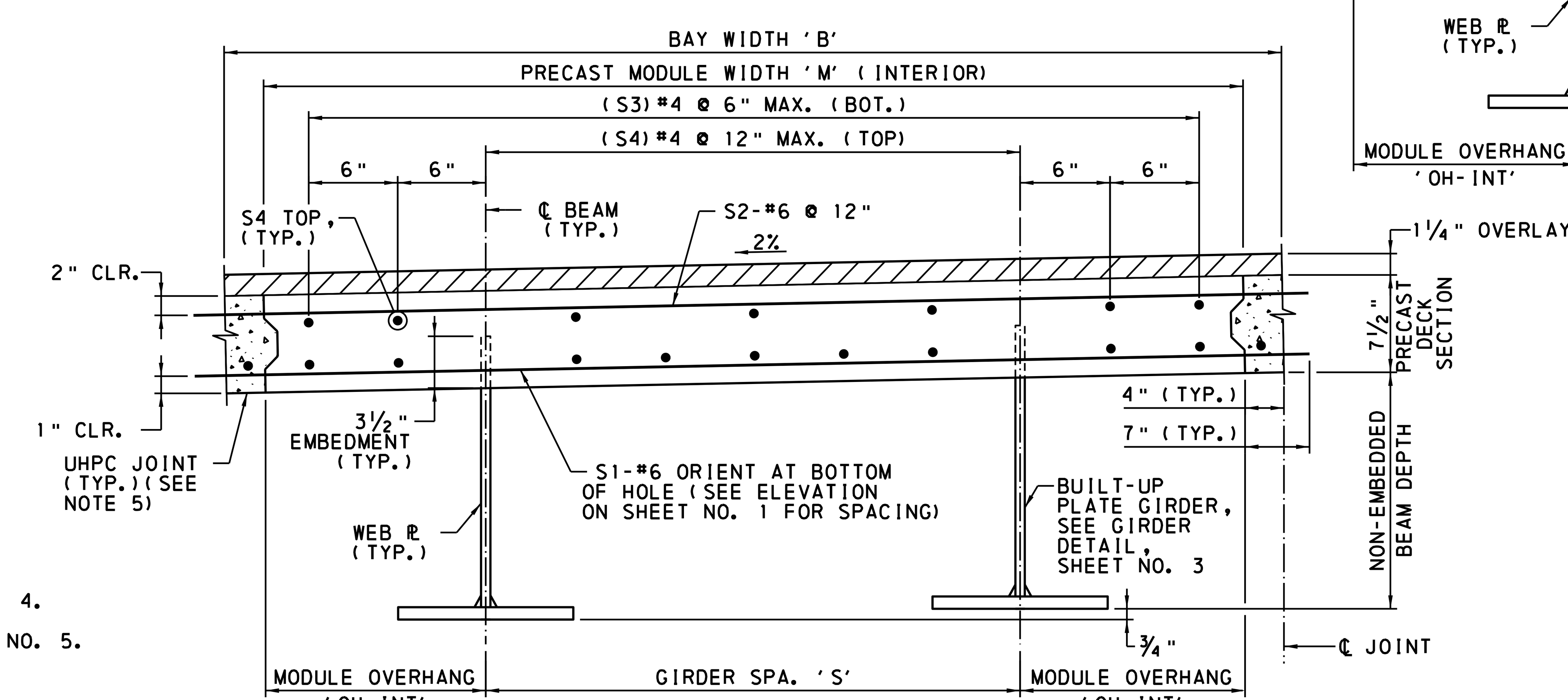
WEARING SURFACE/OVERLAY

* 45" F-SHAPED BARRIER, MODIFIED SHOWN. OTHER MASH COMPLIANT BARRIERS PERMITTED AS PER THE DISTRICT BRIDGE ENGINEER.

** REFER TO BD-601M FOR REINFORCEMENT.

NOTES:

1. FOR DESIGN TABLES, SEE SHEET NO. 3.
2. FOR STEEL END DIAPHRAGM DETAILS, SEE SHEET NO. 4.
3. FOR CONCRETE END DIAPHRAGM DETAILS, SEE SHEET NO. 5.
4. FOR GENERAL AND DESIGN NOTES, SEE SHEET NO. 1.
5. SEE LONGITUDINAL JOINT AND CLOSURE POUR DETAIL, SHEET NO. 3.



