PennDOT Truss Gusset Plate Analysis and Ratings Spreadsheet

Method of Solution for Operating Level Rating

Date: September 25, 2008

1 Notes

- AASHTO Standard Specifications for Highway Bridges, 17th Ed. ("AASHTO")
- AISC Manual of Steel Construction - ASD, 9th Ed. ("ASD AISC")
- Analysis and rating for Allowable Stress and Load Factor Design Methods
- The LFD loads have been factored per Group I in accordance with the Guide Spec. Article 1.2. Only the analysis checks utilize these factored loads

2 General

\[ F_y = \text{specified minimum yield stress of the gusset plate material, ksi} \]
\[ F_u = \text{specified minimum tensile stress of the gusset plate material, ksi (1.4 } F_y \text{ is used as a default value if } F_u \text{ was not input)} \]

3 Allowable Shear

(Tab: Shear@Sec.A-A, …C-C)

- Gusset plates subjected to shear shall be investigated for two conditions: gross shear yielding and net section fracture. The shear capacity is the lesser of the two conditions.
- A factor of 0.74 will be used for the shear stress distribution.

\[ V_g = 0.45 F_y \times 0.74 = 0.33 F_y \text{ allowable stress, ksi MCEB Table 6.6.2.1-2} \]
\[ P_{v,g} = V_g \times A_g = \text{allowable axial force, kips} \]

\[ A_g = \text{area of the gusset plate resisting the shear, in}^2 \]
\[ 0.74 = \text{reduction factor for the flexural shear stress distribution} \]

\[ V_n = 0.45 F_u \times 0.74 = 0.33 F_u \text{ allowable stress, ksi MCEB Table 6.6.2.1-2} \]
\[ P_{v,n} = V_n \times A_n = \text{allowable axial force, kips} \]
where:
\[ A_n = \text{area of the gusset plate (less holes) resisting the shear, in}^2 \]
\[ 0.74 = \text{reduction factor for the flexural shear stress distribution} \]

LFD

Gross Shear Yielding:
\[ F_{v,g} = 0.58 F_y \times 0.74 = 0.42 F_y = \text{shear stress capacity, ksi} \quad \text{FHWA Guidance Eq. 6} \]
\[ P_{v,g} = F_{v,g}A_g = \text{axial force capacity, kips} \]
where:
\[ A_g = \text{area of the gusset plate resisting the shear, in}^2 \]
\[ 0.74 = \text{reduction factor for the flexural shear stress distribution} \]

Net Section Fracture:
\[ F_{v,n} = 0.85 \times 0.58 F_u = 0.49 F_u = \text{shear stress capacity, ksi} \quad \text{FHWA Guidance Eq. 7} \]
\[ P_{v,n} = F_{v,n}A_n = \text{axial force capacity, kips} \]
where:
\[ A_n = \text{area of the gusset plate (less holes) resisting the shear, in}^2 \]
\[ 0.85 = \text{resistance factor for shear fracture on the net section} \]

4 Capacity Equations to Analyze Gusset Plate at End of All Members for Tension and at the End of the Diagonal and Vertical for Compression

(Tab: Ten.&Comp.)

4.1 Gusset Plate In Tension

- Gusset plates subjected to tension shall be investigated for two conditions: yield on the gross section and fracture on the net section. The capacity in tension will be the least of these two values.

ASD

Yield on the Gross Section:

- Use the Whitmore method (i.e. 30° distribution) to determine the “effective width” to be used to calculate the gross area in tension.(See Figure 9)

\[ P_t = 0.75 F_y A_g \quad \text{MCEB Table 6.6.2.1-2} \]

where:
\[ A_g = \text{gross area in tension, in}^2 \]
\[ \quad = \text{effective width x gusset plate thickness} \]
Fracture on Net Section:

- Use the Whitmore method (i.e. 30° distribution) to determine the "effective width" to be used to calculate the net area in tension. (See Figure 9)

\[ P_t = 0.60 F_u A_n \]  

where:
\[ A_n = \text{net area in tension, in}^2 \]
\[ = \text{effective width x gusset plate thickness} \]

LFD

Yield on the Gross Section:

- Use the Whitmore method (i.e. 30° distribution) to determine the "effective width" to be used to calculate the gross area in tension. (See Figure 9)

When \( A_g \leq A_n + \beta A_g \)

\[ P_t = F_y A_g \]

where:
\[ A_g = \text{gross area in tension, in}^2 \]
\[ = \text{effective width x gusset plate thickness} \]
\[ A_n = \text{net area in tension, in}^2 \]
\[ \beta = 0.0 \text{ for M 270 Grade 100/100W steels, or when holes exceed 1\frac{1}{4} \text{ inch in diameter}} \]
\[ = 0.15 \text{ for all other steels and when holes are less than or equal to 1\frac{1}{4} \text{ inch in diameter}} \]