DETAIL A

COMMONWEALTH OF PENNSYLVANIA
DEPARTMENT OF TRANSPORTATION
BUREAU OF PROJECT DELIVERY

STANDARD
CONCRETE DECK SLAB DESIGN & DETAILS FOR FLEX BEAM BRIDGES
TYPICAL SECTIONS

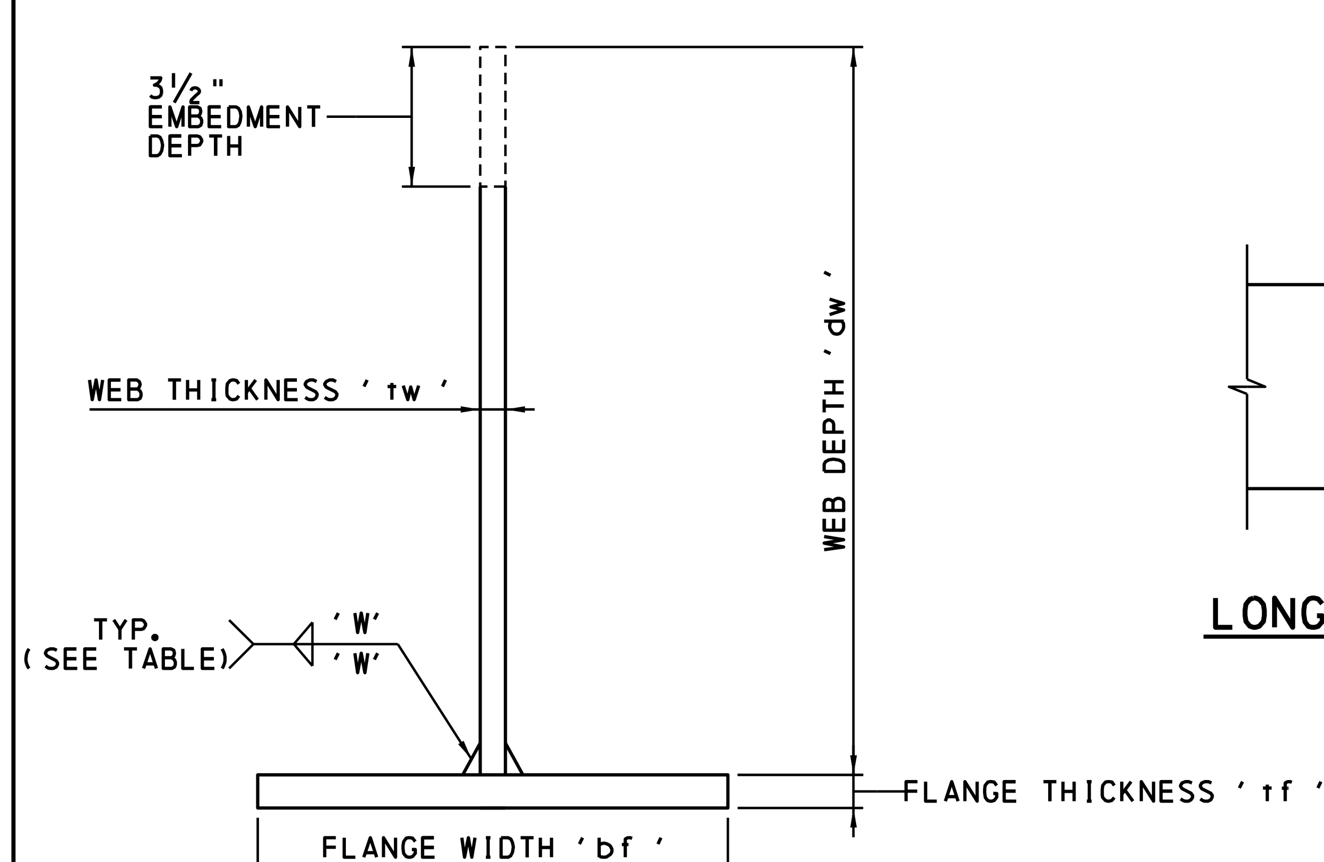
RECOMMENDED ____	RECOMMENDED ____	SHEET 2 OF 6
CHIEF BRIDGE ENGINEER	DIRECTOR, BUR. OF PROJECT DELIVERY	BD-663M

NOTES:

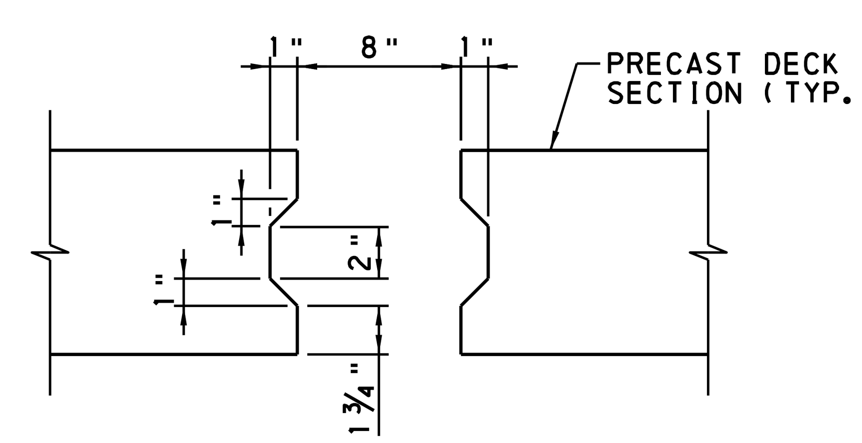
1. FOR STEEL END DIAPHRAGM DETAILS, SEE SHEET NO. 4.
2. FOR CONCRETE END DIAPHRAGM DETAILS, SEE SHEET NO. 5.
3. FOR GENERAL AND DESIGN NOTES, SEE SHEET NO. 1.
4. FOR TYPICAL SECTIONS, SEE SHEET NO. 2.
5. FOR CAMBER TABLES, SEE SHEET NO. 6.

SPAN LENGTH, 'L' (C/C BRGS., FT.)	DESIGN ROADWAY WIDTH, 'W' (FT)	GIRDER SPACING, 'S' (IN)	BAY WIDTH, 'B' (IN)	PRECAST MODULE WIDTH		NUMBER OF MODULES	OUT TO OUT WIDTH, 'Wo' (FT)	ACTUAL CURB TO CURB WIDTH, 'Wc' (FT)	OVERHANG WIDTH		STEEL PLATE GIRDER					SHEAR CONNECTOR SPACING INFORMATION		
											BOTTOM FLANGE		WEB					
				INTERIOR MODULE WIDTH, 'M' (IN)	EXTERIOR MODULE WIDTH, 'M' (IN)				INTERIOR OVERHANG 'OH-INT' (IN)	EXTERIOR OVERHANG, 'OH-EXT' (IN)	THICKNESS, 'tf' (IN)	WIDTH, 'bf' (IN)	DEPTH, 'dw' (IN)	THICKNESS, 'tw' (IN)	STIFFENER DEPTH, 'ds' (IN)	'A' SPACES @ 4"	'B' SPACES @ 8"	'C' SPACES @ 12"
30	24	36	72	64	68	5	30.00	26.63	14	18	0.500	12	17.500	0.375	13.40	25	7	2
	28	40	80	72	76	5	33.33	29.96	16	20	0.500	12	17.500	0.375	13.40	26	8	1
	32	36	72	64	68	6	36.00	32.63	14	18	0.500	12	17.500	0.375	13.40	25	7	2
	36	40	80	72	76	6	40.00	36.63	16	20	0.500	12	17.500	0.375	13.40	26	8	1
	40	40	80	72	76	7	46.67	43.29	16	20	0.500	12	17.500	0.375	13.40	26	8	1
40	24	36	72	64	68	5	30.00	26.63	14	18	0.500	12	20.500	0.375	16.40	30	12	2
	28	40	80	72	76	5	33.33	29.96	16	20	0.750	12	20.250	0.375	16.15	31	10	3
	32	36	72	64	68	6	36.00	32.63	14	18	0.500	12	20.500	0.375	16.40	30	12	2
	36	40	80	72	76	6	40.00	36.63	16	20	0.750	12	20.250	0.375	16.15	31	10	3
	40	40	80	72	76	7	46.67	43.29	16	20	0.750	12	20.250	0.375	16.15	31	10	3
50	24	36	72	64	68	5	30.00	26.63	14	18	0.625	12	24.375	0.500	20.28	30	12	7
	28	40	80	72	76	5	33.33	29.96	16	20	1.000	12	24.000	0.500	19.90	31	13	6
	32	36	72	64	68	6	36.00	32.63	14	18	0.625	12	24.375	0.500	20.28	30	12	7
	36	40	80	72	76	6	40.00	36.63	16	20	1.000	12	24.000	0.500	19.90	31	13	6
	40	40	80	72	76	7	46.67	43.29	16	20	1.000	12	24.000	0.500	19.90	31	13	6
60	24	36	72	64	68	5	30.00	26.63	14	18	0.750	12	30.250	0.625	26.15	30	15	10
	28	40	80	72	76	5	33.33	29.96	16	20	0.875	12	30.125	0.625	26.03	33	18	7
	32	36	72	64	68	6	36.00	32.63	14	18	0.750	12	30.250	0.625	26.15	30	15	10
	36	40	80	72	76	6	40.00	36.63	16	20	0.875	12	30.125	0.625	26.03	33	18	7
	40	40	80	72	76	7	46.67	43.29	16	20	0.875	12	30.125	0.625	26.03	33	18	7
70	24	36	72	64	68	5	30.00	26.63	14	18	0.750	12	33.250	0.625	29.15	38	17	11
	28	40	80	72	76	5	33.33	29.96	16	20	1.125	12	32.875	0.625	28.78	39	15	12
	32	36	72	64	68	6	36.00	32.63	14	18	0.750	12	33.250	0.625	29.15	38	17	11
	36	40	80	72	76	6	40.00	36.63	16	20	1.125	12	32.875	0.625	28.78	39	15	12
	40	40	80	72	76	7	46.67	43.29	16	20	1.125	12	32.875	0.625	28.78	39	15	12

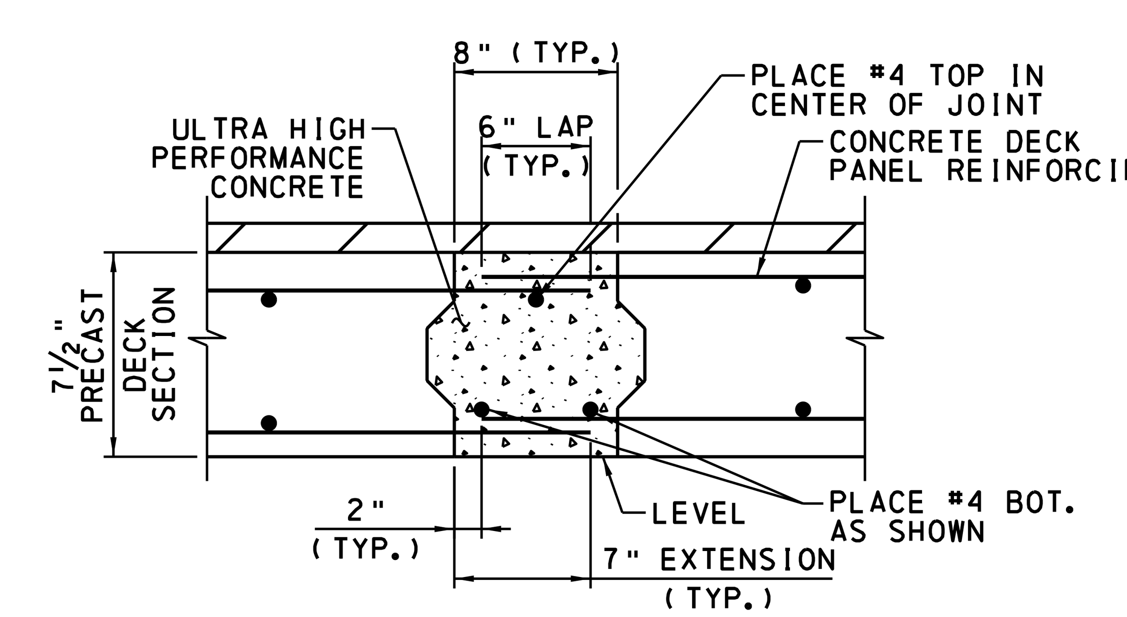
BASE METAL OF THICKER PART JOINED (T)(IN.)	'W', MIN. SIZE OF FILLET WELD (IN.)
$T \leq 3/4$	1/4
$3/4 < T$	5/16