Fracture on Net Section:

- Use the Whitmore method (i.e. 30° distribution) to determine the "effective width" to be used to calculate the net area in tension. (See Figure 9)

When \( A_g > A_n + \beta A_g \)

\[ P_t = F_y (A_n + \beta A_g) \]

where:
\[ A_g = \text{gross area in tension, in}^2 \]
\[ = \text{effective width x gusset plate thickness} \]
\[ A_n = \text{net area in tension, in}^2 \]
\[ \beta = 0.0 \text{ for M 270 Grade 100/100W steels, or when holes exceed 1\frac{1}{4} \text{ inch in diameter}} \]
\[ = 0.15 \text{ for all other steels and when holes are less than or equal to 1\frac{1}{4} \text{ inch in diameter}} \]
4.2 Gusset Plate in Compression

- Gusset plates subjected to compression will be analyzed as an idealized column. (Note: This result is generally conservative when compared to the capacity obtained using plate buckling theory.)
- Investigate the adequacy of the gusset plate at the end of the diagonal due to a compressive diagonal axial force.
- The assumed failure mode in compression is buckling at the end of the compression diagonal, therefore, check compression on the effective width line (Whitmore method, i.e. 30° distribution) at the end of the diagonal. (See Figure 9)
- According to FHWA Guidance, the effective width of the idealized compression member may be determined in accordance with the Whitmore method. The unbraced length, $L_c$, may be determined as the average of three distances ($L_{CB1}$, $L_{CB2}$, $L_{CB3}$) as follows:

Where:

$$ L_{CB2} = \text{distance from the last row of fasteners in the compression member under consideration to the first row of fasteners in the closest adjacent member, measured along the line of action of the compressive axial force (See Figure 9)} $$

$$ L_{CB1}, L_{CB3} = \text{distance from each of the ends of the Whitmore width to the first row of fasteners in the closest adjacent member, measured parallel to the line of action of the compressive axial force. When the Whitmore width enters into the adjacent member, the associated distance at that end should be set to zero (See Figure 9)} $$

- ASD

- Use the allowable stresses provided in AASHTO Standard Specifications Table 10.32.1A for compression in concentrically loaded columns.

$$ P_c = \text{compression strength along effective width based on } A_g F_a, \text{ kips} $$

where:

$$ A_g = \text{effective width x gusset plate thickness, in}^2 $$

- Determine the compressive capacity, $F_a$, using column theory:

When $KL_c/r \leq C_c$

$$ F_a = \left( \frac{F_y}{(F.S.)} \right) [1 - (KL_c/r)^2] \frac{F_y}{(4\pi^2 E)} \text{ MCEB Table 6.6.2.1-2 Eq. 1} $$

When $KL_c/r > C_c$

$$ F_a = \frac{\pi^2 E}{[(F.S.) (KL_c/r)^2]} \text{ MCEB Table 6.6.2.1-2 Eq. 2} $$

where:

$$ K = \text{effective length factor, } K, \text{ used in the column capacity equation for the unsupported length at the end of the compression diagonal is taken to be 2.0 if the gusset plate analysis shows that the plate has yielded due to shear on Section A-A, otherwise it is taken as 1.2. (See AASHTO Standard Specifications Appendix C Table C.1)} $$

$$ L_c = L_{CB1,4,5} = \text{distance between the last row of fasteners in the diagonal / vertical to the closest row of fasteners in the chord measured along the line of action of the diagonal / vertical, in.} $$
\[ r = \text{radius of gyration of the plate, in.} \]
\[ = t / \sqrt{12} \]

\[ F.S. = \text{factor of safety taken as 1.70 from MCEB Table 6.6.2.1-2} \]

\[ E = \text{modulus of elasticity of steel, ksi} \]

\[ C_c = \sqrt{(2 \pi^2 E / F_y)} \]

\[ L_c = L_{c3,4,or5} = \text{distance between the last row of fasteners in the diagonal / vertical to the closest row of fasteners in the chord measured along the line of action of the diagonal / vertical, in.} \]

**LFD**

- Use the capacity equation provided in AASHTO Standard Specifications Article 10.54.1.1 for compression in concentrically loaded columns.

\[ P_c = \text{compression strength along effective width based on } A_g F_a, \text{ kips} \]

where:

\[ A_g = \text{effective width x gusset plate thickness, in}^2 \]