GIRDER DETAIL



LONGITUDINAL JOINT DETAIL



LONGITUDINAL CLOSURE POUR

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DEPARTMENT OF TRANSPORTATION
BUREAU OF PROJECT DELIVERY

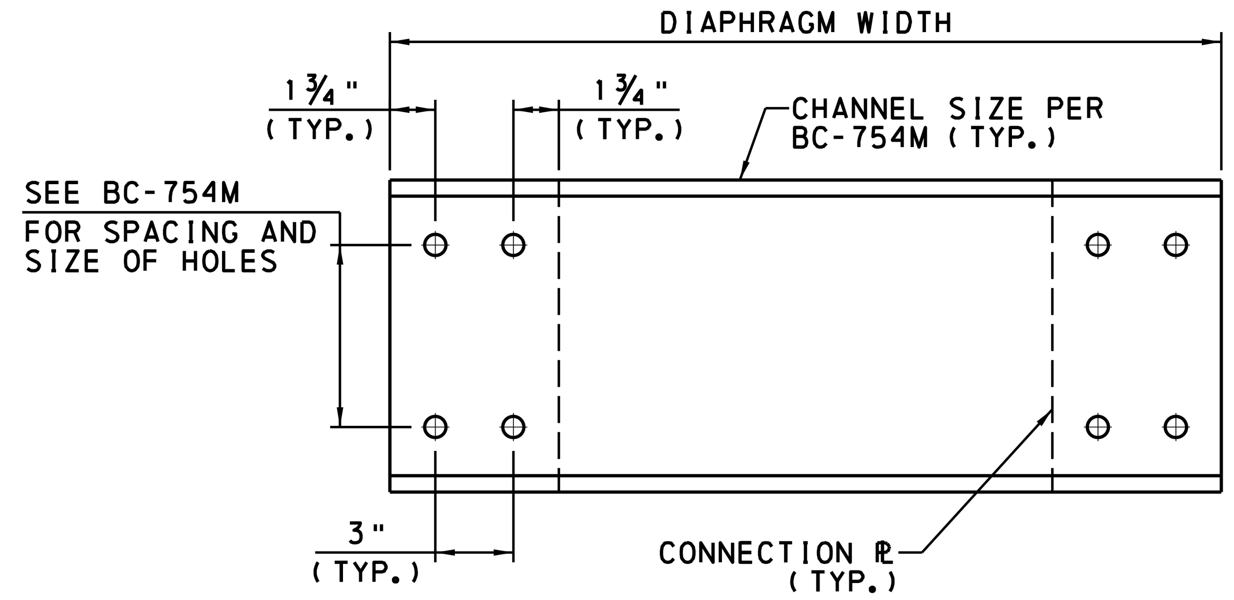
STANDARD
CONCRETE DECK SLAB DESIGN & DETAILS FOR FLEX BEAM BRIDGES
DESIGN TABLES

RECOMMENDED ________ CHIEF BRIDGE ENGINEER

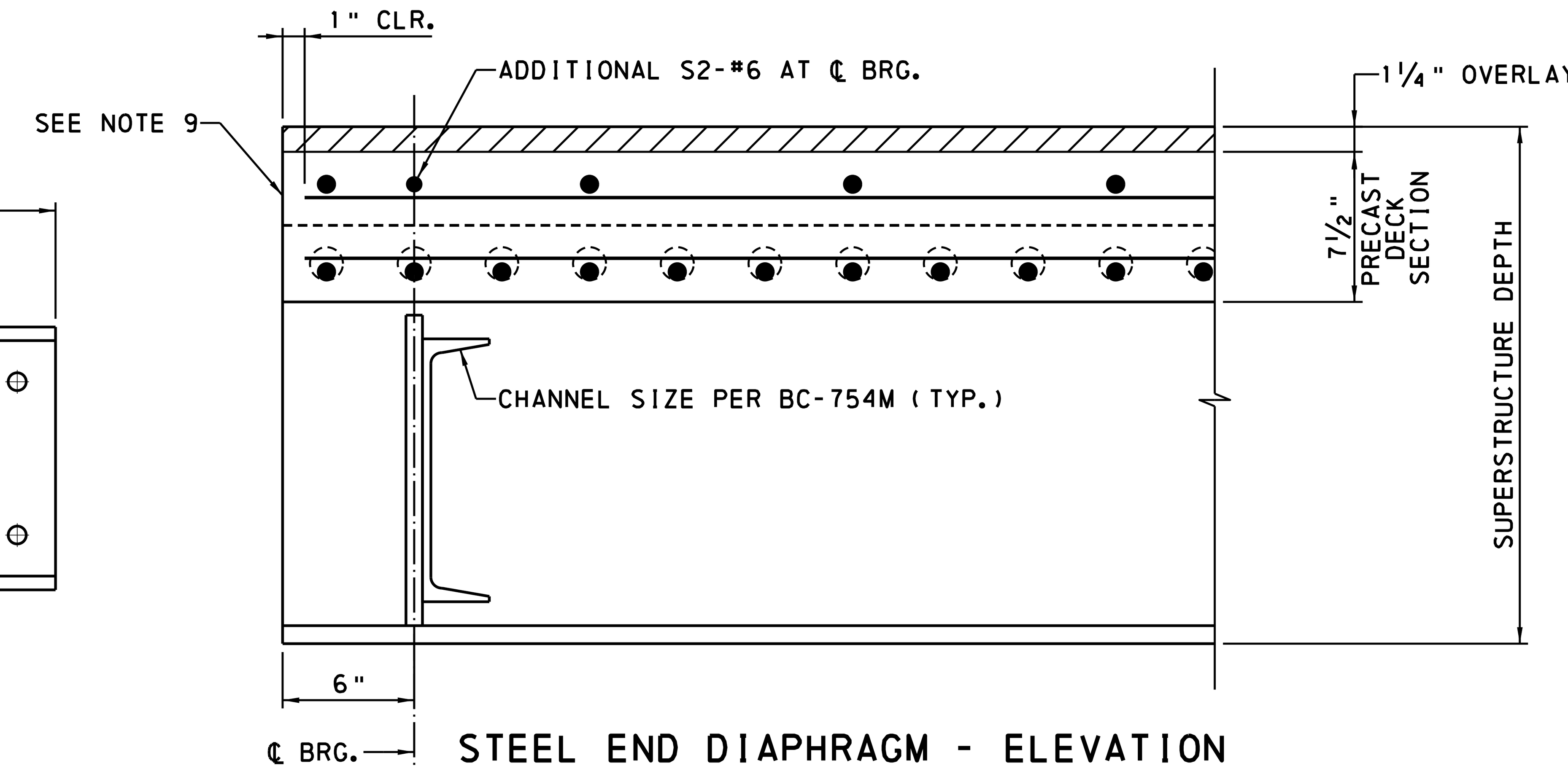
RECOMMENDED ________ DIRECTOR, BUR. OF PROJECT DELIVERY

SHEET 3 OF 6

BD-663M



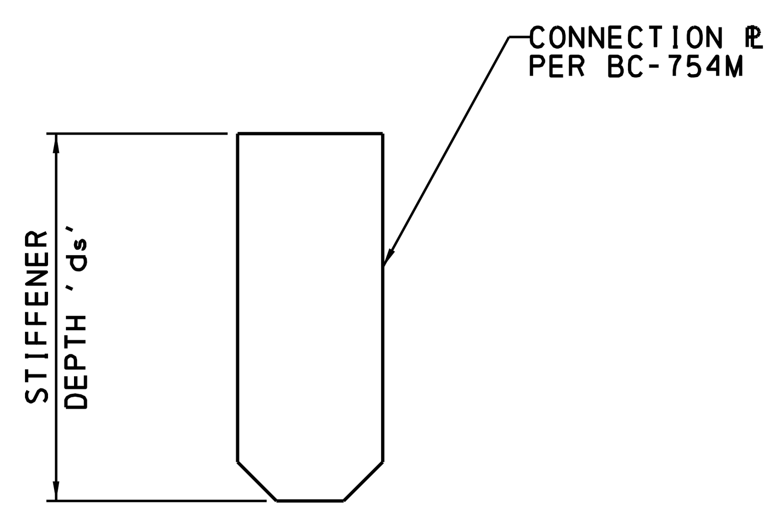
DIAPHRAGM DETAIL



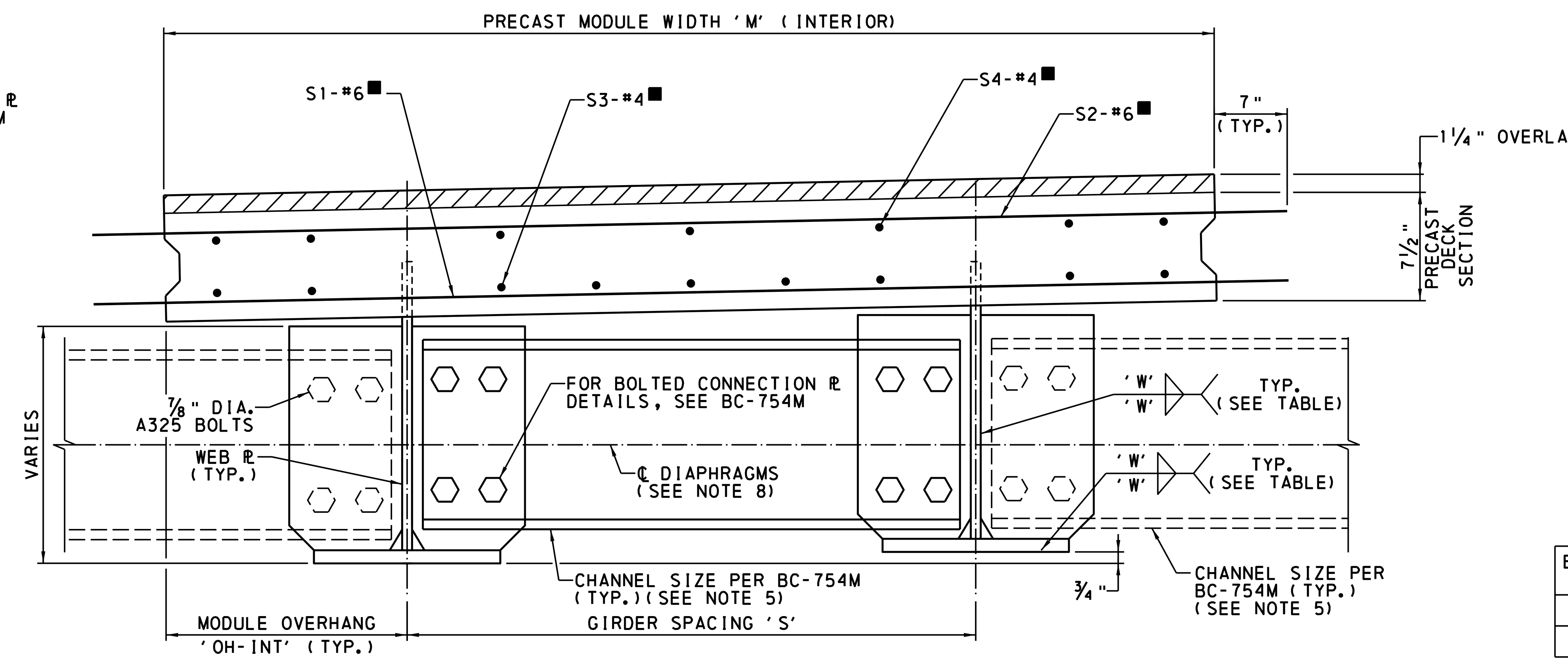
STEEL END DIAPHRAGM - ELEVATION

NOTES:

1. FOR TYPICAL SECTIONS, SEE SHEET NO. 2.
2. FOR GENERAL AND DESIGN NOTES, SEE SHEET NO. 1.
3. FOR DESIGN TABLES, SEE SHEET NO. 3.
4. FOR CAMBER TABLES, SEE SHEET NO. 6.
5. DIAPHRAGMS TO BE PLACED AND CONNECTED TO STIFFENERS PRIOR TO PLACEMENT OF PRECAST MODULES. DESIGNER TO CONFIRM DIAPHRAGM DEPTH IS SUITABLE FOR CONSTRUCTABILITY AND MAINTENANCE PURPOSES FOR EACH INDIVIDUAL DESIGN APPLICATION.
6. DIAPHRAGMS BETWEEN PRECAST MODULES TO BE PLACED AND CONNECTED TO STIFFENERS IN THE FIELD AFTER PLACEMENT OF PRECAST MODULES.
7. FOR CONCRETE END DIAPHRAGMS, SEE SHEET NO. 5.
8. PLACE STEEL END DIAPHRAGMS CENTERED VERTICALLY ABOUT THE ℄ OF THE STIFFENERS.
9. WHEN A NEOPRENE STRIP SEAL IS TO BE USED, ADJUST AND COORDINATE LOCATION OF LAST TWO TRANSVERSE BARS BEYOND ℄ BRG. TO ALLOW PLACEMENT OF STEEL EXTRUSION.



STIFFENER DETAIL



STEEL END DIAPHRAGM - SECTION

BASE METAL OF THICKER PART JOINED (T)(IN.)	'W', MIN. SIZE OF FILLET WELD (IN.)
T≤ ¾	¼
¾<T	5⁄16

LEGEND:

■ FOR SPACING AND LOCATION DETAILS, SEE SHEET NO. 2.