- Determine the compressive capacity, \( F_a \), using column theory:

\[ F_a = 0.85 F_{cr} \quad \text{AASHTO Eq. 10-150} \]

When \( K L_c / r \leq C_c \)

\[ F_{cr} = F_y \left[ 1 - \left( F_y / (4 \pi^2 E) \right) (K L_c / r)^2 \right] \quad \text{AASHTO Eq. 10-151} \]

When \( K L_c / r > C_c \)

\[ F_{cr} = \pi^2 E / (K L_c / r)^2 \quad \text{AASHTO Eq. 10-153} \]

where:

\[ K = \text{effective length factor, } K, \text{ used in the column capacity equation for the unsupported length at the end of the compression diagonal is taken to be 2.0 if the gusset plate analysis shows that the plate has yielded due to shear on Section A-A, otherwise it is taken as 1.2. (See AASHTO Standard Specifications Appendix C Table C.1)} \]

\[ C_c = \sqrt{(2 \pi^2 E / F_y)} \]
5 Block Shear Rupture Capacity for Any Member Connection at a Joint

(Tab: Block Shear)

- Assume all cross-sectional elements are connected to transmit the tensile force (i.e. reduction coefficient, \( U = 1.0 \)).
- See Figure 8 for assumed block shear rupture planes.
- The "1.25" factor is used to convert the capacity to an Operating Level capacity and was determined by dividing the Inventory Euler buckling Factor of Safety (2.12) by the Operating Euler buckling Factor of Safety (1.70). (Refer to AASHTO Manual for Condition Evaluation of Bridges Section 6.6.2.1-1)

- **ASD**

\[
P_{bs} = A_v F_v + A_t F_t \quad \text{ASD AISC Chapter J4}
\]

where:
- \( A_v = \) net shear area, \( \text{in}^2 \)
  \[= \text{total length along shear edge, } L_v \times \text{gusset plate thickness} \]
- \( F_v = \) allowable shear rupture, ksi
  \[= 1.25 \times (0.3) F_u \quad \text{Modified ASD AISC Eq. J4-1} \]
- \( A_t = \) net tension area, \( \text{in}^2 \)
  \[= \text{total length along tension edge, } L_t \times \text{gusset plate thickness} \]
- \( F_t = \) allowable tension rupture, ksi
  \[= 1.25 \times (0.5) F_u \quad \text{Modified ASD AISC Eq. J4-2} \]

- **LFD**

If \( A_{tn} \geq 0.58 A_{vn} \)

\[
P_{bs} = 0.85 \times (0.58 F_y A_{vg} + F_u A_{tn}) \quad \text{FHWA Guidance Eq. 4}
\]

where:
- \( A_{vg} = \) gross area along the plane resisting shear stress, \( \text{in}^2 \)
- \( A_{tn} = \) net area along the plane resisting tension stress, \( \text{in}^2 \)
- 0.85 = resistance factor for block shear

Otherwise,

\[
P_{bs} = 0.85 \times (F_u A_{vn} / \sqrt{3} + F_y A_{tg}) \quad \text{FHWA Guidance Eq. 5}
\]

where:
- \( A_{vn} = \) net area along the plane resisting shear stress, \( \text{in}^2 \)
- \( A_{tg} = \) gross area along the plane resisting tension stress, \( \text{in}^2 \)
- 0.85 = resistance factor for block shear
6 Connection Capacities

- In typical connection design, the connection is designed for the average of the design strength of the diagonals and the factored/service loads they carry, however, the axial force shall not be less than 75% of the design strength. The design strength is the lesser of the member strength, column capacity and strength based on the width-thickness ratio \( b / t \). For simplicity, check the adequacy of the fasteners under the applied axial load.
- Assume the member forces are distributed equally among all fasteners.
- Slip-critical connections are assumed for normal truss connection designs, however, if rivets or low-carbon steel bolts (A307) are specified on the plans, the connection is bearing-type, only high strength bolts can support the high preload necessary for slip-critical connections. In any case, bearing should always be checked in the event that the connection slips due to insufficient tension in the fasteners.
- Bearing strength of the fastener has not been checked as a part of this spreadsheet.