▨ WEARING SURFACE/OVERLAY

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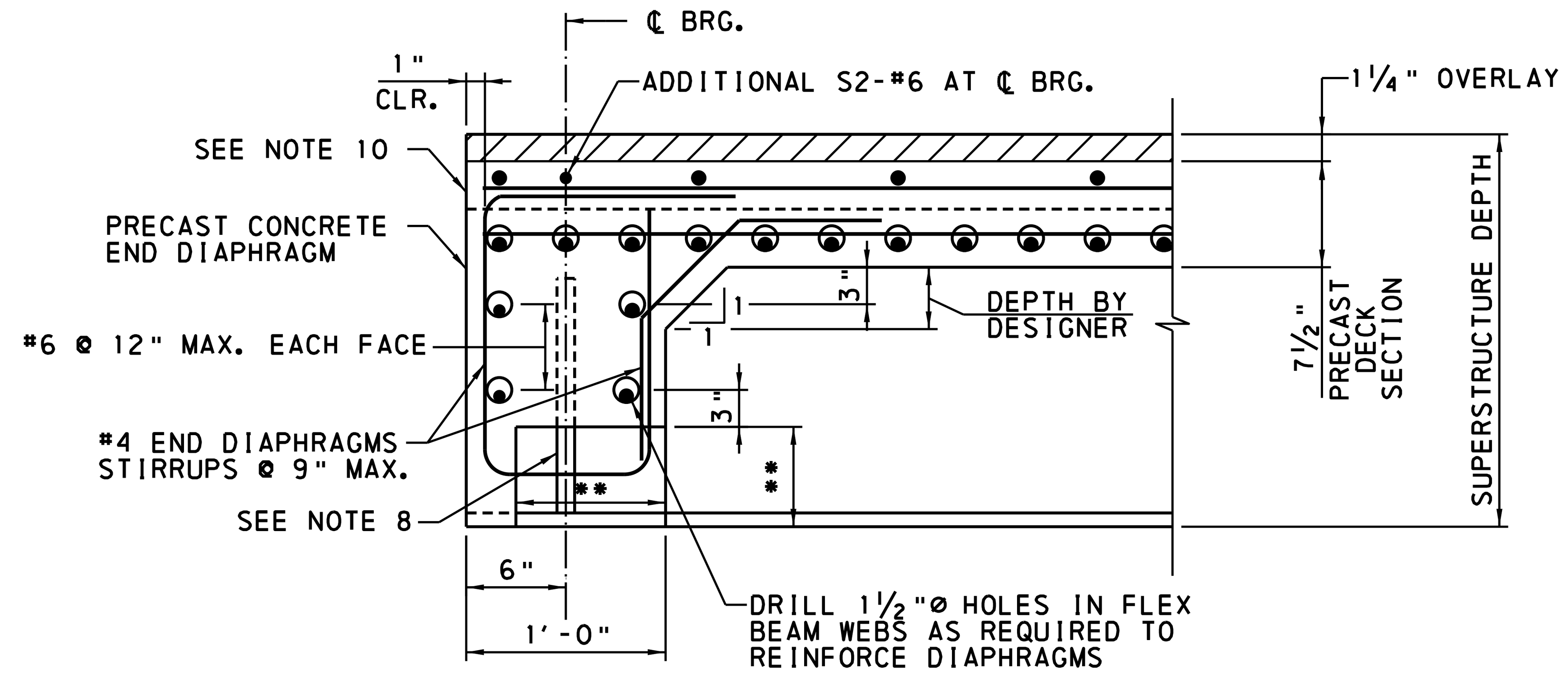
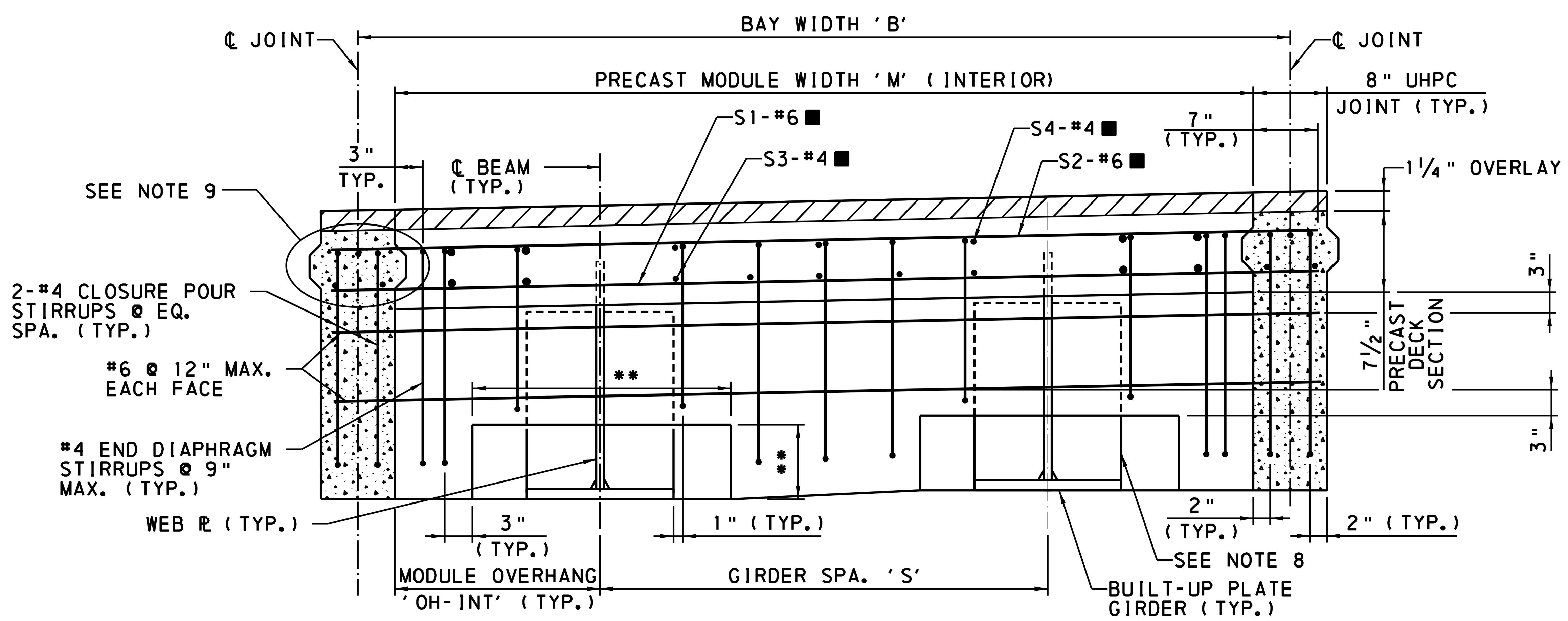
CONCRETE DECK SLAB DESIGN & DETAILS FOR FLEX BEAM BRIDGES
STEEL END DIAPHRAGMS

RECOMMENDED ________ CHIEF BRIDGE ENGINEER	RECOMMENDED ________ DIRECTOR, BUR. OF PROJECT DELIVERY	SHEET 4 OF 6
		BD-663M



CONCRETE END DIAPHRAGM - ELEVATION



CONCRETE END DIAPHRAGM - SECTION

NOTES:

1. BLOCKOUT SIZE TO BE DETERMINED BY THE ENGINEER.
2. FOR TYPICAL SECTIONS, SEE SHEET NO. 2.
3. FOR GENERAL AND DESIGN NOTES, SEE SHEET NO. 1.
4. FOR DESIGN TABLES, SEE SHEET NO. 3.
5. FOR CAMBER TABLES, SHEET NO. 6.
6. CONCRETE END DIAPHRAGMS TO BE PRECAST WITH THE MODULES. ULTRA HIGH PERFORMANCE CONCRETE LONGITUDINAL JOINT TO BE CAST IN THE FIELD AFTER PLACEMENT OF PRECAST MODULES.
7. FOR STEEL END DIAPHRAGMS, SEE SHEET NO. 4.
8. FOR STIFFENER DETAIL, SEE SHEET NO. 4.
9. FOR REINFORCEMENT OF LONGITUDINAL DECK CLOSURE POUR, SEE SHEET NO. 3.
10. WHEN A NEOPRENE STRIP SEAL IS TO BE USED, ADJUST AND COORDINATE LOCATION OF LAST TWO TRANSVERSE BARS BEYOND ℄ BRG. TO ALLOW PLACEMENT OF STEEL EXTRUSION.

LEGEND:

■	FOR SPACING AND LOCATION DETAILS, SEE SHEET NO. 2.
**	FORMED BLOCKOUT FOR BEARING (SEE NOTE 1) (TYP.)
(stippled)	ULTRA HIGH PERFORMANCE CONCRETE
(hatched)	WEARING SURFACE/OVERLAY

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STANDARD
CONCRETE DECK SLAB DESIGN &
DETAILS FOR FLEX BEAM BRIDGES
CONCRETE END DIAPHRAGMS

RECOMMENDED ________	RECOMMENDED ________	SHEET 5 OF 6
CHIEF BRIDGE ENGINEER	DIRECTOR, BUR. OF PROJECT DELIVERY	BD-663M

CAMBER AND THEORETICAL DEAD LOAD DEFLECTIONS, INCHES - FOR L = 30' SPAN																						
DEFLECTION / CAMBER	LOCATION																					
	0		0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0	
GIRDER SPACING 'S' (IN.)	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40
DEAD LOAD DEFLECTIONS (TYPE 1)	0	0	1/16	1/16	1/16	1/16	1/8	1/8	1/8	1/8	1/8	3/16	1/8	1/8	1/8	1/8	1/16	1/16	1/16	1/16	0	0
DEAD LOAD DEFLECTIONS (TYPE 2)	0	0	1/16	1/16	1/16	1/16	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/16	1/16	1/16	1/16	0	0
VERTICAL PROFILE CAMBER (NOTE A)																						
TOTAL CAMBER (NOTE B)																						

CAMBER AND THEORETICAL DEAD LOAD DEFLECTIONS, INCHES - FOR L = 40' SPAN																						
DEFLECTION / CAMBER	LOCATION																					
	0		0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0	
GIRDER SPACING 'S' (IN.)	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40
DEAD LOAD DEFLECTIONS (TYPE 1)	0	0	1/8	1/8	3/16	3/16	5/16	1/4	5/16	5/16	3/8	5/16	5/16	5/16	5/16	1/4	3/16	3/16	1/8	1/8	0	0
DEAD LOAD DEFLECTIONS (TYPE 2)	0	0	1/8	1/16	3/16	3/16	1/4	3/16	5/16	1/4	5/16	1/4	5/16	1/4	1/4	3/16	3/16	3/16	1/8	1/16	0	0
VERTICAL PROFILE CAMBER (NOTE A)																						
TOTAL CAMBER (NOTE B)																						

CAMBER AND THEORETICAL DEAD LOAD DEFLECTIONS, INCHES - FOR L = 50' SPAN																						
DEFLECTION / CAMBER	LOCATION																					
	0		0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0	
GIRDER SPACING 'S' (IN.)	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40
DEAD LOAD DEFLECTIONS (TYPE 1)	0	0	3/16	1/8	5/16	5/16	7/16	3/8	1/2	7/16	9/16	1/2	1/2	7/16	7/16	3/8	5/16	5/16	3/16	1/8	0	0
DEAD LOAD DEFLECTIONS (TYPE 2)	0	0	1/8	1/8	5/16	1/4	3/8	5/16	7/16	3/8	1/2	3/8	7/16	3/8	3/8	5/16	5/16	1/4	1/8	1/8	0	0
VERTICAL PROFILE CAMBER (NOTE A)																						
TOTAL CAMBER (NOTE B)																						