6.1 Investigate the Fasteners in Shear

- ASD
  - \( P \leq R_v \) must be satisfied.
  - \( R_v = \) capacity of all bolts or rivets in connection in shear, kips
    - \( = F_{v,\text{ASD}} A_f N_f \)

  where:
  - \( F_{v,\text{ASD}} = \) shear capacity of one bolt (MCEB Table 6.6.2.1-4 may be used) or rivet (MCEB Table 6.6.2.1-3 may be used), ksi
  - \( A_f = \) area of one fastener, bolt or rivet, in\(^2\)
  - \( N_{T#} = \) total number of fasteners connecting the individual member to the gusset plate
    - \( = N_{L#} x N_{is} \) when not entered under optional input information
  - \( N_t = \) total number of shear planes in fasteners connecting the chord member to the gusset and outside/inside splice plates
    - \( = N_{T#} + N_{is} N_{is} \) for chord member
    - \( = N_{T#} \) for vertical and diagonal member

Where:
- \( L_s = \) distance between the first row of fasteners in member # and the last row of fasteners in member #, in.
- \( l_{is} = \) inside splice plate length, in. (See Figure 6)
- \( N_{L#} = \) total number of fasteners along the length of member # (See Figure 1-4)
- \( N_{is} = \) total number of fasteners along the width of member # (See Figure 1-4)
- \( N_{ls} = \) total number of shear planes in fasteners connecting the chord member to the inside splice plate
  - \( = N_{L#} (0.5 l_{s1} + 0.5 l_{is}) / (L_s + 4 d) \)
- \( d = \) nominal diameter of the fastener, in.
**LFD**

\[ P \leq R_v \text{ must be satisfied.} \]

\[ R_v = \text{shear capacity of all bolts or rivets in connection from AASHTO Article 10.56.1.3.2, kips} \]

\[ = F_{v,LFD} A_f N_f \text{ AASHTO Eq. 10-166a} \]

where:

\[ F_{v,LFD} = \text{shear capacity of one bolt or rivet (AASHTO Table 10.56A may be used), ksi} \]

\[ A_f = \text{area of one fastener, bolt or rivet, in}^2 \]

\[ N_f = \text{total number of shear planes in fasteners connecting the chord member to the gusset and inside splice plates} \]

(See \( N_f \) description under ASD for more information)

### 6.2 Investigate Bearing on the Connected Material

- Assume holes are standard size.
- The allowable bearing force for the connection is equal to the sum of the allowable bearing forces for the individual fasteners in the connection.

**ASD**

\[ P \leq R_b \text{ must be satisfied.} \]

\[ R_b = \text{capacity of connection in bearing, kips} \]

\[ = N_{fe} F_{be} A_{brg} + N_{fo} F_{bo} A_{brg} \]

where:

\[ N_{fe} = \text{total number of fasteners connecting the individual member to the gusset plate along the edge} \]

\[ N_{fo} = \text{remaining fasteners connecting the individual member to the gusset plate} \]

\[ F_{be} \text{ or } F_{bo} = \text{allowable stress of the gusset plate material in bearing along the edge, } F_{be} \text{ or between the holes, } F_{bo}, \text{ psi} \]

where:

\[ F_{be} \text{ or } F_{bo} = 0.7 \frac{L_c F_u}{d} < 1.4 F_u \text{ MCEB Table 6.6.2.1-4 for Standard Holes} \]

\[ F_u = \text{specified minimum tensile strength of the connected material, ksi} \]

\[ d = \text{nominal diameter of the fastener, in.} \]

\[ L_c = L_e \text{ or } L_o = \text{clear distance between the holes, } L_o, \text{ or between the hole and the edge of the material, } L_e, \text{ in the direction of the applied bearing force, in. } \]

\( L_o \) is taken as 2d for edge holes and \( L_c \) is taken as follows for all other holes:
\[ L_{o} = \frac{L_{#}}{(N_{L#} - 1)} - \text{hole diameter} \]

\[ N_{L#} = \text{total number of fasteners along the length of member # (See Figure 1-4)} \]

\[ L_{#} = \text{distance between the first row of fasteners in member # and the last row of fasteners in member #, in.} \]

\[ A_{org} = \text{area of gusset plate in bearing for each hole, in}^2 \]

- **LFD**
  - Use the provisions of AASHTO Article 10.56.1.3.2.

\[ P \leq R_{b} \text{ must be satisfied.} \]

\[ R_{b} = \text{capacity of connection in bearing, kips} \]

\[ = \text{the result of } 0.9 \frac{L_{c} \cdot t}{F_{u}} \leq 1.8 t \cdot F_{u} \text{ multiplied by the respective number of fasteners, } N_{f} \]

\[ \text{AASHTO Eq. 10-166b} \]

\[ \text{(See } N_{f} \text{ description under ASD for more information)} \]

where:

\[ L_{c} = L_{e} \text{ or } L_{o} = \text{clear distance between the holes, } L_{o}, \text{ or between the hole and the edge of the material, } L_{e}, \text{ in the direction of the applied bearing force, in.} \]

\[ L_{e} \text{ is taken as } 2d \text{ for edge holes and } L_{o} \text{ is taken as follows for all other holes:} \]

\[ L_{o} = \frac{L_{#}}{(N_{L#} - 1)} - \text{hole diameter} \]

\[ t = \text{thickness of the connected material, in.} \]

\[ F_{u} = \text{specified minimum tensile strength of the connected material, ksi} \]

\[ d = \text{nominal diameter of the fastener, in.} \]

7 **Unsupported Edge in Compression Adequacy Check**

(Tab: Summary)

- **ASD and LFD**
  - Check whether \( b/t \) is less than or greater than \( 11,000 / \sqrt{F_{y}} \) for every free edge on the gusset plate.