CAMBER AND THEORETICAL DEAD LOAD DEFLECTIONS, INCHES - FOR L = 60' SPAN																						
DEFLECTION / CAMBER	LOCATION																					
	0		0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0	
GIRDER SPACING 'S' (IN.)	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40
DEAD LOAD DEFLECTIONS (TYPE 1)	0	0	3/16	3/16	3/8	3/8	9/16	9/16	5/8	5/8	11/16	11/16	5/8	5/8	9/16	9/16	3/8	3/8	3/16	3/16	0	0
DEAD LOAD DEFLECTIONS (TYPE 2)	0	0	3/16	3/16	5/16	5/16	7/16	7/16	1/2	1/2	9/16	1/2	1/2	1/2	7/16	7/16	5/16	5/16	3/16	3/16	0	0
VERTICAL PROFILE CAMBER (NOTE A)																						
TOTAL CAMBER (NOTE B)																						

CAMBER AND THEORETICAL DEAD LOAD DEFLECTIONS, INCHES - FOR L = 70' SPAN																						
DEFLECTION / CAMBER	LOCATION																					
	0		0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0	
GIRDER SPACING 'S' (IN.)	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40	36	40
DEAD LOAD DEFLECTIONS (TYPE 1)	0	0	5/16	5/16	5/8	9/16	13/16	3/4	15/16	7/8	1	15/16	15/16	7/8	13/16	3/4	5/8	9/16	5/16	5/16	0	0
DEAD LOAD DEFLECTIONS (TYPE 2)	0	0	1/4	1/4	1/2	7/16	11/16	9/16	13/16	11/16	13/16	11/16	13/16	11/16	11/16	9/16	1/2	7/16	1/4	1/4	0	0
VERTICAL PROFILE CAMBER (NOTE A)																						
TOTAL CAMBER (NOTE B)																						

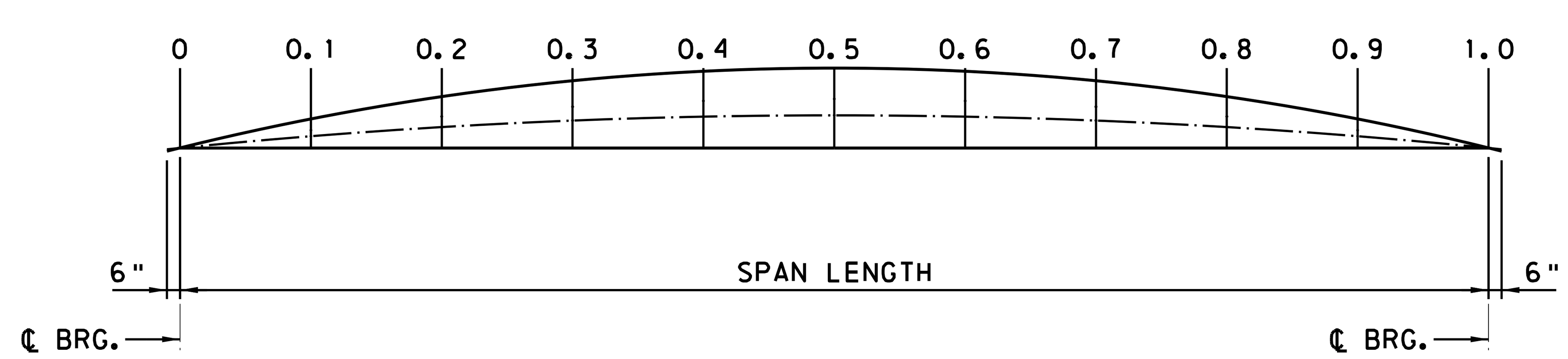
CAMBER DEFLECTION NOTES:

TYPE 1 = THEORETICAL DEAD LOAD DEFLECTION CALCULATED USING THE LONG TERM MOMENT OF INERTIA DUE TO SELF-WT OF STEEL BEAM, (INCLUDING DIAPHRAGMS), AND THE TRIBUTARY WIDTH OF THE DECK (ASSUMING SHORED CONSTRUCTION). ALSO INCLUDES DEAD LOAD DEFLECTION USING THE LONG TERM MOMENT OF INERTIA DUE TO SELF-WT OF 1¼" WEARING SURFACE/OVERLAY PLACED AT TIME OF CONSTRUCTION.

TYPE 2 = THEORETICAL DEAD LOAD DEFLECTION CALCULATED USING THE LONG TERM MOMENT OF INERTIA DUE TO SELF-WT OF BARRIER (ASSUMING SHORED CONSTRUCTION). NOTE THAT TYPE 2 DEAD LOAD DEFLECTIONS ARE ONLY APPLICABLE TO THE OUTERMOST FLEX BEAM UNITS (FASCIA GIRDER AND FIRST INTERIOR GIRDER).

NOTE A: AS REQUIRED BY DESIGN, TO BE PROVIDED BY FINAL DESIGN ENGINEER.

NOTE B: TOTAL CAMBER TO INCLUDE CAMBER REQUIRED FOR DEAD LOAD DEFLECTIONS TYPE 1 AND TYPE 2, AND FOR VERTICAL PROFILE, AS REQUIRED BY DESIGN.



NO-LOAD CAMBER TOLERANCE = 0 TO +0.5 IN. AT MIDSPAN. POSITIVE CAMBER IS UPWARD.

———— INITIAL CAMBER - THE INITIAL, NO LOAD CAMBER.

—·—·— LONG-TERM CAMBER - THE ACTUAL PROFILE OF THE COMPOSITE SPAN UNDER DEAD LOAD OF STEEL WT AND CONCRETE DECK USING THE LONG TERM MOMENT OF INERTIA.

NOTES:

1. FOR GENERAL AND DESIGN NOTES, SEE SHEET NO. 1.
2. FOR DESIGN TABLES, SEE SHEET NO. 3.
3. DEAD LOAD OF STEEL T COMPONENTS AND CONCRETE SLAB IS CARRIED ON A COMPOSITE SECTION.

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STANDARD
CONCRETE DECK SLAB DESIGN & DETAILS FOR FLEX BEAM BRIDGES
BEAM CAMBER

RECOMMENDED ________	RECOMMENDED ________	SHEET 6 OF 6
CHIEF BRIDGE ENGINEER	DIRECTOR, BUR. OF PROJECT DELIVERY	BD-663M