If \( b/t > 11,000 / \sqrt{F_{y}} \), then the edge shall be stiffened. \( \text{Guide Spec. Art.1.11} \)

(Note: this check need not be performed for stiffened edges)

where:

\[ b = \text{maximum length of unstiffened edges, in.} \]

\[ t = \text{gusset plate thickness at edge, in.} \]
8 Reduction Ratios (optional)

- Assume an equal distribution of axial force to all fasteners in the chord.

  Ten.&Comp., Block Shear, and Connections Tab

  Chord Force Reduction Ratio = \( \frac{N_{T_c}}{N_T} \)

  \( N_{T_c} = \) total number of fasteners connecting the chord member to the gusset plate
  \[ = N_{lc} \times N_{tc} \text{ when not entered under optional input information} \]

  \( N_T = \) total number of fasteners in the connection
  \[ = N_{Tc} + N_{ts2} \times N_{ts2} + N_{ts3} \times N_{ts3} \]

  Shear@Sec.B-B, ...C-C Tab

  Chord Force Reduction Ratio = Bolt Reduction Ratio \times Section Reduction Ratio

  Bolt Reduction Ratio = \( 1 - \left( \frac{e_B \text{ or } c}{L_{cs} + 4 \text{ d}} \right) \)

  Section Reduction Ratio = \( \frac{A_e}{A_t} \)

  \( A_e = \) effective gusset plate gross section
  \[ = (8 \text{ t} + w_c + 4 \text{ d}) \text{ t} \]

  \( A_t = \) total gross section including all splice plates, outside, inside, top, and bottom, connecting chords to effective gusset plate
  \[ = 0.5 w_{s2} \times t_{s2} + 0.5 w_{s3} \times t_{s3} + w_c \times (t_{s4} + t_{s1}) + (8 \text{ t} + w_c + 4 \text{ d}) \text{ t} \]

  where:

  \( N_{ts2} = \) determine the number of fasteners in one-half of the top splice plate transversely across the chord member
  \[ = 0.5 \times w_{s2} / (3 \text{ d}) \]

  \( N_{ts2} = \) determine the number of fasteners in one-half of the top splice plate longitudinally along the chord member
  \[ = 0.5 \times t_{s2} / (3 \text{ d}) \]

  \( N_{ts3} = \) determine the number of fasteners in one-half of the bottom splice plate transversely across the chord member
  \[ = 0.5 \times w_{s2} / (3 \text{ d}) \]

  \( N_{ts3} = \) determine the number of fasteners in one-half of the bottom splice plate longitudinally along the chord member

  \( w_{s2} = \) top splice plate width (See Figure 6)

  \( t_{s2} = \) top splice plate length (See Figure 6)

  \( w_{s3} = \) bottom splice plate width (See Figure 6)

  \( t_{s3} = \) bottom splice plate length (See Figure 6)

  \( d = \) nominal diameter of the fastener, in.
\( N_{tc} = \) number of fasteners in a single row transversely across the chord

\( N_{LC} = \) number of fasteners in a single row longitudinally along the line of action of the chord

\( e_B \text{ or } C = \) distance from the line of action in the vertical member to the vertical cut at Section B-B or C-C, in.

\( w_c = \) distance between outermost rows of fasteners measured perpendicular to the line of action on chord, in.

\( L_c = \) distance between the first row of fasteners in chord and the last row of fasteners in chord, in.

\( t = \) gusset plate thickness at edge, in.

9 Operating Level Rating Equations

- ASD

\[
RF = \frac{\text{Operating Capacity} - \text{Dead Load}}{\text{Live Load} + \text{Impact}} \quad \text{MCEB Articles 6.5.1 and 6.5.2}
\]

- LFD

\[
RF = \frac{\text{Nominal Capacity} - 1.3 \text{Dead Load}}{1.3 \text{(Live Load + Impact)}} \quad \text{MCEB Articles 6.5.1 and 6.5.3}
\]